

# Mathematical Techniques and Applications for Engineers

OCR Level 3 Certificate **H865**

## Mark Schemes for the Components

**January 2010**

**H865/MS/R/10J**

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## OCR Level 3 Certificate

### Mathematical Techniques and Applications for Engineers H865

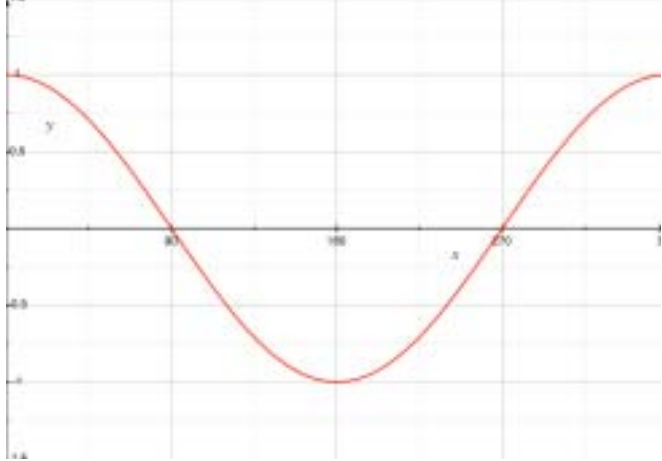
#### REPORTS ON THE COMPONENTS

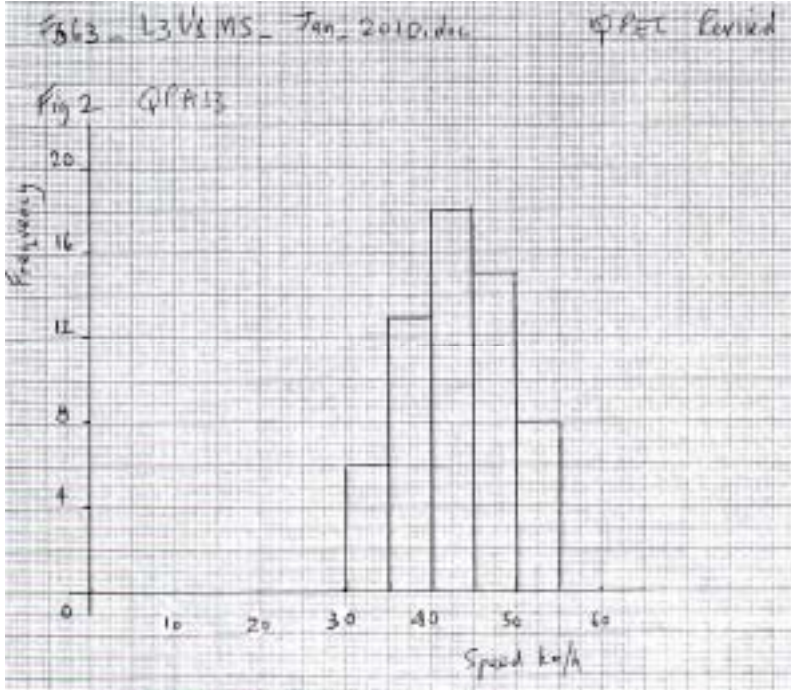
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# 01 Component 1

## Section A

Question	Expected Answers	Marks	Rationale
1	$(x - 5)(x - 2) = x^2 - 2x - 5x + 10$ $= x^2 - 7x + 10$	[1] [1]	[2]
2	$x^2 + 3x - 40 = (x + 8)(x - 5)$ $(x + 8)$ $(x - 5)$	[1] [1]	[2]
3	$(x + 3)/5 - (x + 6)/2 = (2x + 6 - 5x - 30)/10$ $= (-3x - 24)/10$	[1] [1]	[2]
4	$3(x - 2) = 4x + 5,$ $3x - 6 = 4x + 5$ $4x - 3x = -6 - 5$ $x = -11$	[1] [1]	[2]
5	Length of arc (s) = $(\pi x^{\circ} r)/180$ therefore $x^{\circ} = 180 s/\pi r$ $= (180 \times 480)/(\pi \times 100)$ $= 275$	[1] [1]	[2]
6	$\sin 36^{\circ} = 75/b$ therefore $b = 75/\sin 36^{\circ}$ $= 75/0.5878$ $= 127.6$ mm correct to 1 dp	[1] [1]	[2]

