OCR
RECOGNISING ACHIEVEMENT

SAMPLE

OCR Level 3 Certificate in Mathematics for Engineering

UNIT H860

Component 2

Sample Paper

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Write your answers in black ink

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 40
- The use of calculators **is** allowed for this paper
- Include **all** working out where used
- **You are reminded of the need for clear presentation in your answers.**

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.

This document consists of 5 printed pages.

Please read the text below and answer **ALL** the questions.

Railway Safety

The railway system in the UK is growing: more passengers are being carried year on year, new lines are being built (such as the high-speed line from the Channel Tunnel that terminates in St Pancras station shown here) and new rolling stock being added to the train fleet.

Railways offer a relatively low-carbon means of moving a large number of people from point to point quickly and safely. The driver of a commuter train might well be doing the job of 15 bus drivers or 1000 individual car drivers!

Train drivers are subject to rigid external control on how they can drive their trains. This is in contrast to car or bus drivers who are largely autonomous: they watch the other traffic and then decide what action to take. The most obvious difference is that a train does not have freedom of steering but is guided by the rails and points. However, a more significant difference is that the train driver is told what to do by signals; specifically when to slow down and when to stop.

The Stopping Distance of a Train

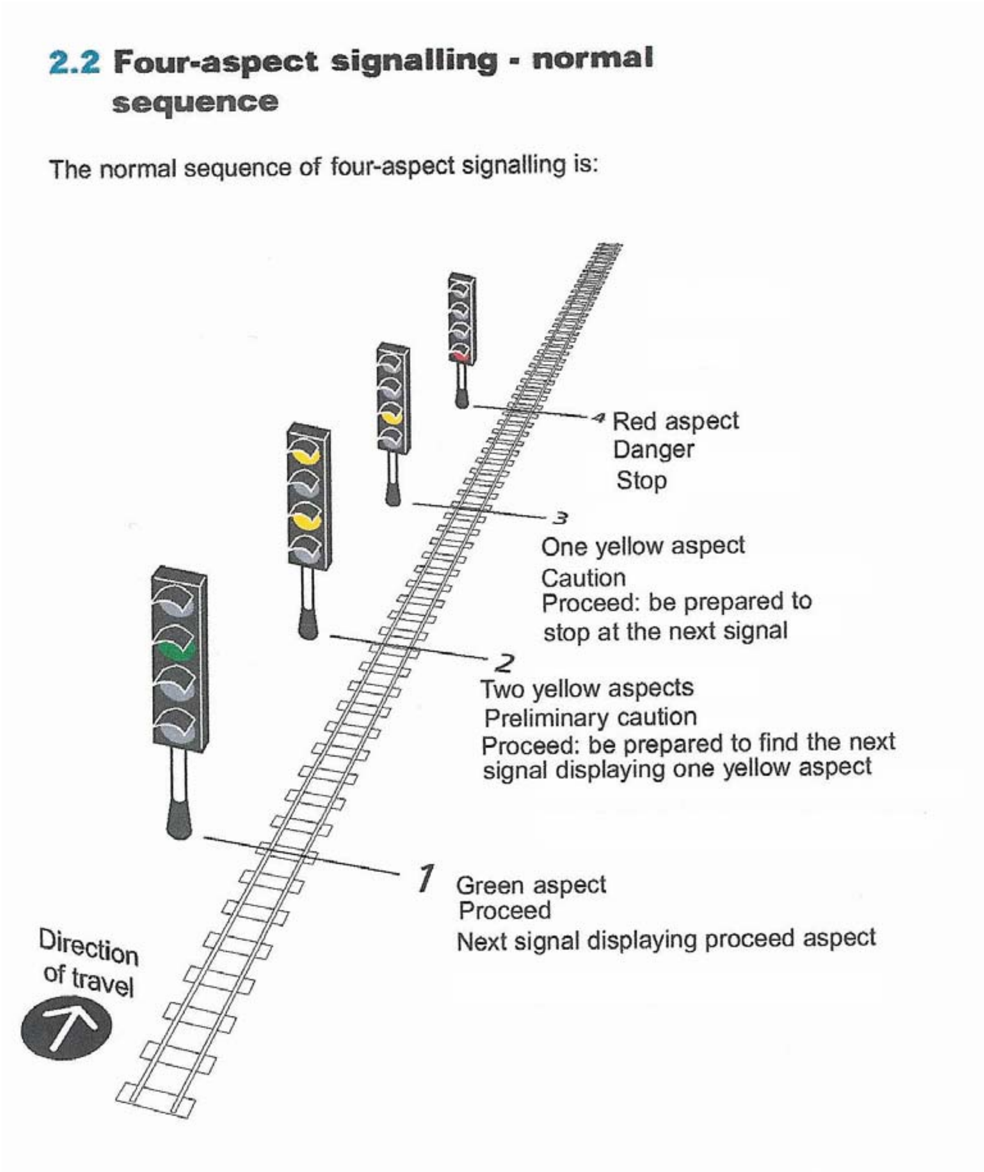
The reason that a train is controlled by signals, rather than relying on the driver reacting to seeing a train up ahead, is that it takes much longer to stop a train than a bus or a car. If a driver waited to see a stationary train ahead before slowing down, their train would almost certainly run into the back of the one ahead.

