Modified Enlarged 18pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Wednesday 8 June 2022 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes
plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet sent with the standard paper or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Booklet.
a scientific or graphical calculator
insert for Question 5a(ii) (with this document)

READ INSTRUCTIONS OVERLEAF

© OCR 2022 [603/0933/7]
DC (PQ/CB) 317774

1040157996

INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.

The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 75.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n$$

$(n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

SECTION A: Pure Mathematics

Answer ALL the questions.

1 Write the solution of the inequality $(x-2)(x+3) > 0$ using set notation. [2]

2 In this question you must show detailed reasoning.

Solve the equation $3x + 1 = 4\sqrt{x}$. [4]

3 Give a counter example to disprove the following statement.

If x and y are both irrational then $x + y$ is irrational. [2]

4 The circle $x^2 + y^2 - 6x + 4y + k = 0$ has radius 5.

Determine the value of k. [3]

5

The diagram shows a curve C for which y is inversely proportional to x. The curve passes through the point $\left(1, -\frac{1}{2}\right)$.

(a) (i) Determine the equation of the GRADIENT FUNCTION for the curve C. [3]

(ii) Sketch this GRADIENT FUNCTION on the axes in the insert or in the Printed Answer Booklet. [1]

(b) The diagram indicates that the curve C has no stationary points.

State what feature of your sketch in part (a)(ii) corresponds to this. [1]

(c) The curve C is translated by the vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$.

Find the equation of the curve after it has been translated. [2]

6

The shape ABC shown in the diagram is a student's design for the sail of a small boat.

The curve AC has equation $y = 2\log_2 x$ and the curve BC has equation $y = \log_2\left(x - \frac{3}{2}\right) + 3$.

(a) State the x-coordinate of point A. [1]

(b) Determine the x-coordinate of point B. [3]

(c) By solving an equation involving logarithms, show that the x-coordinate of point C is 2. [4]

It is given that, correct to 3 significant figures, the area of the sail is 0.656 units^2.

(d) Calculate by how much the area is over-estimated or under-estimated when the curved edges of the sail are modelled as straight lines. [4]

7

The diagram shows the parallelogram $OACB$ where $\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j}$ and $\overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j}$.

(a) Show that $\cos AOB = -\dfrac{2\sqrt{5}}{25}$. [5]

(b) Hence find the exact value of $\sin AOB$. [2]

(c) Determine the area of $OACB$. [2]

8 (a) The quadratic polynomial $ax^2 + bx$, where a and b are constants, is denoted by $\mathrm{f}(x)$.

Use differentiation from first principles to determine, in terms of a, b and x, an expression for $\mathrm{f}'(x)$. [4]

(b)

The diagram shows the quadratic curve $y = ax^2 + bx$, where a and b are constants. The shaded region is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

The tangent to the curve at $x = 4$ intersects the x-axis at the point with coordinates $(k, 0)$.

Given that the area of the shaded region is 9 units^2, and the gradient of this tangent is $-\frac{3}{4}$, determine the value of k. [7]

BLANK PAGE

SECTION B: Mechanics

Answer ALL the questions.

9 Two forces $(3\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}$ N act on a particle P of mass 4 kg.

Given that the acceleration of P is $(-2\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-2}}$, calculate $\mathbf{F}$. [2]

10 A small ball B is projected vertically upwards from a point 2 m above horizontal ground. B is projected with initial speed $3.5\,\mathrm{m\,s^{-1}}$, and takes t seconds to reach the ground.

Find the value of t. [3]

11

A particle P moves along the x-axis. At time t seconds, where $t \geqslant 0$, the velocity of P in the positive x-direction is v m s^{-1}. It is given that $v = t(t-3)(8-t)$.

P attains its maximum velocity at time T seconds. The diagram shows part of the velocity-time graph for the motion of P.

(a) State the acceleration of P at time T. [1]

(b) In this question you must show detailed reasoning.

Determine the value of T. [5]

(c) Find the total distance that P travels between times $t = 0$ and $t = T$. [3]

12

Particles P and Q, of masses 4 kg and 6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is in equilibrium with P hanging 1.75 m above a horizontal plane and Q resting on the plane. Both parts of the string below the pulley are vertical (see diagram).

(a) Find the magnitude of the normal reaction force acting on Q. [1]

The mass of P is doubled, and the system is released from rest. You may assume that in the subsequent motion Q does not reach the pulley.

(b) Determine the magnitude of the force exerted on the pulley by the string before P strikes the plane. [5]

(c) Determine the total distance travelled by Q between the instant when the system is released and the instant when Q first comes momentarily to rest. [4]

When this motion is observed in practice, it is found that the total distance travelled by Q between the instant when the system is released and the instant when Q first comes momentarily to rest is less than the answer calculated in part (c).

(d) State ONE factor that could account for this difference. [1]

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.