OCR
RECOGNISING ACHIEVEMENT

SAMPLE

OCR Level 3 Principal Learning in Engineering

UNIT F563

Unit F563: Mathematical techniques and applications for engineers

Sample Paper

Time: 2 hours

Candidate Name

Centre Number | | | | | |

Candidate Number | | | | |

INSTRUCTIONS TO CANDIDATES

- Write your name in the space above.
- Write your Centre number and candidate number in the boxes above.
- Answer **all** the questions in Section A and **three** out of eight questions in Section B.
- Write your answers in blue or black ink in the spaces provided.
- Read each answer carefully and make sure you know what you have to do before starting your answer.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
  Section A is 30 marks
  Section B is 30 marks
- The use of calculators **is** allowed for this paper
- Include **all** working out where used

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.

For Examiner's Use	
Section A	
Section B	
Total	

This document consists of 24 printed pages.

Section A

Answer **all** questions.

1 Remove the brackets and simplify $(x + 4)(x + 5)$

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…… [2]

2 Simplify the expression $a^2b + a^2c$

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…… [2]

3 Simplify the following expression $(x + 4)/5 + (x + 2)/3$

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…… [2]

4 Solve the equation $5(2 + x) = 8x – 14$

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…… [2]

5 A wheel is rotating at 50 revolutions per minute. Express this angular speed in radians per second

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……..… [2]

6 An alternating e.m.f. is represented by v = 50 sin θ. Determine the value of v when the angle θ = 60^o

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7 Given that sec x = 15/14 from basic principles find cos x

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8 Calculate the area of a triangle ABC given side b = 10 m, side c = 16 m and angle A = 70^o

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9 Differentiate $y = 3x^5$

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…… [2]

10 Differentiate $y = 3 \sin 4x$

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11 Evaluate the integral $\int_0^4 6x^2 . dx$

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…… [2]

12 Find the integral $\int \cos 5x . dx$

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…… [2]

13 State what is meant by the following terms when dealing with statistics

(a) population

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…… [1]

(b) (b) sample

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…… [1]

14 The diameters of six bars were measured with a micrometer giving the following results in millimetres:

16.25 16.10 16.00 16.40 16.60 16.15

Determine the arithmetic mean diameter.

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……..… [2]

15 Determine the probability of selecting at random a 56 ohm resistor from a box containing twelve 56 ohm resistors and twenty-four 47 ohm resistors.

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Section A Total [30]

Section B

Answer any THREE questions

1 When Kirchhoff's laws are applied to a particular electrical circuit the currents i_1 and i_2 are connected by the equations:

$$6\,i_1 + 10\,i_2 = 2$$

$$9\,i_1 + 6\,i_2 = 12$$

Solve the equations to find the values of currents i_1 and i_2.

[10]

2 A rigid beam 20 m. long carries a uniformly distributed total load of 1000 N. The bending moment (BM), is related to its distance (A) from one end of the beam according to $BM = -50A^2 + 500A + 1000$.

Calculate the distance (A) when the bending moment (BM) is 200 Nm.

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3 A water main is 600 mm. diameter and is more than half full of water. The angle subtended at the centre by the horizontal surface of the water is (2/3) π radians.

(a) Draw a labelled diagram showing the cross-sectional area of the water pipe partly filled with water.

[2]

(b) Calculate

(i) the length of the circumference of the water pipe that is wet

(ii) the depth of the water.

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[8]

4 A vertical mast (AB), shown in Fig.1, is 35 m high. The mast is restrained by taut guy wires BC and BD which make angles of 30° to the vertical, such that points C, A and D are in line. The ground plane slopes as shown giving the distance AD as 20 m.