Question	Expected Answers	Marks	Rationale
7	Fig.1 Graph of $y = \cos \theta$ Marks: x axis +1 and -1 points [1] y axis $90^\circ$ and $270^\circ$ points [1] 	[2]	
8	$\sin^2 x + \cos^2 x = \sin^2 150 + \cos^2 150$ [1] $= 0.25 + 0.75$ [1] $= 1$ QED	[2]	
9	$y = 5x^4 - 4x^3$ $dy/dx = 20x^3 - 12x^2$ $20x^3$ [1] $- 12x^2$ [1]	[2]	
10	$y = 4e^x + \ln(2x)$ $dy/dx = 4e^x + 1/x$ $4e^x$ [1] $+ 1/x$ [1]	[2]	
11	$\int (5x^4 + 8x^3 + 3x^2) dx = x^5 + 2x^4 + x^3 + C$ $x^5 + 2x^4$ [1] $+ x^3 + C$ [1]	[2]	

Question	Expected Answers	Marks	Rationale
12	$\int 8 \sin 4x \cdot dx = (-8 \cos 4x)/4 + C$ $= -2 \cos 4x + C$	[1] [1]	[2]
13	<p data-bbox="349 379 515 411">Histogram -</p>  <p data-bbox="349 1171 707 1270">           Marks:            Frequency and speed axes [1]            Five correct positions [1]         </p>	[2]	

Question	Expected Answers	Marks	Rationale
14	Ungrouped data given is 7, 11, 9, 14, 10, 12 and 8 Rearranged becomes 7, 8, 9, [10], 11, 12, 14 showing median as 10 [1] Mean = $(7 + 11 + 9 + 14 + 10 + 12 + 8)/7 = 71/7 = 10.143$ correct to 3dp [1]	[2]	
15	Probability that the lathe is being used = $7/8$ [1] = 87.5% [1]	[2]	

## Section B

Question	Expected Answers	Marks	Rationale
1	(a) Given $T = 2\pi \sqrt{L/g}$ When $L = 5 \text{ m}$ and $g = 9.81 \text{ ms}^{-2}$ Then Given $T = 2\pi \sqrt{5/9.81}$ [1] $= 4.49 \text{ s}$ correct to 2 dp [1]	[2]	
	(b) Given $T = 2\pi \sqrt{L/g}$ . Divide both sides by $2\pi$ then $T/2\pi = \sqrt{L/g}$  Take the square of both sides then $(T/2\pi)^2 = L/g$ [1]  Open the bracket then $T^2/4\pi^2 = L/g$  Multiply both sides $g$ then $L = gT^2/4\pi^2$ [1]  When $T = 4 \text{ s}$ and $g = 9.81 \text{ ms}^{-2}$ then $L = (9.81 \times 4^2)/4\pi^2 = 3.98 \text{ m}$ correct to 2 dp [1]	[3]	
	(c) Given $R_1/R_2 = (1 + \alpha t_1)/(1 + \alpha t_2)$ Cross multiply then $R_1(1 + \alpha t_2) = R_2(1 + \alpha t_1)$ [1] Open the brackets then $R_1 + R_1 \alpha t_2 = R_2 + R_2 \alpha t_1$ [1] Reorganise equation then $R_1 \alpha t_2 - R_2 \alpha t_1 = R_2 - R_1$ [1] Take $\alpha$ and make a bracket then $\alpha (R_1 t_2 - R_2 t_1) = R_2 - R_1$  Divide both sides by $(R_1 t_2 - R_2 t_1)$ then $\alpha = (R_2 - R_1)/(R_1 t_2 - R_2 t_1)$ [1] So $\alpha = (98 - 100)/[(100 \times 70) - (98 \times 30)] = -4.926 \times 10^{-4}$ [1]	[5]	
			[Total:10]



