GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for January 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Marking instructions for GCE Mathematics (MEI): Pure strand

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.

2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.

3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)

4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

9. **Rules for crossed out and/or replaced work**

   If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

   If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

Award NR (no response) if:

- Nothing is written at all in the answer space
- There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
- There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

The hash key [#] on your keyboard will enter NR.

Award 0 if:

- There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

M1	method mark (M2, etc, is also used)
A1	accuracy mark
B1	independent mark
E1	mark for explaining
U1	mark for correct units
G1	mark for a correct feature on a graph
M1 dep*	method mark dependent on a previous mark, indicated by *
cao	correct answer only
ft	follow through
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
sc	special case
soi	seen or implied
www	without wrong working

14. Annotating scripts. The following annotations are available:

✓and ✗

BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working (after correct answer obtained)
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0,1
SC	Special case
^	Omission sign
MR	Misread

Highlighting is also available to highlight any particular points on a script.

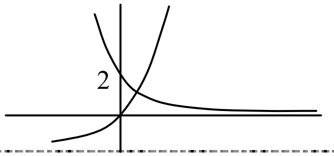
15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

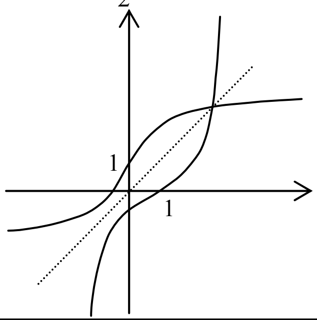
    Please do not type in the comments box yourself. Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.

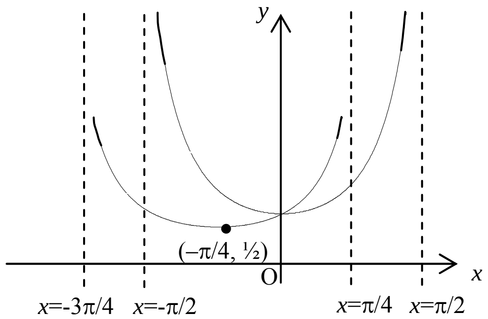
17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see scoris assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.

18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

1 $y=\sqrt[3]{1+x^2}=(1+x^2)^{1/3}$ $\Rightarrow$ $\frac{dy}{dx}=\frac{1}{3}(1+x^2)^{-\frac{2}{3}}.2x$ $=\frac{2}{3}x(1+x^2)^{-\frac{2}{3}}$	M1 M1 B1 A1 [4]	$(1+x^2)^{1/3}$ chain rule $(1/3)\ u^{-2/3}$ (soi) cao, mark final answer	Do not allow MR for square root their $dy/du \times du/dx$ (available for wrong indices) no ft on ½ index oe e.g. $\frac{2x(1+x^2)^{-\frac{2}{3}}}{3}$, $\frac{2x}{3\sqrt[3]{(1+x^2)^2}}$, etc but must combine 2 with 1/3.
2 $\lvert 2x+1\rvert \geq 4$ $\Rightarrow$ $2x+1\geq 4 \Rightarrow x\geq 1½$ or $2x+1\leq -4 \Rightarrow x\leq -2½$	M1 A1 M1 A1 [4]	allow M1 for 1½ seen allow M1 for –2½ seen	Same scheme for other methods, e.g. squaring, graphing Penalise both > and < once only. –1 if both correct but final ans expressed incorrectly, e.g $-2½ \geq x \geq 1½$ or $1½ \leq x \leq -2½$ (or even $-2½ \leq x \leq 1½$ from previously correct work) e.g. SC3
3 $A=\pi r^2$ $\Rightarrow$ $dA/dr = 2\pi r$ When $r = 2$, $dA/dr = 4\pi$, $dA/dt = 1$ $\frac{dA}{dt}=\frac{dA}{dr}.\frac{dr}{dt}$ $\Rightarrow$ $1 = 4\pi.dr/dt$ $\Rightarrow$ $dr/dt = 1/4\pi = 0.0796$ (mm/s)	M1A1 A1 M1 A1 [5]	$2\pi r$ soi (at any stage) chain rule (o.e) cao: 0.08 or better condone truncation	M1A0 if incorrect notation, e.g. dy/dx, dr/dA, if seen. $2r$ is M1A0 must be dA/dr (soi) and dA/dt any correct form stated with relevant variables , e.g. $\frac{dr}{dt}=\frac{dr}{dA}.\frac{dA}{dt}$, $\frac{dr}{dt}=\frac{dr}{dA}/\frac{dt}{dA}$, etc. allow $1/4\pi$ but mark final answer
4 $\sin\theta$ = BC/AC, $\cos\theta$ = AB/AC $AB^2 + BC^2 = AC^2$ $\Rightarrow$ $(AB/AC)^2 + (BC/AC)^2 = 1$ $\Rightarrow$ $\cos^2\theta + \sin^2\theta = 1$ Valid for $(0° <)\ \theta < 90°$	M1 A1 B1 [3]	or a/b, c/b condone taking AC = 1 Must use Pythagoras allow ≤, or 'between 0 and 90' or < 90 allow < π/2 or 'acute'	allow o/h, a/h etc if clearly marked on triangle. but must be stated arguing backwards unless ⇔ used A0
5(i) 	B1 B1 B1 [3]	shape of $y = e^x - 1$ and through O shape of $y = 2e^{-x}$ through (0, 2) (not (2,0))	for first and second B1s graphs must include negative x values condone no asymptote $y = -1$ shown asymptotic to x-axis (shouldn't cross)
(ii) $e^x - 1 = 2e^{-x}$ $\Rightarrow$ $e^{2x} - e^x = 2$ $\Rightarrow$ $(e^x)^2 - e^x - 2 = 0$ $\Rightarrow$ $(e^x - 2)(e^x + 1) = 0$ $\Rightarrow$ $e^x = 2$ (or –1) $\Rightarrow$ $x = \ln 2$ $\Rightarrow$ $y = 1$	M1 M1 B1 B1 B1cao [5]	equating re-arranging into a quadratic in $e^x = 0$ stated www www www	allow one error but must have $e^{2x} = (e^x)^2$ (soi) award even if not from quadratic method (i.e. by 'fitting') provided www allow for unsupported answers, provided www need not have used a quadratic, provided www