This is not because a train is longer or heavier than a lorry but because the coefficient of friction between its steel wheels and the steel rails on which it runs is very much less than that between the rubber tyres of a road vehicle and the road surface. The greatest braking force which either may experience is equal to the weight of the train or lorry multiplied by the coefficient of friction.

A typical coefficient of friction for the train wheel / rail interface is about 0.1, but in good conditions it may be as high as 0.25. By contrast that for a lorry on the road is nearly 1. A deceleration of 0.7 m/sec^2 is usual for a train slowing down normally, and in an emergency 1 m/sec^2 is typical. In a passenger train, there is the danger of people falling over and being injured if the deceleration is too rapid.

In very poor conditions, such as wet rails covered in leaves, the coefficient of friction may be somewhat less than 0.1 and it may be necessary in the interest of safety for it to run at below normal speed, inevitably arriving late. A few trains use rubber tyres but this practice produces many other problems and their use is confined to certain underground systems.

Four Aspect Signalling



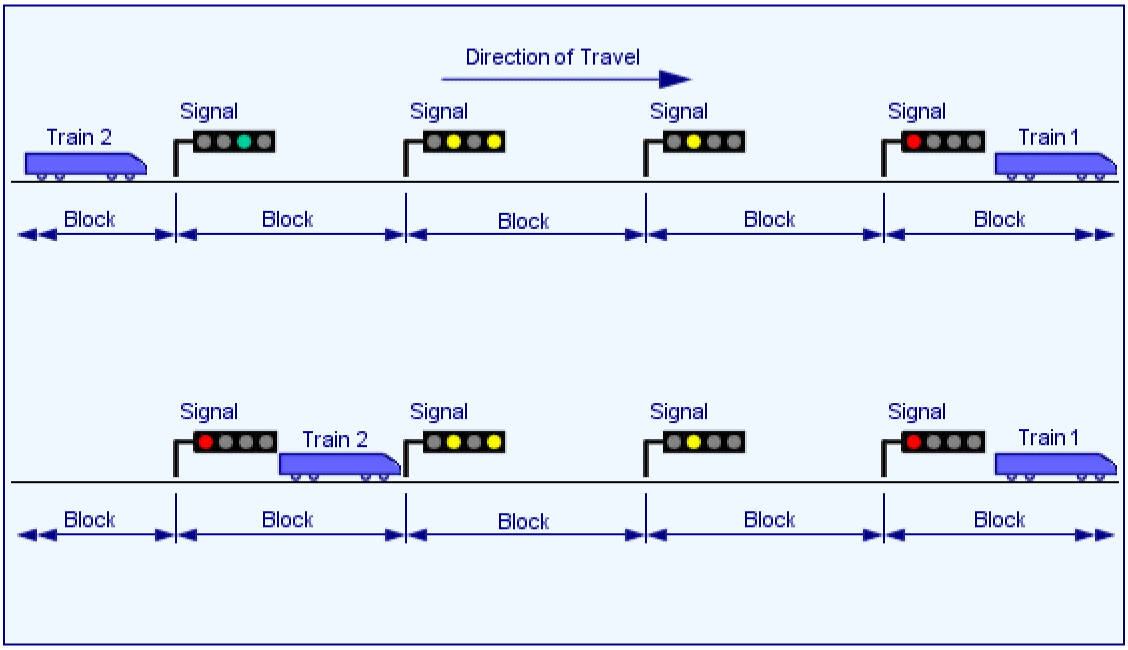
To ensure that one train does not run into the back of another, railway operators divide their lines into blocks the entrance to each one being controlled by a signal. In normal running, there must be at least three clear blocks between trains.