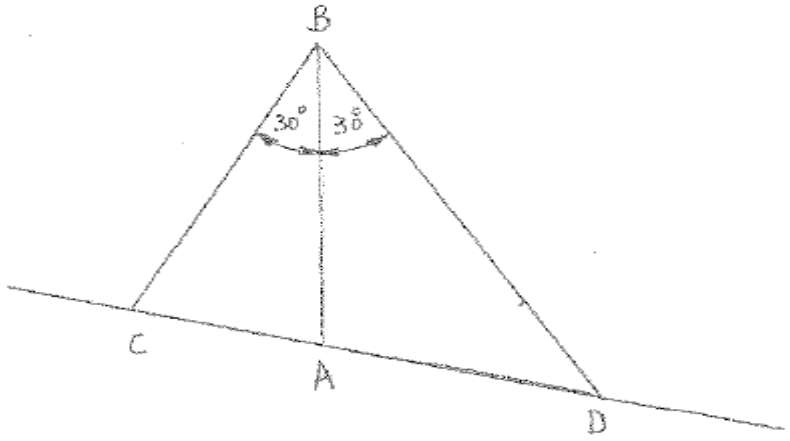

Fig.1

Calculate:

(a) angle D,

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…… [2]

(b) the length of the wire BD,

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…… [2]

(c) angle C

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(d) the distance CD,

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…… [2]

(e) the distance AC

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…… [1]

(f) the length of the wire BC.

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…… [2]

5 The distance (s) metres moved by a vehicle in time (t) seconds is given by the equation

$s = t^3 - 4t^2 + 2t$.

Find

(a) the initial velocity of the vehicle

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…… [4]

(b) the velocity of the vehicle after 8 seconds

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…… [2]

(c) the acceleration of the vehicle after 8 seconds

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…… [2]

(d) the time when the acceleration is zero.

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…… [2]

6 The pressure (P) and volume (V) of a gas are related by $PV^{1.5} = C$. The work done (W) in expanding the gas is given by the integral formula:

$$\text{Work done (W)} = \int P.\, dV$$

(a) Determine a value for C when pressure (P) is 120×10^3 Nm^{-2} and volume (V) is 0.6 m^3

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…… [3]

(b) From $PV^{1.5} = C$ make pressure (P) the subject of the formula

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…… [1]

(c) Calculate the work done when the gas expands from a volume of 0.6 m^3 to 2.0 m^3

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…… [6]

7 A random sample of components was taken from a production line. Each component had a nominal diameter of 8.0 mm ± 1.5 mm. The results of this sample are placed in the categories shown:

Variation (x)	6.6	7.8	8.0	8.2	8.4
Frequency (f)	3	9	20	16	7

Determine for the sample the:

- arithmetic mean;
- variance; and
- the standard deviation.

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…… [10]

8 The probability of an engineering system failing in one year due to excessive temperature is 1/20, due to vibration is 1/30 and due to moisture is 1/40.

(a) Determine the probabilities that over a one-year period the system fails due to extreme:

(i) temperature and vibration - assuming that the failure modes are independent
(ii) vibration or moisture - assuming that the failure modes are mutually exclusive

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(b) Determine the probabilities that over a one-year period the system will not fail because of both extreme temperature and vibration

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…… [3]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Principal Learning

OCR Level 3 Principal Learning in Engineering **[F563]**

Unit F563: Mathematical techniques and applications for engineers

Sample Model Solutions and Mark Scheme

The maximum mark for this paper is [60].