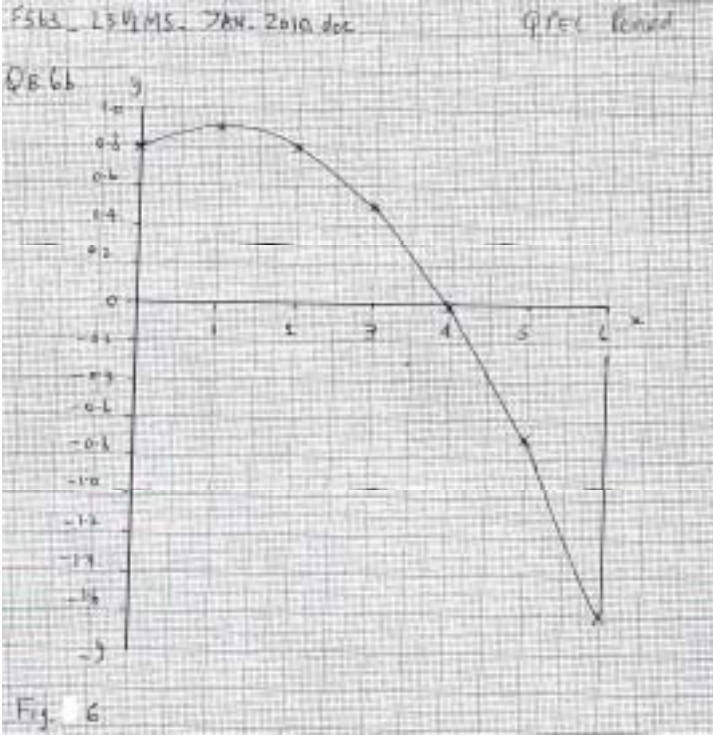
Question	Expected Answers	Marks	Rationale
2	(a) Given equation is $E = aW + b$ When $E = 16 \text{ N}$ , $W = 120 \text{ N}$ and when $E = 34 \text{ N}$ , $W = 480 \text{ N}$  So $16 = 120a + b$ Eq. (1) [1] $34 = 480a + b$ Eq. (2) [1]	[2]	
	(b) Multiply Eq. (1) x 4 $64 = 480a + 4b$ Eq. (3) [1] Subtract Eq.(2) from Eq.(3) So $30 = 3b$ [1] Then $b = 30/3 = 10$ [1] To find 'a' substitute $b = 10$ into Eq. (1) So $16 = 120a + b$ Then $16 = 120a + 10$ Then $120a = 16 - 10$ $120 a = 6$ So $a = 6/120 = 0.05$ [1]	[4]	
	(c) Use Eq. (1) to find load when the effort is 42 N So $E = 0.05W + 10$ $42 = 0.05W + 10$ [1] Then $0.05W = 42 - 10 = 32$ So $W = 32/0.05 = 640 \text{ N}$ [1]	[2]	
	(d) Use Eq. (1) to find the effort when the load is 300 N So $E = 0.05W + 10$ [1] $E = (0.05 \times 300) + 10 = 25 \text{ N}$ [1] Accept any alternative method of solution	[2]	
			[Total:10]

Question	Expected Answers	Marks	Rationale
3	(a) Apply the sine rule to find angle ACB $b/\sin B = c/\sin C$	[1]	[1]
	(b) so $\sin C = (c \sin B)/b$ $\sin C = (15 \sin 52^\circ)/18 = 0.6567$ $C = \sin^{-1} 0.6567 = 41.05^\circ = 41^\circ$ to the nearest degree	[1] [1]	[2]
	(c) (i) Apply the cosine rule to find angle CAB $\cos A = (b^2 + c^2 - a^2)/2bc$ $= (100^2 + 210^2 - 170^2)/(2 \times 100 \times 210)$ $= 0.6$ $A = \cos^{-1} 0.6 = 53.13^\circ = 53^\circ$ to the nearest degree	[1] [1] [1]	[3]
	(ii) To calculate x use the right angle triangle in Fig.3  Angle $x = 90 - 53.13 = 36.87^\circ$ $\sin 36.87 = x/100$ so $x = 100 \sin 36.87 = 60$ mm to 2 sf Accept any alternative method of solution	[1] [1] [1]	[3]
	(iii) To calculate y use the right angle triangle in Fig. 3  $\cos 36.87 = y/100$ $y = 100 \cos 36.87 = 80$ mm to 2sf Accept any alternative methods of solution	[1] [1]	[2]
			[Total:10]

