

ADVANCED GCE
MATHEMATICS (MEI)
Differential Equations

4758/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Insert for Question 2 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Question 2.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 A particle is attached to a spring and suspended vertically from an oscillating platform. The vertical displacement, y , of the particle from a fixed point at time t is modelled by the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0.5 \sin t.$$

- (i) Find the general solution. [9]

Initially the displacement and velocity are both zero.

- (ii) Find the solution. [5]

- (iii) Describe the motion of the particle for large values of t . [2]

- (iv) Find approximate values of the velocity and displacement at $t = 20\pi$. [3]

The motion of the platform is stopped at $t = 20\pi$ and the differential equation modelling the subsequent motion of the particle is

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0.$$

- (v) Write down the general solution. Sketch the solution curve for $t > 20\pi$. [5]

2 **There is an insert for use with part (b)(i) of this question.**

- (a) The differential equation

$$\frac{dy}{dx} - y \tan x = \tan x$$

is to be solved for $|x| < \frac{1}{2}\pi$.

- (i) Find the general solution. [8]

- (ii) Find the equation of the solution curve that passes through the origin and sketch the curve. [4]

- (b) The differential equation

$$\frac{dy}{dx} - y^2 \tan x = \tan x$$

is to be solved approximately, first by using a tangent field and then by Euler's method.

- (i) On the insert is a tangent field for the differential equation. Sketch the solution curves through the origin and through $(0, 1)$. [4]

Euler's method is now used, starting at $x = 0, y = 1$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$.

- (ii) Carry out two steps with a step length of 0.1 to verify that the algorithm gives $x = 0.2$, $y \approx 1.0201$. [5]

- (iii) Explain why it would be inappropriate to extend this numerical solution as far as $x = 1.6$. [2]

- (iv) How could the accuracy of the estimate found in part (b)(ii) be improved? [1]

- 3 Fig. 3 shows a small ball projected from a point O over horizontal ground. The forces acting on the ball are its weight and air resistance. Its initial horizontal component of velocity is v_1 and its subsequent horizontal velocity \dot{x} is modelled by the differential equation

$$\frac{d\dot{x}}{dt} = -k\dot{x},$$

where k is a positive constant.

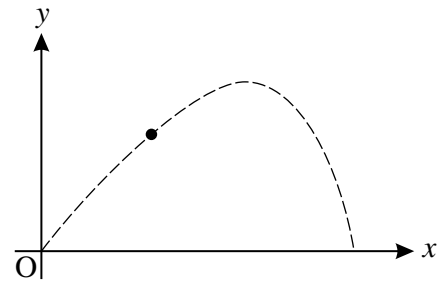


Fig. 3

The units of displacement are metres and the units of time are seconds.

- (i) Solve this differential equation to find \dot{x} in terms of t and hence show that the horizontal displacement from O is given by $x = \frac{v_1}{k}(1 - e^{-kt})$. [8]

The ball's initial vertical component of velocity is v_2 and its subsequent vertical velocity \dot{y} is modelled by the differential equation

$$\frac{d\dot{y}}{dt} = -k\dot{y} - g.$$

- (ii) Solve this differential equation to find \dot{y} in terms of t and hence show that the vertical displacement from O is given by $y = \frac{kv_2 + g}{k^2}(1 - e^{-kt}) - \frac{g}{k}t$. [10]

- (iii) Eliminate t between the expressions for x and y to show that $y = \left(\frac{kv_2 + g}{kv_1}\right)x + \frac{g}{k^2} \ln\left(1 - \frac{kx}{v_1}\right)$. [4]

- (iv) In the case $v_1 = v_2 = 10$, $k = 0.1$, determine whether the ball will pass over a 5 m high wall at a horizontal distance 8 m from O. [2]

- 4 The simultaneous differential equations

$$\frac{dx}{dt} = -3x - 4y + 23,$$

$$\frac{dy}{dt} = 2x + y - 7$$

are to be solved.

- (i) Show that $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 5$. [5]

- (ii) Find the general solution for x . [7]

- (iii) Find the corresponding general solution for y . [4]

When $t = 0$, $x = 8$ and $y = 0$.