As a train enters a new block, it passes a signal. If this is green it means that this block and the two beyond are clear and the train can continue at full speed. If, however, the way ahead is not clear, the driver is instructed to slow down and stop by a sequence of signals:

Green:	Next three blocks are clear. Continue at full speed
Double Yellow:	There is a train three blocks ahead. Slow down.
Single Yellow:	There is a train two blocks ahead. Continue to slow down.
Red:	There is a train in the next block. Stop here.

Consequently, a train must be able to stop within two blocks after passing a double yellow signal. This requirement determines the relationship between the design speed of the track and the length of a block.

The Capacity of a Railway Line



Inspecting the diagram above, it would seem that the most efficient use of a railway line would involve trains travelling one after another at slightly over 3 block intervals. But of course this becomes difficult to maintain every time a train stops at a station which can have an impact on the train behind depending on how many blocks separate the two. Ideally, trains are free to run at or near the design speed for the line in order to minimise journey time and to avoid wasted energy through repeated acceleration and braking cycles. This reduces the number of trains that can run along the line in any given time interval: this number is the capacity of the line.

Questions

1. The railway line between two towns runs horizontally on a straight track and has a design speed of 40 m/sec. The trains using the line have length 100 m. When slowing down, trains have a deceleration of 0.7 m/sec^2.

   (a) Calculate the length of a one block of track.

   [4]

   (b) A train travelling initially at 40 m/sec makes an emergency halt, coming to rest in 1300 metres. Estimate the coefficient of friction between its wheels and the rails.

   [6]

2. The speed, v in m/s, of a train leaving a station can be modelled by the equation:

   $v = 5\sqrt{t}$, where t is time in seconds.

   (a) Sketch a graph of this information for $0 \le t \le 100$.

   [1]

   (b) The train has stopped at a railway station.

   (i) Find the time taken for the train to travel a distance of 1200m after leaving the station.

   [6]

   (ii) Calculate the acceleration at the time when the train has travelled 1200m from the station..

   [3]

3 A train T_1 sets out from a terminus station towards London. This service has 12 scheduled station stops on the way which are equally spaced. An express train T_2 sets off 15 minutes later and is not scheduled to stop at any stations on the way to London. Assume that the line speed is 25 m/sec, and that each train has an acceleration rate of 0.2 m/sec^2, a braking rate of 0.5 m/sec^2, and that train T_1 will wait at every station for 30 seconds.

How many stations will the driver of the express train T_2 pass through before he sees the first 'Double Yellow' signal?

[10]

4 A train operator for a particular line is buying some new rolling stock. The trains supplied by one company accelerate in such a way that their speed, in m/s, varies with time in seconds from a standing start according to the expression:

$$v_1(t) = 35\left[\frac{t^2}{1+t^2}\right].$$

However, the expression for the speed of trains from a competitor supplier is

$$v_2(t) = 50\left[1 - e^{-0.02t}\right].$$

(a) How far will each train travel in 30 seconds from a standing start?