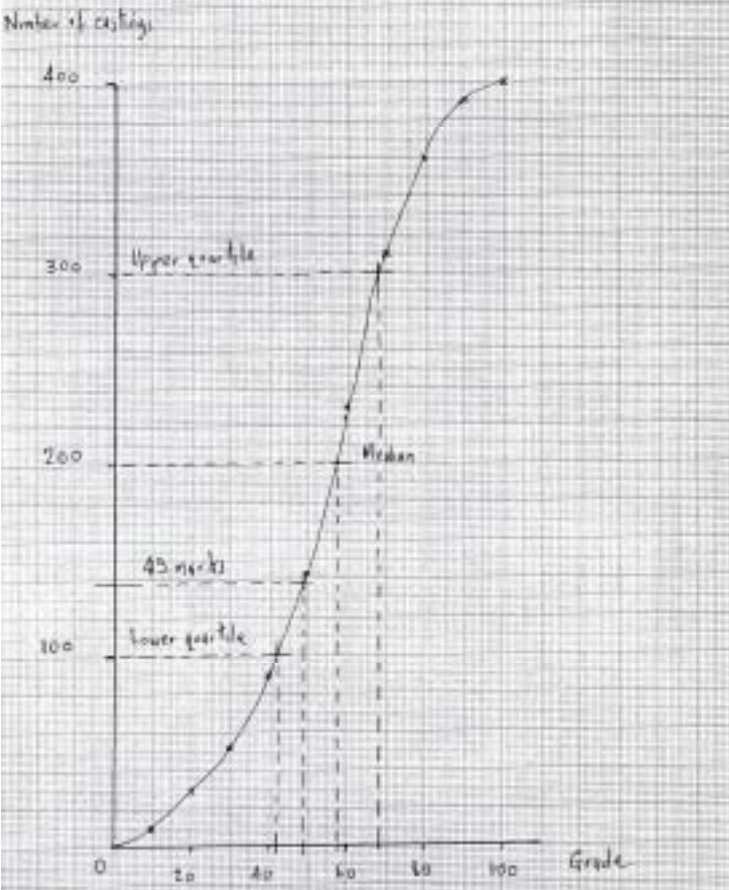
Question	Expected Answers	Marks	Rationale
4 (a) (i)	Use triangle CAD in Fig. 4 Length CD = $410 - 50 = 360$ m Length AD = $375 + 625 = 1000$ m Angle A = $\tan^{-1}(360/1000) = 19.8^\circ$	[1] [1]	[2]
(ii)	Use triangle BAE in Fig. 4 $\tan 19.8 = EB/375$ $EB = 375 \tan 19.8 = 135$ m Height of pylon B = $135 + 50 = 185$ m Accept any alternative method of solution	[1] [1]	[2]
(iii)	Use triangle CAD in Fig. 4 $\cos 19.8 = 1000/AC$ $AC = 1000 / \cos 19.8 = 1062.8$ m Accept any alternative method of solution	[1] [1]	[2]
(iv)	actual length of the cable if 8% of the straight length is added for sagging $= 1062.8 + 8\% \text{ of } 1062.8$ $= 1062.8 + 85.02$ $= 1147.82$ m	[1]	[1]

