RECOGNISING ACHIEVEMENT

Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate

H860/01 Paper 1

Mark Scheme for June 2011

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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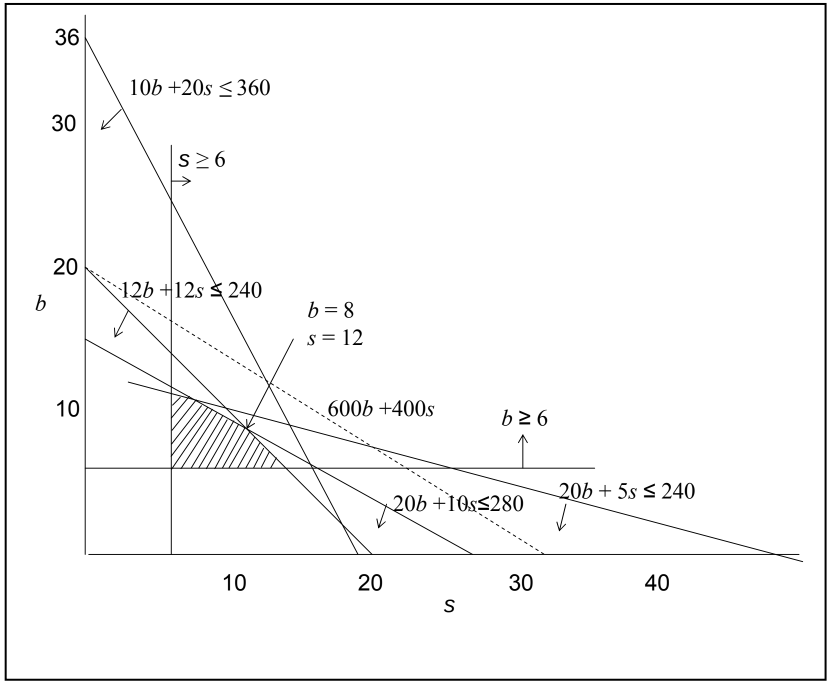
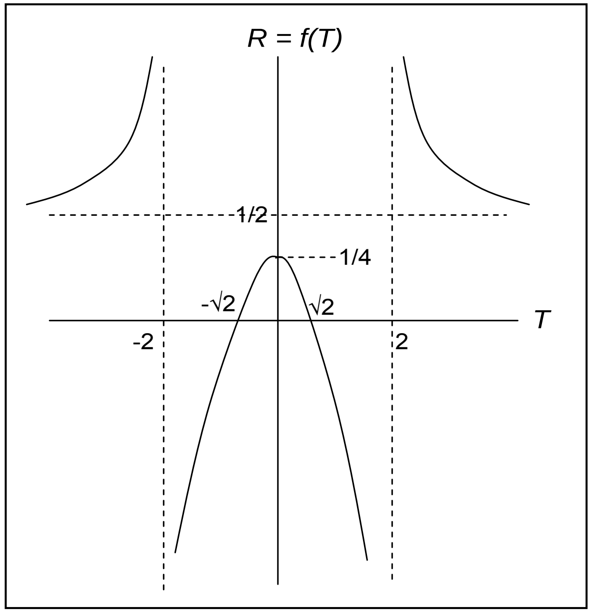
1 a i	Total hours in workshop 1 must be ≤ 480 Total hours in workshop 1 =$10b + 20s + 15a$ Therefore $10b + 20s + 15a \leq 480$	1 **1**	Must provide adequate explanation for one mark.
1 a ii	$12b + 12s + 10a \leq 320$ $20b + 10s + 10a \leq 360$ $20b + 5s + 15a \leq 360$ $b, s, a \geq 6$ Maximise $600b + 400s + 450a$	1 1 1 **3**	1 mark for three ≤ constraints 1 mark for ≥ 6 constraints 1 mark for objective function
1 b i	$10b + 20s \leq 480 - 15 \times 8$ (360) $12b + 12s \leq 320 - 10 \times 8$ (240) $20b + 10s \leq 360 - 10 \times 8$ (280) $20b + 5s \leq 360 - 15 \times 8$ (240) $b, s \geq 6$ Maximise $600b + 400s + 3600$	 1 1 **2**	1 mark for 260, 240, 280 and 240 seen Accept $600b + 400s$
1 b ii	Constraints on graph	3 **3**	1 mark for each two ≤ constraints 1 mark for ≥ 6 constraints
1 b iii	Feasible region indicated	1 **1**	Accept ECF from b ii