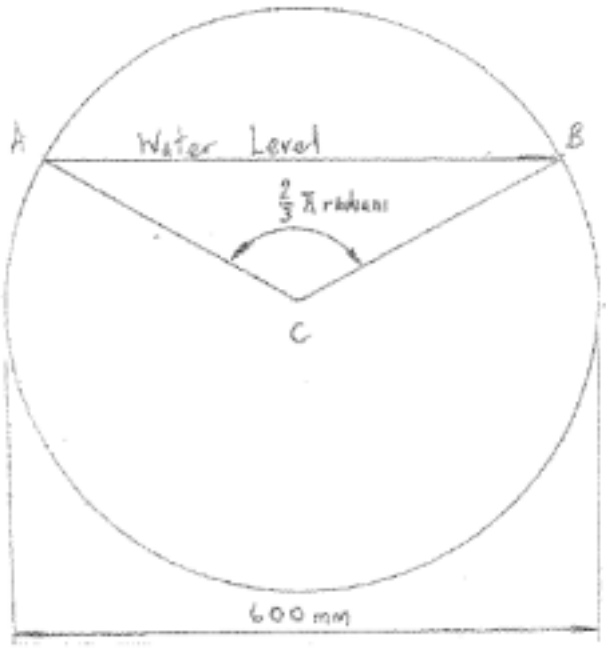
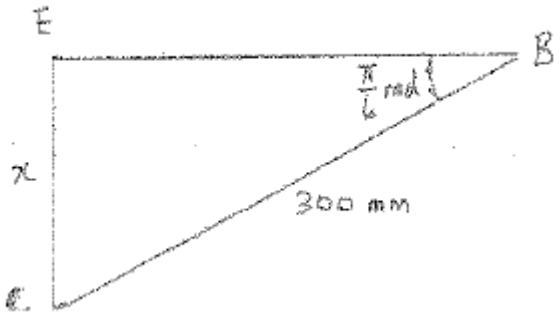
Section A		
Question Number	**Answer**	**Max Mark**
1	$(x+4)(x+5) = x^2 + 5x + 4x + 20 = x^2 + 9x + 20$	**2**
2	$a^2b + a^2c = a^2(b + c)$	**2**
3	$(x + 4)/5 + (x + 2)/3 = (3x + 12 + 5x + 10)/15 = (8x + 22)15$	**2**
4	$5(2 + x) = 8x – 14 \quad 10 + 5x = 8x – 14 \quad 3x = 24 \quad x = 8$	**2**
5	Angular speed = $50 \times 2\pi = 314.2/60 = 5.24$ rad.s^{-1}	**2**
6	$v = 50 \sin \theta = 50 \sin 60^o = 43.3$ V	**2**
7	$\sec x = 15/14 \quad \cos x = 1/\sec x$ so $\cos x = 14/15$	**2**
8	Area = ½ bc sin A = ½ x 10 x 16 x sin $70^o = 75.18$ m^2	**2**
9	$y = 3x^5 \quad dy/dx = 15 x^4$	**2**
10	$y = 3 \sin 4x \quad dy/dx = 12 \cos 4x$	**2**
11	${}_0\int^4 6 x^2 \, dx = [2x^3]^4_0 = 2 \times 4^3 = 128$	**2**
12	$\int \cos 4x \, dx = (\sin 4x /4) +$ constant	**2**
13	Population – the total set of quantities under investigation Sample – a cross section taken from the total population	**2**
14	Mean diameter = (16.25 + 16.10 + 16.00 + 16.40 + 16.60 + 16.15)/6 = 16.25 mm.	**2**
15	Probability of selecting a 56 ohm resistor = 12/(12 + 24) = 1/3	**2**

Section A Total	**[30]**

Section B

Question Number	Answer	Max Mark
1	$6\ i_1 + 10\ i_2 = 2$ Equation 1 $9\ i_1 + 6\ i_2 = 12$ Equation 2 Multiply eqn 1 by 6 $36\ i_1 + 60\ i_2 = 12$ Equation 3 Multiply eqn 2 by 4 $36\ i_1 + 24\ i_2 = 48$ Equation 4 Subtract eqn 4 from eqn 3 $36\ i_2 = -36$ $i_2 = -36/36$ $i_2 = -1$ A Substitute i_2 into eqn 1 $6\ i_1 + (10 \times -1) = 2$ $6\ i_1 - 10 = 2$ $6\ i_1 = 2 + 10$ $6\ i_1 = 12$ $i_i = 2$ A	[10]
2	Given BM = $-50\ A^2 + 500\ A + 1000$. BM = 200 Nm So 200 = $-50\ A^2 + 500\ A + 1000$. Therefore $-50\ A^2 + 500\ A + 1000 = 200$ $-50\ A^2 + 500\ A + 1000 - 200 = 0$ $-50\ A^2 + 500\ A + 800 = 0$ Divide through by 10 then $-5\ A^2 + 50\ A + 80 = 0$ solve a quadratic equation of the type $ax^2 + bx + c = 0$ using the	