Question	Expected Answers	Marks	Rationale
(b)	Inspect the triangle in Fig. 5 then: $\sin A = a/c$ , $\cos A = b/c$ and $\tan A = a/b$ Given item: $\sin A \cos A (1 + \tan^2 A)$ Becomes $(a/c) (b/c) [(1 + a^2/b^2)]$ [1] Becomes $(a/c) (b/c) [(a^2 + b^2)/a^2]$ Pythagorus $a^2 + b^2 = c^2$ Then $(a/c) (b/c) [(a^2 + b^2)/a^2] = (a/c) (b/c)[(c^2/a^2)]$ [1] $= b/a = \tan A$ [1]	[3]	
			[Total:10]
(5)	The angle $\theta$ radians turned through by a flywheel in $t$ seconds is give by: $\theta = t^3 - 8t^2 + 16t + 20$ Angular velocity ( $\omega$ ) = $d\theta/dt = 3t^2 - 16t + 16$ [1]	[1]	
(a)	When $t = 4s$ then $d\theta/dt = 3(4)^2 - 16(4) + 16$ [1] $= 0$ i.e Stationary [1]	[2]	
(b)	Angular velocity ( $\omega$ ) = $d\theta/dt = 3t^2 - 16t + 16$ Angular acceleration ( $\alpha$ ) is $d\omega/dt = 6t - 16$ [1] When $t = 6s$ then angular acceleration $d\omega/dt = 6(6) - 16 = 20 \text{ rad s}^{-2}$ [1]	[2]	
(c)	Ues $d\omega/dt = 6t - 16$ to find the time when the angular acceleration is zero So $0 = 6t - 16$ $6t = 16$ [1] $t = 16/6 = 2.67 \text{ s}$ correct to 2 dp [1]	[2]	

Question	Expected Answers	Marks	Rationale
	<p><b>(d)</b> Use <math>d\theta/dt = 3t^2 - 16t + 16</math> to find the time when the angular velocity is zero            So <math>3t^2 - 16t + 16 = 0</math>            Solution of quadratic equation by factorisation</p> <p>So <math>3t^2 - 16t + 16 = 0</math></p> <p><math>(3t - 4)(t - 4) = 0</math> [1]            Then <math>(3t - 4) = 0</math> or <math>(t - 4) = 0</math> [1]            So <math>3t = 4</math> then <math>t = 4/3</math> s            Or <math>t = 4</math> s [1]</p>	<b>[3]</b>	
			<b>[Total:10]</b>
<b>6 (a) (i)</b>	Integral = Area under the curve $= \int_2^4 6x^2 dx$ [1]	<b>[1]</b>	
	<p><b>(ii)</b> Area under the curve <math>= \int_2^4 6x^2 dx = [2x^3]_2^4</math> [1]</p> <p><math>= 2(4^3) - 2(2^3)</math></p> <p><math>= 128 - 16</math></p> <p><math>= 112</math> square units [1]</p>	<b>[2]</b>	

Question	Expected Answers	Marks	Rationale																
<b>(b)</b>	From $y = 0.1(4 - x)(2 + x)$ a table can be constructed																		
	<table border="1" style="margin-left: 20px;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Y</td> <td>0.8</td> <td>0.9</td> <td>0.8</td> <td>0.5</td> <td>0</td> <td>-0.7</td> <td>-1.6</td> </tr> </table>	X	0	1	2	3	4	5	6	Y	0.8	0.9	0.8	0.5	0	-0.7	-1.6		
X	0	1	2	3	4	5	6												
Y	0.8	0.9	0.8	0.5	0	-0.7	-1.6												
																			
	<p>Correct x axis and correct y axis [1]            Correct shape above the x axis [1]            Correct shape below the x axis [1]</p>	<b>[3]</b>																	