6 $(x+y)^2 = 4x$ $\Rightarrow \quad 2(x+y)(1+\frac{dy}{dx}) = 4$ $\Rightarrow \quad 1+\frac{dy}{dx} = \frac{4}{2(x+y)} = \frac{2}{x+y}$ $\Rightarrow \quad \frac{dy}{dx} = \frac{2}{x+y} - 1$ *	M1 A1 A1	Implicit differentiation of LHS correct expression = 4 www (**AG**)	Award no marks for solving for y and attempting to differentiate allow one error but must include dy/dx ignore superfluous dy/dx = … for M1, and for both A1s if not pursued condone missing brackets A0 if missing brackets in earlier working
or $x^2 + 2xy + y^2 = 4x$ $\Rightarrow \quad 2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 4$ $\Rightarrow \quad \frac{dy}{dx}(2x+2y) = 4 - 2x - 2y$ $\Rightarrow \quad \frac{dy}{dx} = \frac{4}{2x+2y} - 1 = \frac{2}{x+y} - 1$ *	M1dep A1 A1	Implicit differentiation of LHS dep correct expansion correct expression = 4 (oe after re-arrangement) www (**AG**)	allow 1 error provided 2xdy/dx and 2ydy/dx are correct, but must expand $(x + y)^2$ correctly for M1 (so $x^2 + y^2 = 4x$ is M0) ignore superfluous dy/dx = … for M1, and for both A1s if not pursued A0 if missing brackets in earlier working
When $x = 1$, $y = 1$, $\frac{dy}{dx} = \frac{2}{1+1} - 1 = 0$ *	B1 [4]	(**AG**) oe (e.g. from $x + y = 2$)	or e.g $2/(x + y) - 1 = 0 \Rightarrow x + y = 2, \Rightarrow 4 = 4x, \Rightarrow x = 1, y = 1$ (oe)
7 (i) bounds $-\pi + 1$, $\pi + 1$ $\Rightarrow \quad -\pi + 1 < f(x) < \pi + 1$	B1B1 B1cao [3]	or … < y < … or $(-\pi + 1, \pi + 1)$	not … < x < …, not 'between …'
(ii) $y = 2\arctan x + 1 \quad x \leftrightarrow y$ $x = 2\arctan y + 1$ $\Rightarrow \quad \frac{x-1}{2} = \arctan y$ $\Rightarrow \quad y = \tan(\frac{x-1}{2}) \Rightarrow f^{-1}(x) = \tan(\frac{x-1}{2})$ 	M1 A1 A1 B1 B1 [5]	attempt to invert formula or $\frac{y-1}{2} = \arctan x$ reasonable reflection in $y = x$ (1, 0) intercept indicated.	one step is enough, i.e. $y - 1 = 2\arctan x$ or $x - 1 = 2\arctan y$ need not have interchanged x and y at this stage allow y = … curves must cross on $y = x$ line if present (or close enough to imply intention) curves shouldn't touch or cross in the third quadrant

