

Mathematics for Engineering

OCR Level 3 Certificate in Mathematics for Engineering **H860/02**

Paper 2

Mark Scheme for June 2010

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Question		Answer	Marks
1	(a)	Both tables show a final velocity of 15.59 m/s for the 2 m parachute. Since the object in the second table has travelled 200 m further than in the first table without an increase in velocity, it is reasonable to assume that 15.59 m/s is approximately the terminal velocity. The same argument cannot be applied to the 1 m and 1.5 m parachutes. However, the two final velocity values for the 1.5 m parachute are very close and it would be reasonable to conclude that the terminal velocity is very little more than 27.66 m/s. The terminal velocity for the 1 m parachute is inconclusive.	2 3 [5]
	(b)	k can be deduced directly from $\frac{d^2x}{dt^2} = g - kS \frac{dx}{dt}$ when $\frac{d^2x}{dt^2} = 0$ and $\frac{dx}{dt}$ is known. Using the fact that the 2 m parachute provides a terminal velocity of 15.59 m/s $g - k \times \pi \times 15.59 = 0$ $k = g/(\pi \times 15.59) = 9.8/(3.14159 \times 15.59) = 0.2$	2 2 [4]
Total			[9]

Question		Answer	Marks
2	(a)	(i) $\frac{d^2h}{dt^2} = -g$ (acceleration)	1
		(ii) $\frac{dh}{dt} = -gt + V_0$ where V_0 is the initial upward velocity	2
		(iii) $h = \frac{-gt^2}{2} + V_0t + A_0$ where A_0 is the initial height (= 0)	2 [5]
(b)	Maximum height is reached when $\frac{dh}{dt} = 0$ $-gt + V_0 = 0 \quad t = \frac{V_0}{g}$ Maximum height = 200 $\frac{-gt^2}{2} + V_0t = 200$ $\frac{-g}{2} \left(\frac{V_0}{g}\right)^2 + V_0 \left(\frac{V_0}{g}\right) = 200$ $-\frac{V_0^2}{2} + V_0^2 = 200g$ $V_0^2 = 400g$ $V_0 = \sqrt{400g} = 62.61 \text{ m s}^{-1}$	1 1 1 1 1 [5]	

		Alternative acceptable solution (final velocity) ² = (initial velocity) ² - 2gs $0 = u^2 - 2gs$ $u = \sqrt{2gs} = \sqrt{2 \times 9.8 \times 200} = 62.61 \text{ m s}^{-1}$	
			Total [10]

Question			Answer	Marks
3	(a)	(i)	$\frac{d^2x}{dt^2}$ is acceleration i.e. the rate of change of velocity v wrt t i.e. $\frac{d^2x}{dt^2} = \frac{dv}{dt}$ $\frac{dx}{dt} = v$ velocity Replacing in $\frac{d^2x}{dt^2} = g - kS \frac{dx}{dt}$ gives $\frac{dv}{dt} = g - kSv$	1 1 [2]
		(ii)	Using integrating factor method : $\frac{dv}{dt} + Pv = Q$ $P = kS$ $Q = g$ $IFv = \int Q \cdot IF dt$ $IF = e^{\int P dt} = e^{\int kS dt} = e^{kSt}$ $e^{kSt} v = \int g e^{kSt} dt$ $e^{kSt} v = \frac{g}{kS} e^{kSt} + A$ $v = \frac{g}{kS} + A e^{-kSt}$ when $t = 0, v = 0 \Rightarrow A = -\frac{g}{kS}$ $v = \frac{g}{kS} - \frac{g}{kS} e^{-kSt}$ $v = \frac{g}{kS} (1 - e^{-kSt})$ Alternative acceptable solution using separation of variables $\frac{dv}{dt} = g - ksv$ $\int \frac{dv}{g - ksv} = \int dt$ $-\frac{1}{ks} \ln(g - ksv) = t + C$	1 1 1 1 1 1 1 [6]

		$\ln(g - ksv) = -kst + B$ $g - ksv = Ae^{-kst}$ $ksv = g - Ae^{-kst}$ $v = \frac{g}{sk} - De^{-kst}$ <p>when $t = 0, v = 0 \Rightarrow D = \frac{g}{kS}$</p> $v = \frac{g}{kS}(1 - e^{-kSt})$	
	(b)	$v = \frac{g}{kS}(1 - e^{-kSt})$ $x = \int v dt$ $x = \int \frac{g}{kS}(1 - e^{-kSt}) dt$ $x = \frac{g}{kS} \left(t + \frac{e^{-kSt}}{kS} \right) + B$ <p>When $t = 0, x = 0$</p> $0 = \frac{g}{kS} \left(\frac{1}{kS} \right) + B \Rightarrow B = -\frac{g}{(kS)^2}$ $x = \frac{g}{kS} \left(t + \frac{e^{-kSt}}{kS} \right) - \frac{g}{(kS)^2}$ $x = \frac{g}{kS} \left(t + \frac{1}{kS}(e^{-kSt} - 1) \right)$	<p>1</p> <p>1</p> <p>1</p> <p>1 [4]</p>
Total			[12]

Question	Answer	Marks
4 (a)	<p>Using $v = \frac{g}{kS}(1 - e^{-kSt})$</p> <p>Terminal velocity reached as $t \rightarrow \infty$ i.e. as $e^{-kSt} \rightarrow 0$</p> $v = \frac{g}{kS}$ <p>Also allow use of $\frac{dv}{dt} = g - kSv$ from Q3a i.e. $0 = g - ksv$</p>	1 [1]
(b) (i)	$S = \frac{g}{5k} = \pi \left(\frac{d}{2} \right)^2$ $d = 2 \sqrt{\frac{g}{5k\pi}}$ $d = 2 \sqrt{\frac{9.8}{5 * 0.25\pi}} = 3.16 \text{ m}$	3 [3]

		<p>(ii) Half terminal velocity</p> $\frac{g}{Sk} (1 - e^{-kSt}) = 2.5$ $(1 - e^{-kSt}) = \frac{2.5Sk}{g}$ $e^{-kSt} = 1 - \frac{2.5kS}{g}$ $-kSt = \ln\left(1 - \frac{2.5kS}{g}\right)$ $t = \frac{-\ln\left(1 - \frac{2.5kS}{g}\right)}{kS}$ <p>When $k = 0.25$,</p> $S = \pi\left(\frac{d}{2}\right)^2 = 7.84$ $t = 0.353647 \text{ s}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>[5]</p>
			Total	[9]

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