Question	Expected Answers	Marks	Rationale																																																							
(c)	$y = 0.1(4 - x)(2 + x) = 0.1(8 + 2x - x^2)$ <p>By inspection of the table and the graph we find that</p> <p>Area enclosed above the x axis = <math>0.1 \int_0^4 (8 + 2x - x^2) dx</math></p> $= 0.1[(8x + x^2 - x^3/3)]_0^4$ $= 0.1(32 + 16 - 21\frac{1}{3}) - 0$ $= 2.66667$ <p>Area enclosed below the x axis = <math>0.1 \int_4^6 (8 + 2x - x^2) dx</math></p> $= 0.1[(8x + x^2 - x^3/3)]_4^6$ $= 0.1(48 + 36 - 72) - 0.1(32 + 16 - 21\frac{1}{3})$ $= 1.2 - 2.66667$ $= -1.46667 \text{ square units}$ <p>(Negative sign indicates under the x axis)</p> <p>Total area = <math>2.66667 + 1.46667 = 4.13334</math> square units</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[4]</p>																																																								
			<b>[Total:10]</b>																																																							
7	<p>(a) Cumulative frequency table:</p> <table border="0" style="width: 100%;"> <tr> <td>Grades not more than</td> <td>10</td> <td>20</td> <td>30</td> <td>40</td> <td>50</td> <td>60</td> <td>70</td> <td>80</td> <td>90</td> <td>100</td> </tr> <tr> <td>Frequency</td> <td>11</td> <td>19</td> <td>22</td> <td>38</td> <td>52</td> <td>88</td> <td>80</td> <td>50</td> <td>30</td> <td>10</td> </tr> <tr> <td>Cumulative frequency</td> <td>11</td> <td>30</td> <td>52</td> <td>90</td> <td>142</td> <td>230</td> <td>310</td> <td>360</td> <td>390</td> <td>400</td> </tr> <tr> <td>Grades not more than</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>Cumulative frequency</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	Grades not more than	10	20	30	40	50	60	70	80	90	100	Frequency	11	19	22	38	52	88	80	50	30	10	Cumulative frequency	11	30	52	90	142	230	310	360	390	400	Grades not more than											Cumulative frequency											<p>[1]</p> <p>[1]</p> <p>[2]</p>	
Grades not more than	10	20	30	40	50	60	70	80	90	100																																																
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Question	Expected Answers	Marks	Rationale
(b)	<p>F 563, 2303 MS, JAN. 2010, Inc Q/FEC Revised</p> <p>Q67</p> <p>Number of castings</p>  <p>Fig. 7</p>	<p>[1] [1]</p>	<p>[2]</p>
	<p>Correct axes Correct plot</p>		



Question	Expected Answers	Marks	Rationale
<b>(c)</b>	From the graph: (i) median mark = $57 \pm 2$ [1] (ii) Lower quartile = $42 \pm 2$ [1] Upper quartile = $68 \pm 2$ [1]  (i) The number of castings with not more than 49 marks is $140 \pm 2$ . [1] Therefore $140 \pm 2$ out of 400 do not make the grade [1] but $260 \pm 2$ out of the 400 do make the grade ie $65 \pm 0.5\%$ [1]	<b>[6]</b>	
			<b>[Total:10]</b>
<b>8 (a) (i)</b>	The probability of something happening is the likelihood or chance of it happening [1]	<b>[1]</b>	

Question	Expected Answers	Marks	Rationale
(ii)	An independent event is one in which the probability of an event happening does not affect the probability of another event happening.	[1] [1]	[2]
(b) (i)	The probability of selecting at random a resistor, $p$ , is given by the ratio: Number of resistors/Total number of components $p = \text{Number of resistors/Total number of components}$ $= 36/(36 + 39)$ $= 36/75$	[1] [1]	[2]
(ii)	The probability of selecting at random a capacitor, $q$ , is given by the ratio: Number of capacitors/Total number of components  $q = \text{Number of capacitors/Total number of components}$ $= 39/(36 + 39)$ or $(1 - 36/75)$ $= 39/75$	[1]	[1]
(c)	Total number of components = $36 + 39 = 75$ The probability of randomly selecting a resistor on the first draw is $36/75$ [1] There are now 35 resistors in a batch of 74. The probability of randomly selecting a resistor on the second draw is $35/74$  The probability of selecting a resistor on the first and second draw is $(36/75) \times (35/74)$ $= 0.227$	[1] [1] [1]	[4]
			[Total:10]

# **Grade Thresholds**

**OCR Level 3 Certificate Mathematical Techniques and Applications for Engineers H865  
January 2010 Examination Series**

**This exam was not awarded in January 2010, therefore there are no Grade Thresholds.**

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