## Monday 4 October 2021 - Afternoon

## AS Level Further Mathematics B (MEI)

## Y410/01 Core Pure

## Time allowed: 1 hour 15 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator


## INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer all the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do not send this Question Paper for marking. Keep it in the centre or recycle it.


## INFORMATION

- The total mark for this paper is $\mathbf{6 0}$.
- The marks for each question are shown in brackets [ ].
- This document has 4 pages.


## ADVICE

- Read each question carefully before you start your answer.


## Answer all the questions.

1 Using standard summation formulae, find $\sum_{r=1}^{n}\left(r^{2}-3 r\right)$, giving your answer in fully factorised form.

2 The equation $3 x^{2}-4 x+2=0$ has roots $\alpha$ and $\beta$.
Find an equation with integer coefficients whose roots are $3-2 \alpha$ and $3-2 \beta$.

3 Three planes have the following equations.

$$
\begin{aligned}
2 x-3 y+z & =-3, \\
x-4 y+2 z & =1, \\
-3 x-2 y+3 z & =14 .
\end{aligned}
$$

(a) (i) Write the system of equations in matrix form.
(ii) Hence find the point of intersection of the planes.
(b) In this question you must show detailed reasoning.

Find the acute angle between the planes $2 x-3 y+z=-3$ and $x-4 y+2 z=1$.

4 Anika thinks that, for two square matrices $\mathbf{A}$ and $\mathbf{B}$, the inverse of $\mathbf{A B}$ is $\mathbf{A}^{-1} \mathbf{B}^{-1}$. Her attempted proof of this is as follows.

$$
\begin{aligned}
(\mathbf{A B})\left(\mathbf{A}^{-1} \mathbf{B}^{-1}\right) & =\mathbf{A}\left(\mathbf{B} \mathbf{A}^{-1}\right) \mathbf{B}^{-1} \\
& =\mathbf{A}\left(\mathbf{A}^{-1} \mathbf{B}\right) \mathbf{B}^{-1} \\
& =\left(\mathbf{A A}^{-1}\right)\left(\mathbf{B B}^{-1}\right) \\
& =\mathbf{I} \times \mathbf{I} \\
& =\mathbf{I} \\
\text { Hence }(\mathbf{A B})^{-1} & =\mathbf{A}^{-1} \mathbf{B}^{-1}
\end{aligned}
$$

(a) Explain the error in Anika's working.
(b) State the correct inverse of the matrix $\mathbf{A B}$ and amend Anika's working to prove this.

5 Prove by induction that $\sum_{r=1}^{n} r \times 2^{r-1}=1+(n-1) 2^{n}$ for all positive integers $n$.

6 A transformation T of the plane has associated matrix $\mathbf{M}=\left(\begin{array}{cc}1 & \lambda+1 \\ \lambda-1 & -1\end{array}\right)$, where $\lambda$ is a non-zero
(a) (i) Show that T reverses orientation.
(ii) State, in terms of $\lambda$, the area scale factor of T .
(b) (i) Show that $\mathbf{M}^{2}-\lambda^{2} \mathbf{I}=\mathbf{0}$.
(ii) Hence specify the transformation equivalent to two applications of T.
(c) In the case where $\lambda=1, \mathrm{~T}$ is equivalent to a transformation S followed by a reflection in the $x$-axis.
(i) Determine the matrix associated with S .
(ii) Hence describe the transformation S .

7 (a) (i) Find the modulus and argument of $z_{1}$, where $z_{1}=1+\mathrm{i}$.
(ii) Given that $\left|z_{2}\right|=2$ and $\arg \left(z_{2}\right)=\frac{1}{6} \pi$, express $z_{2}$ in $a+b$ i form, where $a$ and $b$ are exact real numbers.
(b) Using these results, find the exact value of $\sin \frac{5}{12} \pi$, giving the answer in the form $\frac{\sqrt{m}+\sqrt{n}}{p}$, where $m, n$ and $p$ are integers.

## 8 In this question you must show detailed reasoning.

The equation $x^{3}+k x^{2}+15 x-25=0$ has roots $\alpha, \beta$ and $\frac{\alpha}{\beta}$. Given that $\alpha>0$, find, in any order,

- the roots of the equation,
- the value of $k$.

9 (a) On a single Argand diagram, sketch the loci defined by

- $\quad \arg (z-2)=\frac{3}{4} \pi$,
- $|z|=|z+2-\mathrm{i}|$.
(b) In this question you must show detailed reasoning.

The point of intersection of the two loci in part (a) represents the complex number $w$.
Find $w$, giving your answer in exact form.

## END OF QUESTION PAPER

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