[8]

(b) Which train would you recommend to the operator and why?

[2]

Paper Total: [40]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Level 3 Certificate in Mathematics for Engineering

Component 2

Unit H860

Sample Mark Scheme

The maximum mark for this paper is 40.

Question	Marks

Q1 Train's design speed = 40/msec
Deceleration = 0.7 m/sec^2

(a) We need to calculate the length of a one block of track.

With u = 40 m/sec, if train has deceleration of 0.7 m/sec^2, it will come to rest, ie $\upsilon = 0$ after covering distance s.

$v^2 = u^2 - 2as$

$0 = 40^2 - 2 \times 0.7 \times s$ M1

$\Rightarrow \quad s = \frac{40 \times 40}{2 \times 0.7} = 1142.86$ A1

According to the text, the distance covered by the train before stopping, should be at least 3 blocks. B1

$\therefore$ length of one block = $\frac{1142.86}{3} \cong 380.95$

$\cong 381$ m accept 400m A1 ft

AC1.1/AC4.1 (4)

(b) u = 40 m/sec, v = 0, distance covered before stopping = 750m.

$v^2 = u^2 + 2as$

$0 = 1600 - 2 \times a \times 750$ M1

$a = \frac{1600}{1500} = 1.07$ A1

To find the coefficient of friction:-

Use of friction = μN M1

Use of F = ma M1

Combine to get

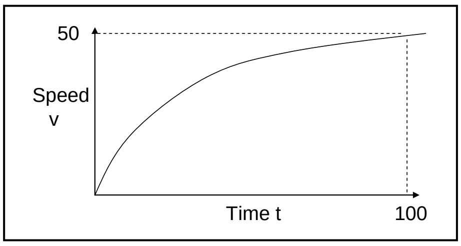
$\mu = \frac{a}{g} = \frac{1.07}{9.8}$ M1

= $\cong 0.109$ (quite reasonable) A1

AC1.1/AC7.1 (6)

Question	**Marks**

2. (a)



B1

AC4.1 (1)

(b) (i) $\int_0^T 5\sqrt{t}\,dt$ (= 1200) M1

$\left[5\times\frac{2}{3}\times t^{\frac{3}{2}}\right]_0^T = \frac{10}{3}T^{\frac{3}{2}} = 1200$ dealing with limits M1

$\frac{2}{3}\times t^{\frac{3}{2}}$ seen A1

expression for ‘T’ = 1200 A1

$T = (360)^{\frac{2}{3}}$ attempt to solve M1

= 50.6 accept 50 or 51 A1

(ii) Acceleration = $\frac{dv}{dt} = \frac{5}{2}\times t^{-\frac{1}{2}} = \frac{5}{2\times\sqrt{50.6}} = 0.3514$ m/s^2 differentiation M1

$\frac{5}{2}\times t^{-\frac{1}{2}}$ seen A1

= 0.351(4) A1

AC 6.3 (6),AC6.2 (3)

Question	Marks

3 Accept alternative approaches.

Method – work out the 'delay' to train T_1 in braking to a stop, waiting 30 seconds at a station and accelerating back up to normal speed — M1

Braking $u = 25$, $a = -0.5$, $v = 0$

$v = u + at$

$0 = 25 - 0.5t$, $t = 50$ — B1

$v^2 = u^2 + 2as$

$0 = 625 - s \qquad s = 625$ — B1

Waiting $t = 30$, $s = 0$

Accelerating $25 = 0 + .2t$, $t = 125$

$625 = 0 + 2 \times .2 \times s \qquad s = \frac{625}{.4} = 1562.5$ — B1 B1

T_1 distance 2187.5 m in 205 seconds — M1

T_2 travels 2187.5 m in 87.5 seconds — M1

T_1 Takes 117.5 seconds extra to stop at station — A1

T_2 is 900 seconds later, number of stations to catch up is $\frac{900}{117.5}$ — M1