1 b iv	$b = 8$, $s = 12$ from graph 	1 **1**	Must indicate objective line but accept ECF from b ii and b iii
2 a	$p = 0.2$, $q = 0.8$ P(0) = 10!/(0! × (10 – 0)!) × 0.2^0 × 0.8^{10-0} = 0.1074 P(1) = 10!/(1! × (10 – 1)!) × 0.2^1 × 0.8^{10-1} = 0.2684 P(2 or more) = 1 – (0.1074 + 0.2684) = 0.6242	1 1 1 **3**	1 mark for use of $P(X) = \frac{N!}{X!(N-X)!} 0.2^X 0.8^{N-X}$ 1 mark for correct answer to one or more of P(0) and P(1) 1 mark for final result with ECF

2 b i	Mean = Np = 20 Variance = Npq = 16	1 **1**	1 mark for 20 and 16 seen
2 b ii	$N(Np, Npq) = N(20, 16)$ $P(17.5 \le x \le 24.5)$ $P((17.5 - 20)/4 \le z \le (24.5 - 20)/4)$ $P(-0.625 \le z \le 1.125)$ = 0.234 + 0.3696 = 0.6036	1 1 1 **3**	Allow ECF from b i Allow $P((18 - 20)/4 \le z \le (24 - 20)/4)$ With final answer 0.1915 + 0.3413 = 0.5328
2 c	Poisson distribution $\lambda = 5$ $P(x) = 5^x e^{-5}/x!$, P(0) = 0.0067, P(1) = 0.0337, P(2) = 0.0842 $P(x \le 2)$ = P(0) + P(1) + P(2) = 0.1246	1 1 1 **3**	1 mark for use of $\frac{\lambda^x e^{-\lambda}}{x!}$ 1 mark for at least two correct results Accept 0.1247 directly from table Accept ECF from previous step

3 a	$h=(e^{3/2}+e^{-3/2})\ =4.7048$	1 **1**	1 mark for 4.4817 + 0.2231 seen
3 b	Total Area $=2\int_0^3(e^{x/2}+e^{-x/2})\mathrm{d}x$ $=2\left[2e^{x/2}-2e^{-x/2}\right]_0^3$ $2\left[(2e^{3/2}-2e^{-3/2})-(2-2)\right]$ $4(e^{3/2}-e^{-3/2})=17.0342$	1 1 1 1 **4**	Hyperbolic functions do not appear in the specification, however, if candidates recognize that $y=(e^{x/2}+e^{-x/2})=$ 2 cosh x then use the standard integral (2 sinh x) and arrive at the correct answer, 4 marks should be awarded. Allow 1 mark for reasonable approximation using straight lines for roof or trapezoidal/Simpson's rule etc.
3 c i	$y=(e^{x/2}+e^{-x/2})$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}(e^{x/2}-e^{-x/2})$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4}(e^{x}+e^{-x}-2)$ $1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4}(e^{x}+e^{-x}+2)=\frac{1}{4}(e^{x/2}+e^{-x/2})^2$ $\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}=\frac{1}{2}(e^{x/2}+e^{-x/2})$	1 1 1 **3**	Correct solutions using hyperbolic functions should also gain full marks

3 c ii	$\frac{1}{2}\int_0^3 (e^{x/2}+e^{-x/2})\mathrm{d}x = \frac{17.0342}{4} = 4.2585$	1 **1**	Allow FT from part b
4 a	$\mathrm{f}(T) = A + \frac{B}{(T-2)} + \frac{C}{(T+2)}$ $A = \frac{1}{2}\quad B = \frac{1}{4}\quad C = -\frac{1}{4}$ $\frac{1}{2} + \frac{1}{4(T-2)} - \frac{1}{4(T+2)}$	2 **2**	OE
4 b i	$\frac{dR}{dT} = -\frac{1}{4(T-2)^2} + \frac{1}{4(T+2)^2} = 0$ when $T = 0$, $R = 1/4$	2 **2**	Allow ECF from 4 a
4 b ii	Poles at $T = \pm 2$ Zeros at $T = \pm\sqrt{2}$	2 **2**	

4 b iii		1 1 **2**	1 mark for general shape of upper part of graph 1 mark for general shape of lower part of graph
4 c	$T^2 - 2 = 2(T^2 - 4)R$ $T^2 - 2 = 2T^2R - 8R$ $T^2(1-2R) = 2 - 8R$ $T = \pm\sqrt{\dfrac{2-8R}{1-2R}}$ $T = \pm\sqrt{\dfrac{2-8R}{1-2R}}$ *T* is real when R >1/2 or R ≤ 1/4	1 2 **3**	