8(i) $\int_0^1 \frac{x^3}{1+x}\,dx$ let $u = 1+x,\ du = dx$ when $x = 0$, $u = 1$, when $x = 1$, $u = 2$ $= \int_1^2 \frac{(u-1)^3}{u}\,du$ $= \int_1^2 \frac{(u^3-3u^2+3u-1)}{u}\,du$ $= \int_1^2 (u^2-3u+3-\frac{1}{u})\,du$ * $\int_0^1 \frac{x^3}{1+x}\,dx = \left[\frac{1}{3}u^3-\frac{3}{2}u^2+3u-\ln u\right]_1^2$ $= (\frac{8}{3}-6+6-\ln 2)-(\frac{1}{3}-\frac{3}{2}+3-\ln 1)$ $= \frac{5}{6}-\ln 2$	B1 B1 M1 A1dep B1 M1 A1cao [7]	$a = 1$, $b = 2$ $(u-1)^3/u$ expanding (correctly) dep du = dx (o.e.) **AG** $\left[\frac{1}{3}u^3-\frac{3}{2}u^2+3u-\ln u\right]$ substituting correct limits dep integrated must be exact – must be 5/6	seen anywhere, e.g. in new limits e.g. du/dx = 1, condone missing dx's and du's, allow du = 1 upper – lower; may be implied from 0.140… must have evaluated ln 1 = 0
(ii) $y = x^2\ln(1+x)$ $\Rightarrow$ $\frac{dy}{dx} = x^2.\frac{1}{1+x}+2x.\ln(1+x)$ $= \frac{x^2}{1+x}+2x\ln(1+x)$ When $x = 0$, dy/dx = 0 + 0.ln 1 = 0 ($\Rightarrow$ Origin is a stationary point)	M1 B1 A1 M1 A1cao [5]	Product rule d/dx (ln(1 + x)) = 1/(1 + x) cao (oe) mark final ans substituting x = 0 into **correct** deriv www	 or d/dx (ln u) = 1/u where u = 1 + x ln1+x is A0 when x = 0, dy/dx = 0 with no evidence of substituting M1A0 but condone missing bracket in ln(1+x)
(iii) $A = \int_0^1 x^2\ln(1+x)\,dx$ let $u = \ln(1+x)$, $dv/dx = x^2$ $\frac{du}{dx} = \frac{1}{1+x}$, $v = \frac{1}{3}x^3$ $\Rightarrow$ $A = \left[\frac{1}{3}x^3\ln(1+x)\right]_0^1 - \int_0^1 \frac{1}{3}\frac{x^3}{1+x}\,dx$ $= \frac{1}{3}\ln 2-(\frac{5}{18}-\frac{1}{3}\ln 2)$ $= \frac{1}{3}\ln 2-\frac{5}{18}+\frac{1}{3}\ln 2$ $= \frac{2}{3}\ln 2-\frac{5}{18}$	B1 M1 A1 B1 B1ft A1 [6]	Correct integral and limits parts correct $= \frac{1}{3}\ln 2 - ...$ … – 1/3 (result from part (i)) cao	condone no dx, limits (and integral) can be implied by subsequent work u, du/dx, dv/dx and v all correct (oe) condone missing brackets condone missing bracket, can re-work from scratch oe e.g. $= \frac{12\ln 2-5}{18}, \frac{1}{3}\ln 4-\frac{5}{18}$, etc but must have evaluated ln 1 =0 Must combine the two ln terms

9(i) $\frac{d}{dx}(\frac{\sin x}{\cos x}) = \frac{\cos x.\cos x - \sin x.(-\sin x)}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ *	M1 A1 A1 [3]	Quotient (or product) rule (**AG**)	product rule: $\frac{1}{\cos x}.\cos x + \sin x(-\frac{1}{\cos^2 x})(-\sin x)$ but must show evidence of using chain rule on 1/cos x (or d/dx (sec x) = sec x tan x used)
(ii) Area = $\int_0^{\pi/4} \frac{1}{\cos^2 x} dx$ $= [\tan x]_0^{\pi/4}$ = tan(π/4) – tan 0 = 1	B1 M1 A1 [3]	correct integral and limits (soi) $[\tan x]$ or $\left[\frac{\sin x}{\cos x}\right]$	condone no dx; limits can be implied from subsequent work unsupported scores M0
(iii) f(0) = 1/cos^2(0) = 1 g(x) = 1/2cos^2(x + π/4) g(0) = 1/2cos^2(π/4) = 1 (⇒ f and g meet at (0, 1))	B1 M1 A1 [3]	must show evidence	or f(π/4) = 1/cos^2(π/4) = 2 so g(0) = ½ f(π/4) = 1
(iv) Translation in x-direction through –π/4 Stretch in y-direction scale factor ½ 	M1 A1 M1 A1 B1ft B1ft B1 B1dep [8]	must be in x-direction, or $\begin{pmatrix}-\pi/4\\0\end{pmatrix}$ must be in y-direction asymptotes correct min point (–π/4, ½) curves intersect on y-axis correct curve, dep B3, with asymptote lines indicated and correct, and TP in correct position	'shift' or 'move' for 'translation' M1 A0; $\begin{pmatrix}-\pi/4\\0\end{pmatrix}$ alone SC1 'contract' or 'compress' or 'squeeze' for 'stretch' M1A0; 'enlarge' M0 stated or on graph; condone no x = …, ft π/4 to right only (viz. –π/4, 3π/4) stated or on graph; ft π/4 to right only (viz. (π/4, ½)) 'y-values halved', or 'x-values reduced by π/4, are M0 (not geometric transformations), but for M1 condone mention of x- and y- values provided transformation words are used.
(v) Same as area in (ii), but stretched by s.f. ½ . So area = ½ .	B1ft [1]	½ area in (ii)	or $\int_{-\pi/4}^{0} g(x)\,dx = \frac{1}{2}\int_{-\pi/4}^{0} \frac{1}{\cos^2(x+\pi/4)}dx = \frac{1}{2}[\tan(x+\pi/4]_{-\pi/4}^{0}$ = ½ allow unsupported

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