OCR
RECOGNISING ACHIEVEMENT

LEVEL 3 CERTIFICATE

MATHEMATICS FOR ENGINEERING

H860/01

Paper 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 2 hours

H86001

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

1 Consider the framed structure shown in Fig. 1 which consists of eleven light, rigid structural members. The structure consists of five equilateral triangular sections supported at points A and B. The whole structure is arranged in the vertical plane and each joint is freely pin-jointed. Loads of 4 kN, 3 kN, 2 kN and 1.5 kN are applied at points W, X, Y and Z respectively.

(a) Determine the reaction forces of the supports on the framework at points A and B. **[3]**

(b) Determine the forces in the member M_1 and the member M_2. **[3]**

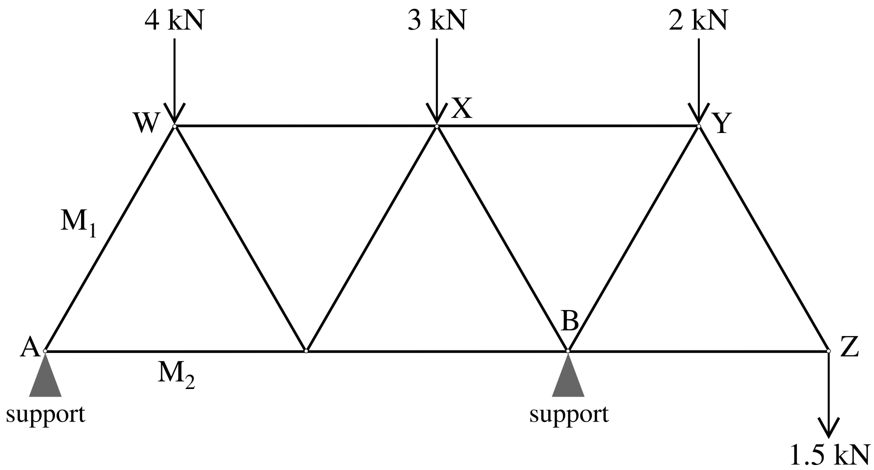

Fig. 1

2 In a certain manufacturing process the times, in minutes, for Operation A to be completed by a machine operator during a morning shift period were observed and recorded. The results are summarised in Table 2.

Operation time (minutes)	Number of observations
> 1 and $\leqslant 2$	26
> 2 and $\leqslant 3$	22
> 3 and $\leqslant 4$	18
> 4 and $\leqslant 5$	14
> 5 and $\leqslant 6$	10
> 6 and $\leqslant 7$	7
> 7 and $\leqslant 8$	3

Table 2

(a) Draw a histogram of the observations summarised in the table. **[2]**

It has been suggested that the distribution of the actual times taken for Operation A over a long period can be approximated by the probability density function

$$\mathrm{f}(t) = \tfrac{1}{161}(50 - 6t) \qquad \text{for } 1 < t \leqslant 8,$$

where t is the time in minutes.

It is assumed that Operation A will take more than 1 minute and will take no more than 8 minutes.

(b) **(i)** Use this probability density function to calculate the proportion of the times taken for Operation A that are predicted to be greater than 4 minutes. **[4]**

(ii) Calculate the median of this probability density function. **[5]**

(c) Based on your answers to part **(b)**, state, with reasons, whether the suggested probability density function provides a good approximation to the data given in Table 2. **[2]**

3 For this question you may assume the following laws regarding the total resistance, R, for an electrical circuit.

For resistors connected in series, $R = R_1 + R_2 + \ldots + R_n$.

For resistors connected in parallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$.

An electrical cable of length 20 km contains two copper wires each with a resistance of $10\,\Omega\,\text{km}^{-1}$. The cable is known to have slight damage at a position D km from one end which is causing a small current leakage. At the point of damage the two copper wires are effectively joined with a resistance of $R\,\Omega$, as shown in Fig. 3.

