

ADVANCED GCE
MATHEMATICS
Further Pure Mathematics 2

4726

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Friday 22 May 2009
Morning

Duration: 1 hour 30 minutes



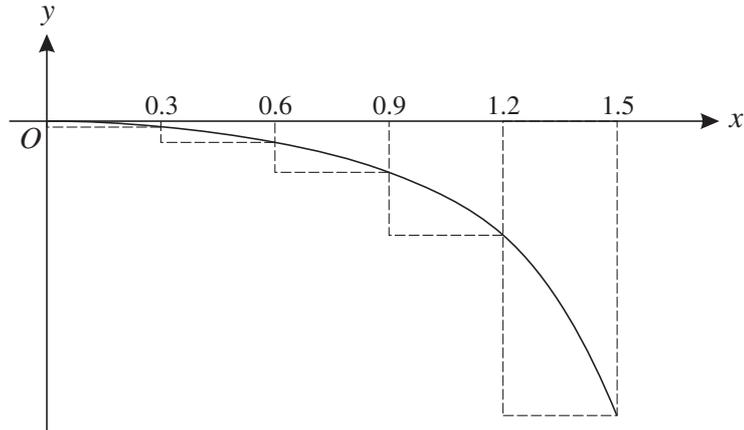
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



The diagram shows the curve with equation $y = \ln(\cos x)$, for $0 \leq x \leq 1.5$. The region bounded by the curve, the x -axis and the line $x = 1.5$ has area A . The region is divided into five strips, each of width 0.3.

(i) By considering the set of rectangles indicated in the diagram, find an upper bound for A . Give the answer correct to 3 decimal places. [2]

(ii) By considering another set of five suitable rectangles, find a lower bound for A . Give the answer correct to 3 decimal places. [2]

(iii) How could you reduce the difference between the upper and lower bounds for A ? [1]

2 Given that $y = \frac{x^2 + x + 1}{(x - 1)^2}$, prove that $y \geq \frac{1}{4}$ for all $x \neq 1$. [4]

3 (i) Given that $f(x) = e^{\sin x}$, find $f'(0)$ and $f''(0)$. [4]

(ii) Hence find the first three terms of the Maclaurin series for $f(x)$. [2]

4 Express $\frac{x^3}{(x - 2)(x^2 + 4)}$ in partial fractions. [6]

5 It is given that $I = \int_0^{\frac{1}{2}\pi} \frac{\cos \theta}{1 + \cos \theta} d\theta$.

(i) By using the substitution $t = \tan \frac{1}{2}\theta$, show that $I = \int_0^1 \left(\frac{2}{1+t^2} - 1 \right) dt$. [5]

(ii) Hence find I in terms of π . [2]

6 Given that

$$\int_0^1 \frac{1}{\sqrt{16+9x^2}} dx + \int_0^2 \frac{1}{\sqrt{9+4x^2}} dx = \ln a,$$

find the exact value of a .

[6]

7 (i) Sketch the graph of $y = \coth x$, and give the equations of any asymptotes.

[3]

(ii) It is given that $f(x) = x \tanh x - 2$. Use the Newton-Raphson method, with a first approximation $x_1 = 2$, to find the next three approximations x_2 , x_3 and x_4 to a root of $f(x) = 0$. Give the answers correct to 4 decimal places.

[4]

(iii) If $f(x) = 0$, show that $\coth x = \frac{1}{2}x$. Hence write down the roots of $f(x) = 0$, correct to 4 decimal places.

[3]

8 (i) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x} , show that

(a) $\cosh(\ln a) \equiv \frac{a^2 + 1}{2a}$, where $a > 0$,

[3]

(b) $\cosh x \cosh y - \sinh x \sinh y \equiv \cosh(x - y)$.

[3]

(ii) Use part (i)(b) to show that $\cosh^2 x - \sinh^2 x \equiv 1$.

[1]

(iii) Given that $R > 0$ and $a > 1$, find R and a such that

$$13 \cosh x - 5 \sinh x \equiv R \cosh(x - \ln a).$$

[5]

(iv) Hence write down the coordinates of the minimum point on the curve with equation $y = 13 \cosh x - 5 \sinh x$.

[2]

9 (i) It is given that, for non-negative integers n ,

$$I_n = \int_0^{\frac{1}{2}\pi} \sin^n \theta d\theta.$$

Show that, for $n \geq 2$,

$$nI_n = (n-1)I_{n-2}.$$

[4]

(ii) The equation of a curve, in polar coordinates, is

$$r = \sin^3 \theta, \quad \text{for } 0 \leq \theta \leq \pi.$$

(a) Find the equations of the tangents at the pole and sketch the curve.

[4]

(b) Find the exact area of the region enclosed by the curve.

[6]

There are no questions printed on this page.



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