Question Number	Answer	Max Mark
	formula $x = [- b \pm \sqrt{(b^2 – 4ac)}]/ 2a$ Where a = - 5, b = 50 and c = 80 So in this case $A = [- 50 \pm \sqrt{(50^2 – 4 \times -5 \times 80)}]/ 2 \times -5$ $A = [- 50 \pm \sqrt{(2500 + 1600)}]/ -10$ $A = [- 50 \pm \sqrt{(4100)}]/ - 10$ $A = [- 50 \pm \sqrt{(4100)}]/ - 10$ $A = [- 50 \pm 64.03]/ - 10$ $A = [- 50 \pm 64.03]/ - 10$ Case one: $A = [- 50 + 64.03]/ - 10$ $A = 14.03/ -10$ $A = - 1.403$ Invalid response because of negative sign. Case two: $A = [- 50 - 64.03]/ - 10$ $A = -114.03/ -10$ $A = +11.403$ m Valid response because of the positive sign. The distance from the end of the beam is 11.403 metres.	[10]

3	**(a)** Figure 2  (**b)(i)** Angle C = 2π - (2/3)π = (4/3) π radians Wet length AB = Cr where r = radius of water main = 300 mm Wet length AB = (4/3) π x 300 = 400 π = 1256.6 mm The length of the circumference that is wet is 1256.6 mm long (**b)(ii)** Figure 3 - triangle BCE  Angle B = π/2 - π/3 = π/6 radians Angle C = π/3 radians cos (π/3) = x / 300	[2] [8]

	Therefore x = 300 cos (π/3) = 300 x 0.5 = 150 mm Depth of water = x + radius = 150 + 300 = 450 mm The depth of the water is 450 mm.	
4	**(a)** From triangle ABD 20/ sin 30° = 35/ sin D So sin D = (35 sin 30°)/ 20 = 0.875 Angle D = $\sin^{-1}0.875 = 61.05°$ Right hand side triangle Angle A = 180 – 30 – 61.05 = 88.95° **(b)** From triangle ABD BD/ sin 88.95° = 20/ sin 30° So BD = (20 sin 88.95°)/ sin 30° = 40 m **(c)** From triangle BCD Angle C = 180 – 60 – 61.05 = 58.95° **(d)** 40/ sin 58.95° = CD/ sin 60° So CD = (40 sin 60°)/ sin 58.95° = 40.44 m **(e)** Therefore Length AC = 40.44 – 20 = 20.44 m **(f)** From triangle BCD $BC^2 = BD^2 + CD^2 – 2$ BD. CD. cos D $BC^2 = 40^2 + 40.44^2 – (2 \times 40 \times 40.44) \cos 61.05$ $BC^2 = 1600 + 1635.4 – 1566$ $BC^2 = 1669.4$ BC = 40.86 m	[2] [2] [1] [2] [1] [2]
5	**(a)** Equation is: $s = t^3 - 4t^2 + 2t$ Velocity = ds/dt So $ds/dt = 3t^2 - 8t + 2$	