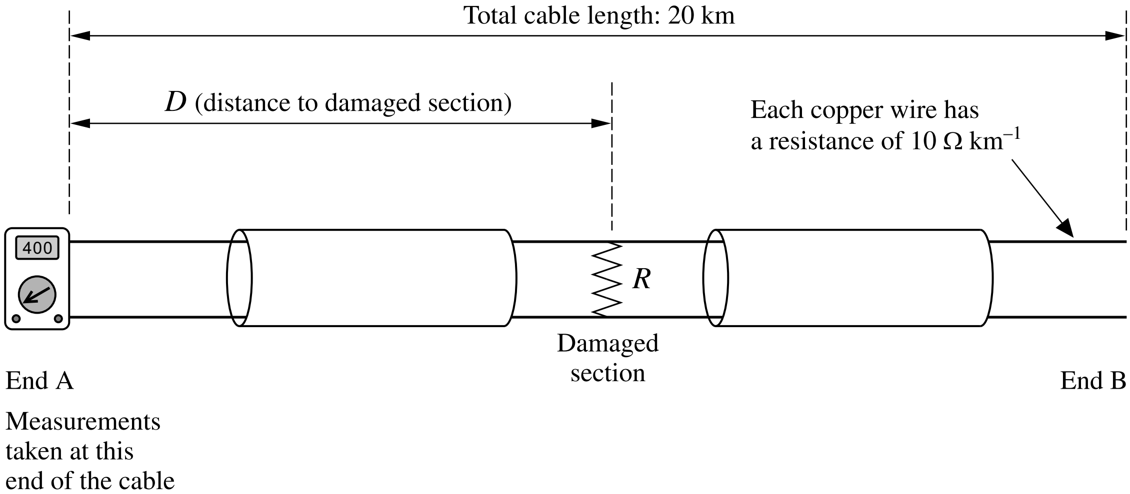

Fig. 3

In order to determine the position of the damaged section, engineers have observed that the DC resistance across the wires at end A is $400\,\Omega$ when the wires at end B are open-circuited. When the wires at end B are short-circuited, the DC resistance across the wires at end A is $250\,\Omega$.

(a) When the two wires at end B of the cable are open-circuited, the resistance measured at end A corresponds to three resistors in series consisting of the two sections of cable up to the point of damage and the resistance $R\,\Omega$.

Write down an equation which relates the measured resistance, $400\,\Omega$, to resistance $R\,\Omega$ and distance D km. **[1]**

(b) When the two wires at end B of the cable are short-circuited, the total resistance measured at end A involves the resistance of the whole length of the cable and the resistance $R\,\Omega$ in a combined parallel and series arrangement. In this case show that

$$250 = \frac{R(400 - 20D)}{400 - 20D + R} + 20D.$$ **[4]**

(c) Determine the values of R and D. **[4]**

4 In this question Ox, Oy and Oz are three mutually perpendicular axes where O is the centre of the Earth, as shown in Fig. 4a. The units of these axes are km, and the Earth is assumed to be a perfect sphere of radius 6360 km.

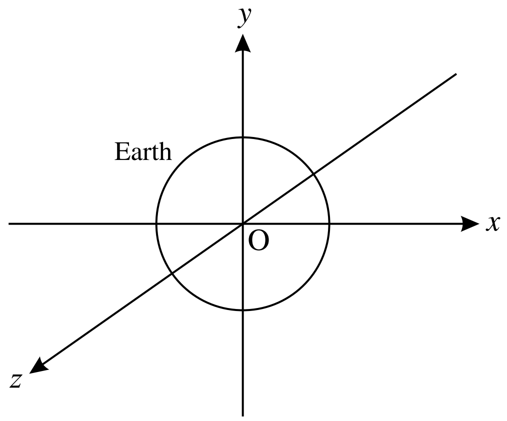

Fig. 4a

Global positioning satellites orbit the Earth at a constant altitude of 20 200 km from the Earth's surface. Fig. 4b shows a satellite at the highest position within its orbital plane, which is inclined at an angle of 55° to the plane of the equator.

