OCR
RECOGNISING ACHIEVEMENT

Friday 13 January 2012 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/01 Paper 1

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 8 page Answer Booklet (sent with general stationery)
- Graph paper
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** A manufacturer mass produces filament lamps and experience shows that the standard deviation of the lifetimes of the lamps remains constant.
Last year the manufacturer conducted a test on a random sample of 2000 lamps. The times, t hours, that the lamps lasted before they failed are summarised in Table 1.

Time in hours	Number failing
$t \leqslant 200$	0
$200 < t \leqslant 400$	50
$400 < t \leqslant 600$	300
$600 < t \leqslant 800$	420
$800 < t \leqslant 1000$	500
$1000 < t \leqslant 1200$	400
$1200 < t \leqslant 1400$	250
$1400 < t \leqslant 1600$	50
$1600 < t \leqslant 1800$	30
$1800 < t$	0

Table 1

(i) Draw a histogram to represent the results shown in the table. **[2]**

(ii) Estimate the mean and standard deviation of the lifetime of the lamps in this sample. Give your answers correct to two significant figures. **[5]**

(b) The quality control department now suspects that the mean lifetime of the lamps has increased since the test last year. They select a random sample of 50 lamps and find that the average lifetime is 975 hours.

Based on these results, test the hypothesis, at the 1% significance level, that lamps produced now have an increased mean lifetime.

You may assume that the mean and standard deviation calculated in part **(a)** are the values for the entire population. **[4]**

2 **(a)** A right-angled triangle, HAG, has fixed hypotenuse HG = h. Prove that the area of HAG is maximised when the lengths of the two perpendicular sides, AH and AG, are equal so that

$$AH = AG = \frac{h}{\sqrt{2}}.$$ **[4]**

(b) A diagram of a roof-supporting structure containing straight, rigid members is shown in Fig. 2. The lengths of members HE, GE and HG are 3 m, 2 m and 4 m respectively. The structure is symmetrical about the line AH, and such that the area of the triangle FHG is maximised.

(i) Determine the height of the structure, AH. **[1]**

(ii) Calculate the angles $G\hat{H}E$ and $H\hat{G}E$. **[3]**

(iii) Calculate the length BC. **[2]**

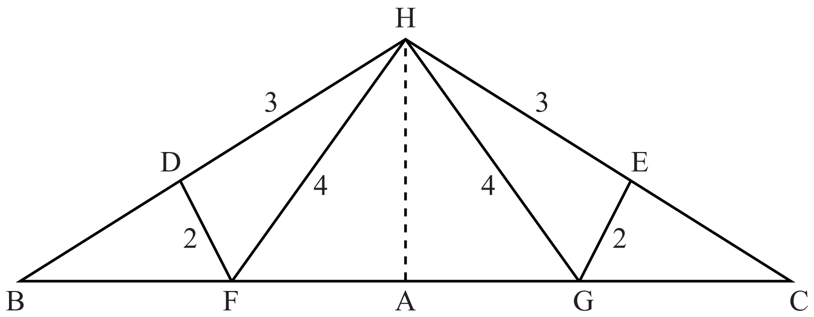

Fig. 2

3 A racing car team is testing a set of new tyres on a car which has a mass of 650 kg (including tyres).

(a) The team needs to determine the coefficient of friction between the four tyres and a test track. In order to do this they place the car on a straight, level section of track with the wheels firmly locked and tow the car with a tractor and rope for a short distance at a constant velocity. A gauge indicates that the tension in the rope during the test remains constant at 7000 N.

By modelling the situation as the whole weight of the car acting through a single tyre, calculate the coefficient of friction between the tyres and the track. **[2]**

(b) The racing car with the same tyres is now driven along a straight, level section of the track until it reaches a speed of 40 m s^{-1}. At this speed the brakes are fully applied and the wheels lock, causing the car to slide in a straight line in the same direction it was originally travelling.

Assuming that the coefficient of friction between the tyres and the track remains constant at the value found in part **(a)**, determine the distance the car will slide along the track. **[3]**

(c) In another test, the car is driven at a constant speed of 25 m s^{-1} down a straight section of the track that slopes uniformly at 10° to the horizontal. The brakes are then applied and the wheels lock, causing the car to slide in a straight line in the same direction it was originally travelling.

Assuming that the coefficient of friction continues to have the value found in part **(a)**, determine the distance the car will slide after the brakes are applied and the time it will take for the car to come to rest. **[4]**

4 In this question take acceleration due to gravity, g, to be 10.

Fig. 4 shows a projectile launched at an angle of $\alpha°$ from the edge of a cliff which is H m above sea level; the initial velocity is $v_o \text{ m s}^{-1}$. It is assumed that the only force acting on the projectile while it is in motion is due to gravity (hence air resistance can be neglected).