	To find the initial velocity of the vehicle $t = 0$ So if $ds/dt = 3t^2 - 8t + 2$ Then when $t = 0$ So $ds/dt = (3 \times 0^2) - (8 \times 0) + 2$ $= 0 - 0 + 2$ $= 2$ Initial velocity of the vehicle is 2 ms^{-1}	[4]
	(b) Equation is: $s = t^3 - 4t^2 + 2t$ $ds/dt = 3t^2 - 8t + 2$ When $t = 8$ seconds then $ds/dt = (3 \times 8^2) - (8 \times 8) + 2$ $= 192 - 64 + 2$ $= 130$ Velocity of the vehicle after 8 seconds is 130 ms^{-1}	[2]
	(c) Velocity = $ds/dt = 3t^2 - 8t + 2$ Acceleration = d^2s/dt^2 so $d^2s/dt^2 = 6t - 8$ When $t = 8$ seconds Then $d^2s/dt^2 = (6 \times 8) - 8$ $= 48 - 8$ $= 40$ Acceleration of the vehicle after 8 seconds is 40 ms^{-2}	[2]
	(d) Acceleration = $d^2s/dt^2 = 6t - 8$ In this case $d^2s/dt^2 = 0$ So $0 = 6t - 8$ $6t = 8$ $t = 8/6$ $= 1.333$ The time when the acceleration is zero is 1.333 s.	[2]

6

(a)

Pressure P = 120 x 10^3 Nm^{-2} and Volume (V) = 0.6 m^3

From $PV^{1.5} = C$.

$C = PV^{1.5}$

$= 120 \times 10^3 \times 0.6^{1.5}$

$= 120 \times 10^3 \times 0.4648$

$= 55771$

The constant C is 55771 [3]

(b)

From $PV^{1.5} = C$. $P = C/ V^{1.5} = CV^{-1.5}$ [1]

(c)

When the gas expands from 0.6 m^3 to 2.0 m^3 then

Work done (W) = $\int P.\, dV$

So Work done (W) = $\int CV^{-1.5}.\, dV$

Work done (W) = $_{0.6}{}^{2.0}\int CV^{-1.5}.\, dV$

$= [\, CV^{-0.5}/ - 0.5\,].\ _{0.6}{}^{2.0}$

$= C[(2.0^{-0.5}/ - 0.5) - (0.6^{-0.5}/ - 0.5)]$

$= C[- 1.4142 - (- 2.5812)]$

$= C[- 1.4142 + 2.5812]$

$= C\,[1.167]$

$= 55771 \times 1.167$

$= 65084.8$

The work done is 65084.8 J

7

Raw score (x)	Frequency (f)	fx	x – mean	$(x - \text{mean})^2$	$f(x - \text{mean})^2$
6.6	3	19.8	- 1.4	1.96	5.88
7.8	9	70.2	- 0.2	0.04	0.36
8.0	20	160	0	0	0
8.2	16	131.2	0.2	0.04	0.64
8.4	7	58.8	0.4	0.16	1.12

$\Sigma f = 55$

$\Sigma fx = 440$

	Mean = 440/55 = 8 $\Sigma f(x - \text{mean})^2 = 8$ Variance = $[\Sigma f(x - \text{mean})^2] / \Sigma f = 8 / 55 = 0.145$ Standard Deviation = $\sqrt{}$ variance = $\sqrt{}$ 0.145 = 0.38 correct to 2dp	[10]
8	Let P_t be the probability of failure due to temperature Then $P_t = 1/20$ and $P_t = 19/20$ Let P_v be the probability of failure due to vibration Then $P_v = 1/30$ and $P_v = 29/30$ Let P_m be the probability of failure due to moisture Then $P_m = 1/40$ and $P_m = 39/40$ **(a)** **(i)** The probability of the system failing due to extreme temperature and vibration is given by $P_t \times P_v$ Which equals 1/20 x 1/30 = 1/600 = 0.0016` **(ii)**The probability of the system failing due to extreme vibration or moisture is given by $P_v + P_m$ Which equals 1/30 + 1/40 = 7/120 = 0.0583 **(b)** The probability that the system will not fail due to extreme temperature and extreme vibration is given by $P_t \times P_m$ Which equals 19/20 x 29/30 = 551/600 = 0.9183	[7] [3]
	Any three questions for Section B Total	**[30]**
	Paper Total	**[60]**