5 a i	$\overrightarrow{AB} = b - a = (0,0,12) - (6,4,9) = (-6,-4,3)$ $\overrightarrow{AD} = d - a = (4,14,9) - (6,4,9) = (-2,10,0)$	1 **1**	Also accept $\overrightarrow{AB} = -6i - 4j + 3k$ $\overrightarrow{AD} = -2i + 10j$ Also accept (6, 4, –3) and (2, –10, 0)
5 a ii	$\cos A = \frac{AB.AD}{\lvert AB\rvert\lvert AD\rvert} = \frac{12-40}{\sqrt{6^2+4^2+3^2}\sqrt{2^2+10^2}} = -0.3515$ $BAD = 110.5816^{o}$	1 1 **2**	1 mark for any reasonable attempt using 2-D geometry.
5 b	$ax + by + cz = d$ $a = \begin{vmatrix}1 & 4 & 9\\1 & 0 & 12\\1 & 14 & 9\end{vmatrix} = -30$ $b = \begin{vmatrix}6 & 1 & 9\\0 & 1 & 12\\4 & 1 & 9\end{vmatrix} = -6$ $c = \begin{vmatrix}6 & 4 & 1\\0 & 0 & 1\\4 & 14 & 1\end{vmatrix} = -68$ $d = \begin{vmatrix}6 & 4 & 9\\0 & 0 & 12\\4 & 14 & 9\end{vmatrix} = -816$ $-30x - 6y - 68z = -816$ $15x + 3y + 34z = 408$	1 1 1 1 1 **5**	Accept alternative solution using vector product $\begin{vmatrix}i & j & k\\-6 & -4 & 3\\-2 & 10 & 0\end{vmatrix} = -30i - 6j - 68k$ $-30x - 6y - 68z = d$ From point B $d = -68\times 12 = -816$ $-30x - 6y - 68z = -816$ $15x + 3y + 34z = 408$

5 c	$\cos(90) = \dfrac{\overrightarrow{DC}\,\overrightarrow{DA}}{\lvert\overrightarrow{DC}\rvert \times \lvert\overrightarrow{DA}\rvert} = 0$ $\overrightarrow{DC} = (x-4, y-14, z-9)$ $\overrightarrow{DA} = (2, -10, 0)$ $\overrightarrow{DC}\,\overrightarrow{DA} = (2(x-4) - 10(y-14)) = 0$ $2x - 10y + 132 = 0$ when $x = 2$ $y = 13.6$ From $15x + 3y + 34z = 408$ $z = (408 - 30 - 40.8)/34 = 9.92$ to 3 s.f.	1 1 1 1 **4**	 Allow ECF
6 a	Output from A = $\cos(\omega_c t)\cos(\omega_o t)$ Output from B = $\sin(\omega_c t)\sin(\omega_o t)$ $\cos(\omega_c t)\cos(\omega_o t) + \sin(\omega_c t)\sin(\omega_o t) = \cos((\omega_c - \omega_o)t)$ (Using $\cos(A - B) = \cos A \cos B + \sin A \sin B$ from list MF1)	1 1 **2**	
6 b	Output from A = $\cos(\omega_c t)\cos(\omega_o t)$ $= \cos(((\omega_c + \omega_o) + (\omega_c - \omega_o))t/2)\cos(((\omega_c + \omega_o) - (\omega_c - \omega_o))t/2)$ $= \frac{1}{2}\cos(\omega_c + \omega_o)t + \frac{1}{2}\cos(\omega_c - \omega_o)t$ Output from B = $\sin(\omega_c t)\sin(\omega_o t)$ $\sin(((\omega_c + \omega_o) + (\omega_c - \omega_o))t/2)\sin(((\omega_c + \omega_o) - (\omega_c - \omega_o))t/2)$ $= -\frac{1}{2}\cos(\omega_c + \omega_o)t + \frac{1}{2}\cos(\omega_c - \omega_o)t$	1 1 1 **3**	

6 c	Output from B = $\sin(\omega_c t + \varphi)\ \sin\omega_o t$ $= (\sin\omega_c t\cos\varphi + \cos\omega_c t\sin\varphi)\sin\omega_o t$ $= \sin\omega_c t\sin\omega_o t\cos\varphi + \cos\omega_c t\sin\omega_o t\sin\varphi$	1 1 **2**	

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