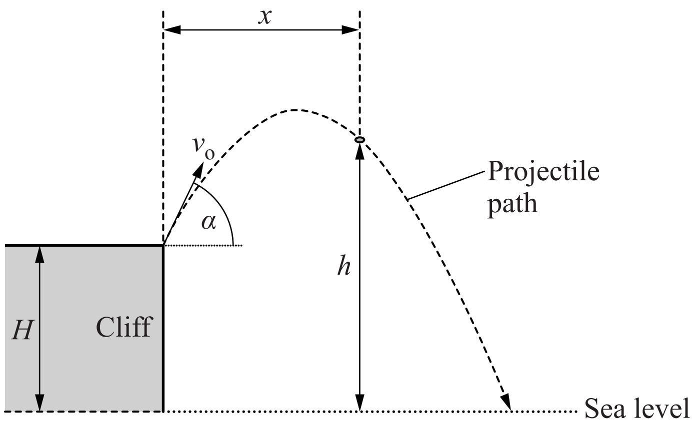

Fig. 4

(a) Show that the height of the projectile above sea level, h m, at time t s after projection, while it is in motion, is given by

$$h = -\frac{gt^2}{2} + v_o t \sin\alpha + H.$$ **[3]**

(b) The relationship between the horizontal distance, x m, travelled by the projectile and the time, t s, after projection is $x = v_o t \cos\alpha$.

Eliminate t from the expression for h given in part **(a)** and determine a formula in the form $h = f(x)$ that relates the height of the projectile, h m, to the horizontal distance travelled, x m, while it is in motion. **[2]**

(c) The projectile is launched at an angle of 30° from the top of a 20 m high cliff and reaches sea level at a distance of 100 m from the base of the cliff.

(i) At what velocity is the projectile launched? **[3]**

(ii) What is the maximum height of the projectile above sea level during its flight? **[2]**

(iii) At what time after its launch does the projectile reach sea level? **[3]**

5 A large, circular container with a curved profile, approximately that shown in Fig. 5, is being filled with liquid at a constant rate. The container is initially empty and the height of the liquid, h m, is measured each minute for three minutes. The measurements recorded are shown in Table 5.

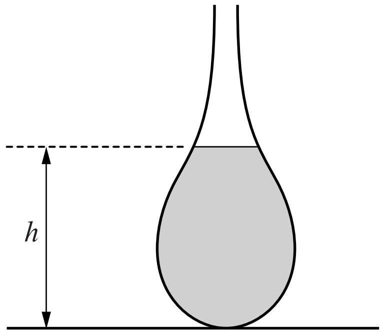

Fig. 5

Time t minutes	1	2	3
Height h metres	0.3	0.5	1.2

Table 5

The relationship between the height of the liquid, h, and time, t, is defined by the cubic equation

$$h = at^3 + bt^2 + ct + d \qquad \text{for } 0 \leqslant t \leqslant 3.$$

(a) Use the information provided to determine the values of the constants a, b, c and d. **[4]**

(b) Explain why $\frac{dh}{dt}$ has a minimum value when the diameter of the container is a maximum. **[1]**

(c) By finding the time at which $\frac{dh}{dt}$ is a minimum, calculate the height of the container where the diameter is a maximum. **[3]**

(d) The average value, $\overline{h}$, of the function $h = \mathrm{f}(t)$ in the interval $t_1 \leqslant t \leqslant t_2$ is given by

$$\overline{h} = \frac{\int_{t_1}^{t_2} \mathrm{f}(t)\,dt}{t_2 - t_1}\ .$$

Use this definition to calculate the average height of the liquid during the three-minute interval. **[2]**

Turn over

6 For this question you are given that the volume, V, created by rotating the region of the plane shaded in Fig. 6a, through 360° about the y-axis is given by

$$V = 2\pi\int_0^b x\mathrm{f}(x)\,\mathrm{d}x$$

where $\mathrm{f}(x)$ is the function that describes the perimeter of the shaded region from A to B.

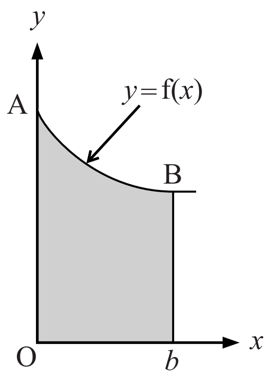

Fig. 6a

A small, circular paperweight has a cross section as shown in Fig. 6b and is symmetrical about its centre line. The profile of the cross section has the equation

$$y = h\cos^2\frac{x\pi}{w}$$

where $h = 40$ mm and $w = 80$ mm.

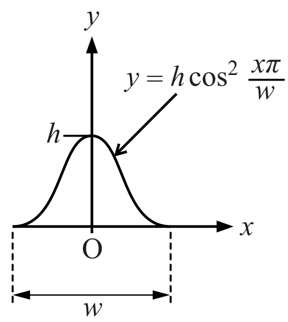

Fig. 6b

(a) Show that the volume of the paperweight, V mm^3, is given by

$$V = 80\pi\int_0^{40} x\cos^2\frac{\pi x}{80}\,\mathrm{d}x.$$ **[1]**

(b) Calculate the volume of the paperweight.

You should use the identity $\cos^2 \mathrm{A} = \frac{1}{2}(\cos 2\mathrm{A} + 1)$. **[6]**

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