RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4726/01

MATHEMATICS

Further Pure Mathematics 2

THURSDAY 7 JUNE 2007

Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The equation of a curve, in polar coordinates, is

$$r = 2\sin 3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \tfrac{1}{3}\pi.$$

Find the exact area of the region enclosed by the curve between $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [4]

2 **(i)** Given that $f(x) = \sin(2x + \frac{1}{4}\pi)$, show that $f(x) = \frac{1}{2}\sqrt{2}(\sin 2x + \cos 2x)$. [2]

(ii) Hence find the first four terms of the Maclaurin series for $f(x)$. [You may use appropriate results given in the List of Formulae.] [3]

3 It is given that $f(x) = \dfrac{x^2 + 9x}{(x-1)(x^2+9)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Hence find $\displaystyle\int f(x)\,dx$. [2]

4 **(i)** Given that

$$y = x\sqrt{1 - x^2} - \cos^{-1} x,$$

find $\dfrac{dy}{dx}$ in a simplified form. [4]

(ii) Hence, or otherwise, find the exact value of $\displaystyle\int_0^1 2\sqrt{1-x^2}\,dx$. [3]

5 It is given that, for non-negative integers n,

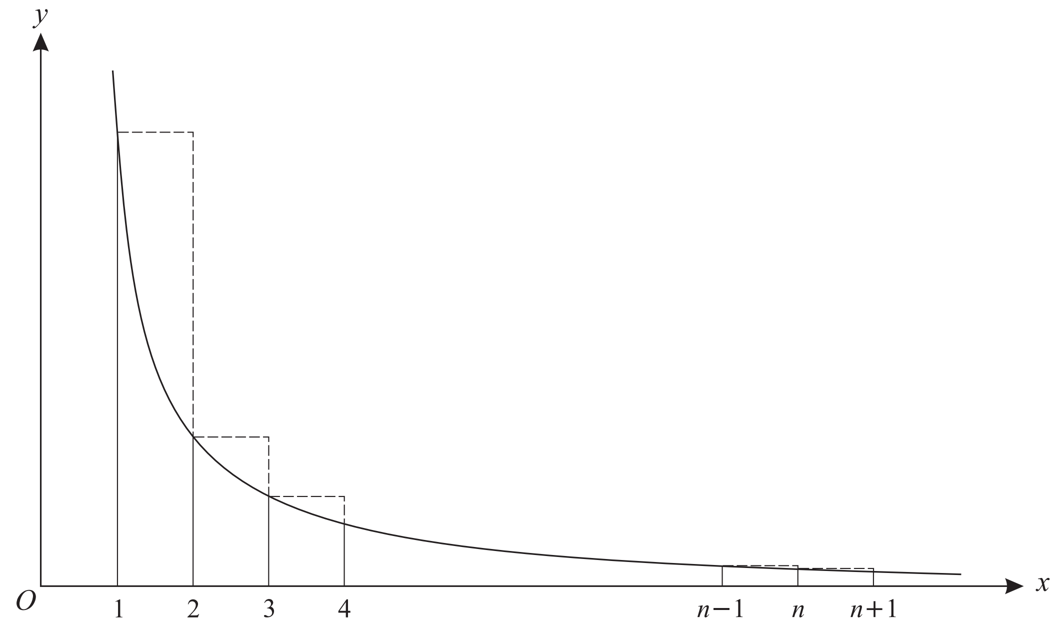
$$I_n = \int_1^{e} (\ln x)^n \, dx.$$

(i) Show that, for $n \geqslant 1$,

$$I_n = e - nI_{n-1}.$$ [4]

(ii) Find I_3 in terms of e. [4]

6



The diagram shows the curve with equation $y = \frac{1}{x^2}$ for $x > 0$, together with a set of n rectangles of unit width, starting at $x = 1$.

(i) By considering the areas of these rectangles, explain why

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} > \int_1^{n+1} \frac{1}{x^2}\,\mathrm{d}x. \qquad [2]$$

(ii) By considering the areas of another set of rectangles, explain why

$$\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots + \frac{1}{n^2} < \int_1^{n} \frac{1}{x^2}\,\mathrm{d}x. \qquad [3]$$

(iii) Hence show that

$$1 - \frac{1}{n+1} < \sum_{r=1}^{n} \frac{1}{r^2} < 2 - \frac{1}{n}. \qquad [4]$$

(iv) Hence give bounds between which $\sum_{r=1}^{\infty} \frac{1}{r^2}$ lies. [2]

7 **(i)** Using the definitions of hyperbolic functions in terms of exponentials, prove that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(x - y). \qquad [4]$$

(ii) Given that $\cosh x \cosh y = 9$ and $\sinh x \sinh y = 8$, show that $x = y$. [2]

(iii) Hence find the values of x and y which satisfy the equations given in part **(ii)**, giving the answers in logarithmic form. [4]

[Turn over

8 The iteration $x_{n+1} = \frac{1}{(x_n + 2)^2}$, with $x_1 = 0.3$, is to be used to find the real root, α, of the equation $x(x+2)^2 = 1$.

(i) Find the value of α, correct to 4 decimal places. You should show the result of each step of the iteration process. [4]

(ii) Given that $f(x) = \frac{1}{(x+2)^2}$, show that $f'(\alpha) \neq 0$. [2]

(iii) The difference, δ_r, between successive approximations is given by $\delta_r = x_{r+1} - x_r$. Find δ_3. [1]

(iv) Given that $\delta_{r+1} \approx f'(\alpha)\delta_r$, find an estimate for δ_{10}. [3]

9 It is given that the equation of a curve is

$$y = \frac{x^2 - 2ax}{x - a},$$

where a is a positive constant.

(i) Find the equations of the asymptotes of the curve. [4]

(ii) Show that y takes all real values. [4]

(iii) Sketch the curve $y = \frac{x^2 - 2ax}{x - a}$. [3]