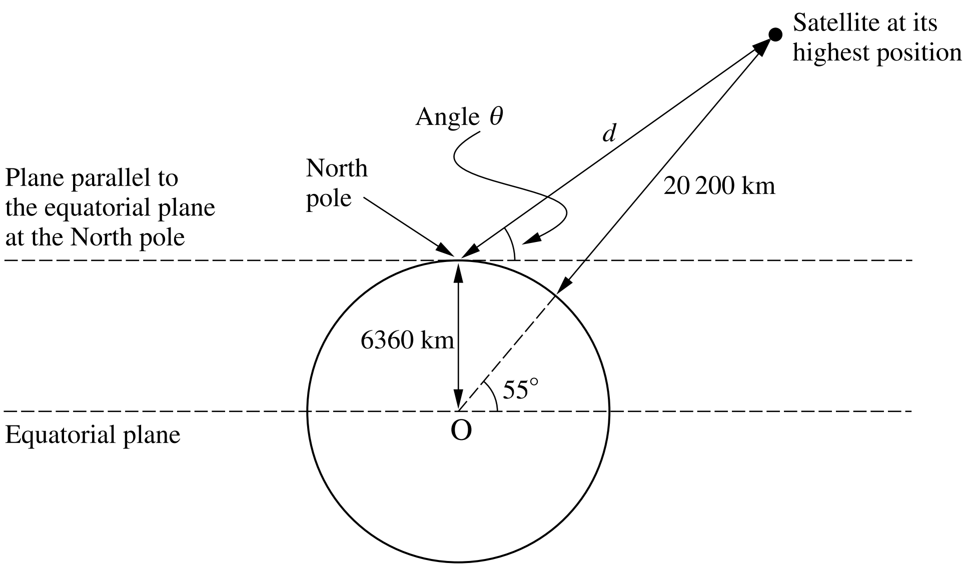

Fig. 4b

(a) Calculate the distance d and the angle θ indicated in Fig. 4b. **[4]**

(b) A satellite at (10 700, 15 350, 1870) transmits a signal. Calculate the time taken by the signal to reach a receiver at (6050, −1100, 1620), assuming that signal propagation speed is 3×10^5 km s^{-1}. **[2]**

(c) Three satellites have positions

(7600, 21 700, 13 200), (−23 000, 0, 13 300), (−7600, −21 700, −13 200).

Determine the equation of the plane in which these satellites lie, given that this plane passes through the origin. Express your answer in the form $Ax + By + Cz = 0$, where A, B, and C are constants. **[6]**

5 A continuous function $f(t)$ is defined on a time interval $a \leqslant t \leqslant b$. In this question you may assume that

the mean value of $f(t)$ is $\dfrac{1}{b-a}\displaystyle\int_a^b f(t)\,dt$,

the root mean square value (r.m.s.) of $f(t)$ is $\sqrt{\dfrac{1}{b-a}\displaystyle\int_a^b \left(f(t)\right)^2 dt}$.

The output of a particular electrical device is an alternating current which may be represented by the function $f(t) = \sin(\omega t)$, where ω is the fundamental frequency in rad s^{-1}.

(a) Determine the mean value of $f(t)$ over the time interval $0 \leqslant t \leqslant \dfrac{\pi}{\omega}$. **[3]**

(b) Determine the root mean square value of $f(t)$ over the time interval $0 \leqslant t \leqslant \dfrac{\pi}{\omega}$. **[4]**

6 **(a)** Starting with the definition

$$y = a^x \Leftrightarrow \log_a y = x, \text{ where } a > 1,$$

prove that

(i) $\ln a - \ln b = \ln\left(\dfrac{a}{b}\right)$, **[2]**

(ii) $\log_{10} a = \dfrac{\ln a}{\ln 10}$. **[3]**

(b) The power of a digitally transmitted signal, P_x, is given by

$$P_x = \sigma_x^2 + \mu_x^2$$

where μ_x is the mean value of the signal,
σ_x^2 is the variance of the signal.

The signal is contaminated by noise. The power of the noise is similarly given by $P_v = \sigma_v^2 + \mu_v^2$.

The signal-to-noise ratio (*SNR*), in decibels, of the transmitted signal is defined as

$$SNR = 10\log_{10}\left(\frac{P_x}{P_v}\right).$$

Prove that, if the mean values of both the noise and the signal are zero, then

$$SNR = \frac{20}{\ln 10}\left(\ln \sigma_x - \ln \sigma_v\right).$$ **[2]**

7 The behaviour of a simple car suspension unit can be modelled by the differential equation

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + c\frac{\mathrm{d}x}{\mathrm{d}t} + kx = 0,$$

where t represents time,
x represents a displacement from the equilibrium position,
m is the mass supported by the suspension unit,
c is the damping coefficient,
k is the spring stiffness.

Given that $m = 4$, $c = 4$ and $k = 1$, verify that

$$x = \mathrm{e}^{-\frac{1}{2}t}(A + Bt)$$

satisfies the differential equation, where A and B are constants. **[6]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.