- (iv) Find the particular solutions for x and y . [4]

- (v) Show that for sufficiently large t , y is always greater than x . [4]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



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**ADVANCED GCE
MATHEMATICS (MEI)**

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INSERT for Question 2

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**Wednesday 27 January 2010
Afternoon**

Duration: 1 hour 30 minutes



Candidate Forename		Candidate Surname	
Centre Number		Candidate Number	

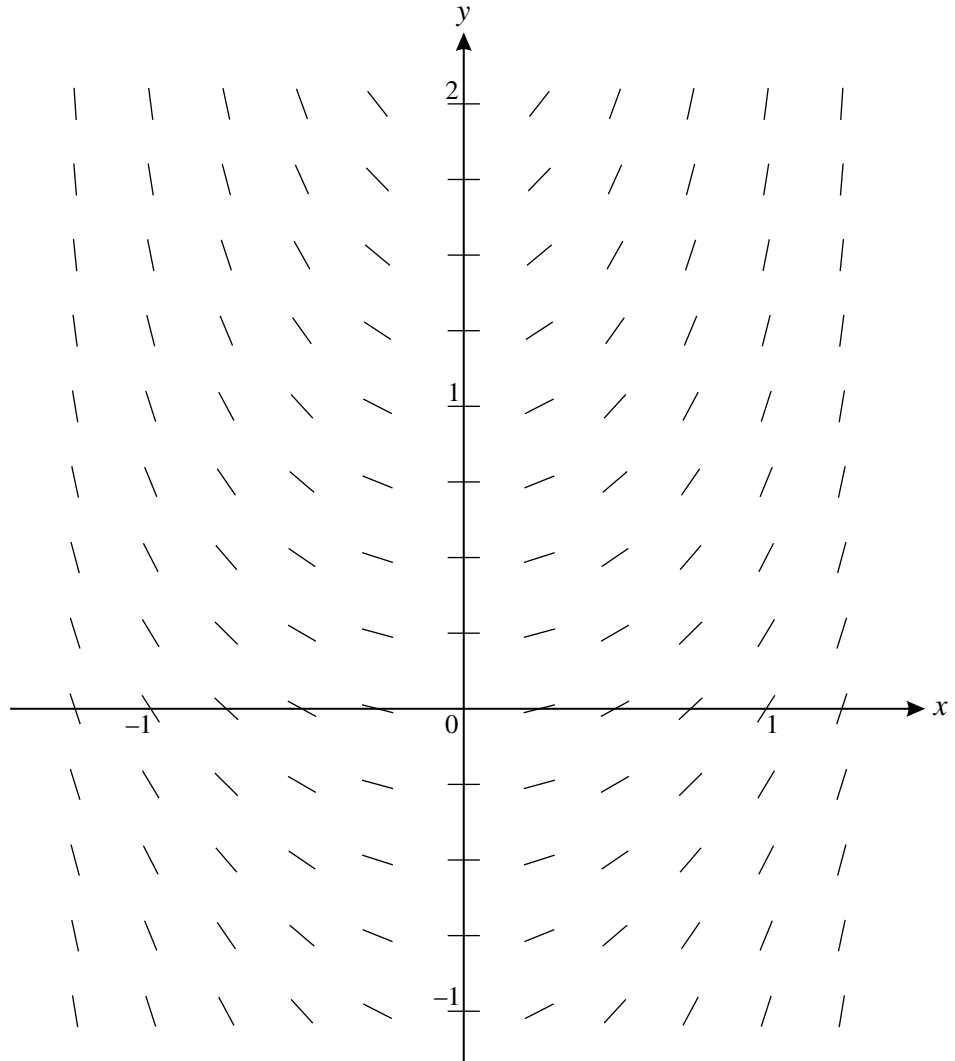
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Question 2 part **(b)(i)**.
- Write your answers to Question 2 part **(b)(i)** in the spaces provided in this insert, and **attach it to your Answer Booklet**.

INFORMATION FOR CANDIDATES

- This document consists of **2** pages. Any blank pages are indicated.

2 (b) (i)

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