Oxford Cambridge and RSA

# Wednesday 6 October 2021 - Afternoon AS Level Mathematics B (MEI) 

H630/01 Pure Mathematics and Mechanics
Time allowed: 1 hour 30 minutes

## You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator


## INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer all the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $\mathrm{gm} \mathrm{s}^{-2}$. When a numerical value is needed use $g=9.8$ unless a different value is specified in the question.
- Do not send this Question Paper for marking. Keep it in the centre or recycle it.


## INFORMATION

- The total mark for this paper is 70.
- The marks for each question are shown in brackets [ ].
- This document has 8 pages.


## ADVICE

- Read each question carefully before you start your answer.


## Formulae AS Level Mathematics B (MEI) (H630)

## Binomial series

$(a+b)^{n}=a^{n}+{ }^{n} \mathrm{C}_{1} a^{n-1} b+{ }^{n} \mathrm{C}_{2} a^{n-2} b^{2}+\ldots+{ }^{n} \mathrm{C}_{r} a^{n-r} b^{r}+\ldots+b^{n} \quad(n \in \mathbb{N})$,
where ${ }^{n} \mathrm{C}_{r}={ }_{n} \mathrm{C}_{r}=\binom{n}{r}=\frac{n!}{r!(n-r)!}$
$(1+x)^{n}=1+n x+\frac{n(n-1)}{2!} x^{2}+\ldots+\frac{n(n-1) \ldots(n-r+1)}{r!} x^{r}+\ldots \quad(|x|<1, n \in \mathbb{R})$

## Differentiation from first principles

$\mathrm{f}^{\prime}(x)=\lim _{h \rightarrow 0} \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

## Sample variance

$s^{2}=\frac{1}{n-1} S_{x x}$ where $S_{x x}=\sum\left(x_{i}-\bar{x}\right)^{2}=\sum x_{i}^{2}-\frac{\left(\sum x_{i}\right)^{2}}{n}=\sum x_{i}^{2}-n \bar{x}^{2}$
Standard deviation, $s=\sqrt{\text { variance }}$

## The binomial distribution

If $X \sim \mathrm{~B}(n, p)$ then $\mathrm{P}(X=r)={ }^{n} \mathrm{C}_{r} p^{r} q^{n-r}$ where $q=1-p$
Mean of $X$ is $n p$

## Kinematics

Motion in a straight line
$v=u+a t$
$s=u t+\frac{1}{2} a t^{2}$
$s=\frac{1}{2}(u+v) t$
$v^{2}=u^{2}+2 a s$
$s=v t-\frac{1}{2} a t^{2}$

## Answer all the questions.

1 Find the coordinates of the point of intersection of the lines $y=3 x-2$ and $x+2 y=10$.

2 An unmanned craft lands on the planet Mars. A small bolt falls from the craft onto the surface of the planet. It falls 1.5 m from rest in 0.9 s .

Calculate the acceleration due to gravity on Mars.

3 Forces $\mathbf{F}_{1}=(2 \mathbf{i}+9 \mathbf{j}) \mathrm{N}$ and $\mathbf{F}_{2}=(-\mathbf{i}+\mathbf{j}) \mathrm{N}$ act on a particle. A third force $\mathbf{F}_{3}$ acts so that the particle is in equilibrium under the action of the three forces.

Find the force $\mathbf{F}_{3}$.

4 (a) Show that 4 ! $<4^{4}$.
(b) Nina believes that the statement $n!<n^{n}$ is true for all positive integers $n$.

Prove that Nina is not correct.

5 The diagram shows the triangle ABC in which $\mathrm{AC}=13 \mathrm{~cm}$ and AB is the shortest side. The perimeter of the triangle is 32 cm . The area is $24 \mathrm{~cm}^{2}$ and $\sin B=\frac{4}{5}$.


Determine the lengths of AB and BC .

6 The displacement of a particle is modelled by the equation $s=7+4 t-t^{2}$, where $s$ metres is the displacement from the origin at time $t$ seconds. The diagram shows part of the displacement-time graph for the particle. The point $(2,11)$ is the maximum point on the graph.

(a) Kai argues that the point $(2,11)$ is on the graph, so the particle has travelled a distance of 11 metres in the first 2 seconds.

Comment on the validity of Kai's argument.
(b) Determine the total distance the particle travels in the first 10 seconds.
(c) Find an expression for the velocity of the particle at time $t$.
(d) Find the speed of the particle when $t=10$.

7 The diagram shows part of a curve which passes through the point $(1,0)$.


The gradient of the curve is given by $\frac{\mathrm{d} y}{\mathrm{~d} x}=6 x+\frac{8}{x^{3}}$.
Determine whether the curve passes through the point $(2,12)$.

8 In this question you must show detailed reasoning.
(a) Use differentiation to find the coordinates of the stationary point on the curve with equation $y=2 x^{2}-3 x-2$.
(b) Use the second derivative to determine the nature of the stationary point.
(c) Show by shading on a sketch the region defined by the inequality $y \geqslant 2 x^{2}-3 x-2$, indicating clearly whether the boundary is included or not.
(d) Solve the inequality $2 x^{2}-3 x-2>0$ using set notation for your answer.

9 (a) Sketch both of the following on the axes provided in the Printed Answer Booklet.
(i) The curve $y=\frac{12}{x}$, stating the coordinates of at least one point on the curve.
(ii) The line $y=2 x+8$, stating the coordinates of the points at which the line crosses the axes.
(b) In this question you must show detailed reasoning.

Determine the exact coordinates of the points of intersection of the curve and the line.

10 A rescue worker is lowered from a helicopter on a rope. She attaches a second rope to herself and to a woman in difficulties on the ground. The helicopter winches both women upwards with the rescued woman vertically below the rescue worker, as shown in the diagram.


The model for this motion uses the following modelling assumptions:

- each woman can be modelled as a particle;
- the ropes are both light and inextensible;
- there is no air resistance to the motion;
- the motion is in a vertical line.
(a) Explain what it means when the women are each 'modelled as a particle'.
(b) Explain what 'light' means in this context.

The tension in the rope to the helicopter is 1500 N . The rescue worker has a mass of 65 kg and the rescued woman has a mass of 75 kg .
(c) Draw a diagram showing the forces on the two women.
(d) Write down the equation of motion of the two women considered as a single particle.
(e) Calculate the acceleration of the women.
(f) Determine the tension in the rope connecting the two women.

11 On the day that a new consumer product went on sale (day zero), a call centre received 1 call about it. On the 2nd day after day zero the call centre received 3 calls, and on the 10th day after day zero there were 200 calls.

Two models were proposed to model $N$, the number of calls received $t$ days after day zero.
Model 1 is a linear model $N=m t+c$.
(a) Determine the values of $m$ and $c$ which best model the data for 2 days and 10 days after day zero.
(b) State the rate of increase in calls according to model 1.
(c) Explain why this model is not suitable when $t=1$.

Model 2 is an exponential model $N=\mathrm{e}^{0.53 t}$.
(d) Verify that this is a good model for the number of calls when $t=2$ and $t=10$.
(e) Determine the rate of increase in calls when $t=10$ according to model 2 .

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