T_2 would catch up in 7.65 stations which is approximately 7 stations — A1

Note:
More accurate answers may take account of the double yellow showing 2 blocks ahead,
So distance travelled by T_2 is 2 blocks less than T_1
1 block is 210 m, T_2 does not travel 420m
Double yellow is 17 seconds earlier
$\frac{873}{117.5} = 7.42$ which is approximately 7 stations

AC1.1 (10)

Question	Marks

Q4

Let $\upsilon\ (t) = 35\left[\frac{t^2}{1+t^2}\right]$ and

$\upsilon\ (t) = 50\left[1 - e^{-0.02t}\right]$

(a) Distance travelled by train 1,

$S_1 = {}_0\int^{30} \upsilon_1 1\ (t)\ dt$ — M1 for unit

M1 for limit, either train

$= 35\ {}_0\int^{30}\left(\frac{t^2}{1+t^2}\right)dt$

Let $t = \tan\ \theta \quad \Rightarrow \quad t^2 = \tan^2\theta$

$\Rightarrow \quad 1+t^2 = 1 + \tan^2\theta = \sec^2\ \theta$

Also $dt = \sec^2\ \theta\ \mathrm{d}\theta$

when $t = 0 \quad \Rightarrow \quad \tan\ \theta = 0 \Rightarrow \theta = 0$

when $t = 30 \Rightarrow \tan\ \theta = 30 \Rightarrow \theta = \tan^{-1}(30)$ — M1

accept any alternative method.

$\therefore\ S_1 = 35\ {}_0\int^{\tan^{-1}(30)} \frac{\tan^2\theta}{\sec^2\theta}\ \sec^2\theta\ d\theta$

$= 35\ {}_0\int^{\tan^{-1}(30)} \tan^2\theta\ d\theta$ — M1

$= 35\ {}_0\int^{\tan^{-1}(30)} (\sec^2\theta - 1)\ d\theta$

$= 35\left[\tan\theta - \theta\right]_0^{30}$

$= 35\left[\tan\ \tan^{-1}(30) - \tan^{-1}(30) - \tan\ 0 + 0\right]$

$= 35\left[30 - \tan^{-1}(30)\right]$

$\cong\ 996$ m — A1

$S_2 = {}_0\int^{30} \upsilon_2(t)\ dt$

$= 50\ {}_0\int^{30}\left(1 - e^{-0.02t}\right)\ dt$

$$= 50\left[t + \frac{e^{-0.02t}}{0.02}\right]_0^{30}$$ M1

$$= 50\left[30 + \frac{e^{-0.6}}{0.02}\right] - 0 - \frac{e^{0}}{0.02}$$

$$= 50\left[30 + \frac{1}{0.02}\left(e^{-0.6} - 1\right)\right]$$ M1

$$= 50\left[30 + 50\left(0.5488 - 1\right)\right]$$

$\cong 372$ m A1

AC 6.3 (8)

(b) The first train can travel 996m in 30 seconds whereas the second train travels approximately 372m. M1

So, I will recommend the first company rolling stock to the operator as with these product train can travel more in same time. A1

AC1.1 (2)

	LO1		LO2			LO3		Lo4		LO5			LO6				LO7					LO8			LO9			LO10	
AC →	**1.1**	**1.2**	**2.1**	**2.2**	**2.3**	**3.1**	**3.2**	**4.1**	**4.2**	**5.1**	**5.2**	**5.3**	**6.1**	**6.2**	**6.3**	**6.4**	**7.1**	**7.2**	**7.3**	**7.4**	**7.5**	**8.1**	**8.2**	**8.3**	**9.1**	**9.2**	**9.3**	**10.1**	**Total marks for question**
Question																													
1 (a)	X							X																					**4**
1 (b)	X																X												**6**
2 (a)								X																					**1**
2 (b)														X	X													X	**9**
3	X																												**10**
4 (a)															X													X	**8**
4 (b)	X																												**2**
Total marks for AC	17							3						3	4		3											10	**40**