OCR
RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/02 Paper 2

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 8 page Answer Booklet (sent with general stationery)
- Insert (inserted)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **40**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** Fig. 1 below shows a diagram of a circuit consisting of five resistors and a 20 V DC voltage source.

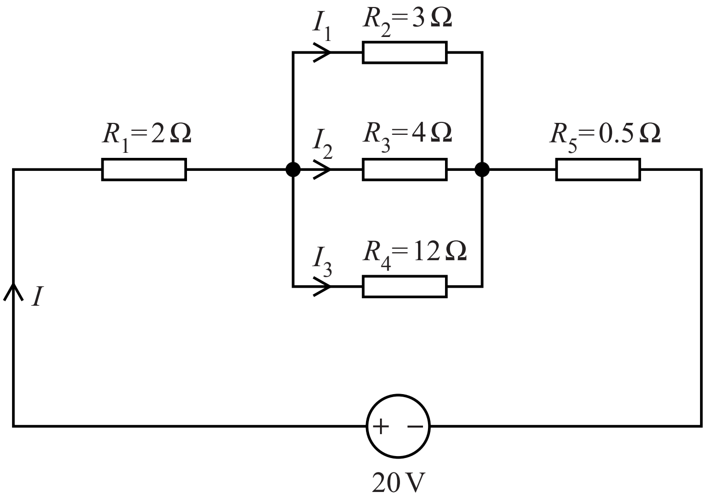

Fig. 1

Calculate

(i) the total resistance, R, **[2]**

(ii) the total current, I, **[1]**

(iii) the potential difference across each resistor, **[2]**

(iv) the currents I_1, I_2 and I_3. **[2]**

(b) The total current, I, flowing through the circuit shown in Fig. 1 is controlled by changing only the value of resistance R_3. The supply voltage remains at 20 V.

(i) What are the maximum and minimum currents possible for the whole circuit for different values of R_3? **[2]**

(ii) What is the value of R_3 when the current for the whole circuit is 6 A? **[3]**

2 **(a)** Fig. 2 below shows a diagram of a circuit containing a resistor with resistance R, a capacitor with capacitance C, a constant DC supply voltage, V, and a single-pole two-way switch.

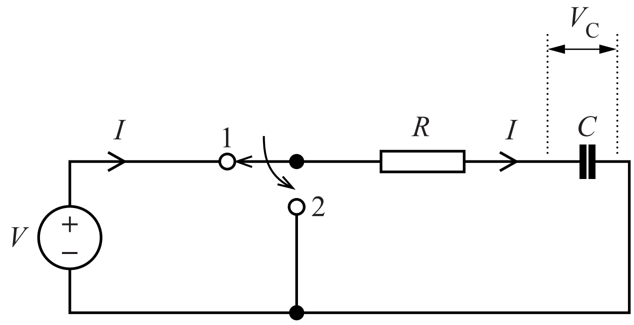

Fig. 2

The differential equation connecting V_C and t is

$$\frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{V}{RC}$$

where t is time in seconds and V_C is the potential difference across the capacitor.

By separating variables and integrating show that

$$V_C = V - Ke^{-\frac{t}{RC}}$$

where K is a constant. **[4]**

(b) Assume that in Fig. 2, $C = 5\,\mu F$, $R = 20\,k\Omega$ and $V = 20$ volts. With the switch in position 1 the capacitor is fully charged and $V_C = 20$ volts. At time $t = 0$, the switch is moved to position 2 and the capacitor is allowed to discharge fully through the resistor.

How long will it take for the potential difference across the capacitor to reduce to 10 volts? **[4]**

3 Fig. 3 shows a diagram of a circuit containing a resistor with resistance R, a capacitor with capacitance C, an inductor with inductance L, a constant DC voltage source, V, and a single-pole two-way switch.
Initially the switch is in position 2; the capacitor is completely discharged and there is no current flowing.

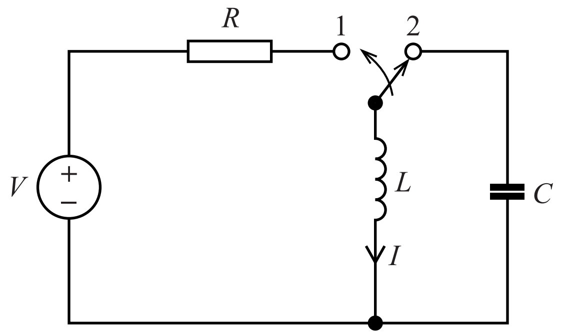

Fig. 3

(a) At time $t = 0$ the switch is moved to position 1 and a current, I, begins to flow through the resistor and the inductor. This current can be modelled by the differential equation

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{V}{L}.$$

Solve this equation **using the integrating factor method** and show that, while the switch remains in position 1,

$$I = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right).$$ **[5]**

(b) As time passes, the change in the flow of current becomes negligible. When this occurs the switch is moved back to position 2. At this time the inductor and capacitor are connected in a closed loop as described in the pre-release document and can be modelled by the differential equation

$$L\frac{d^2I}{dt^2} + \frac{I}{C} = 0. \qquad (*)$$

Given $\frac{dI}{dt} = 0$ when $t = 0$ and by finding the initial condition on I, use the appropriate solution, given in the appendix of the pre-release document, of the differential equation (*) to show that

$$I = \frac{V}{R}\cos\omega t$$

where $\omega = \frac{1}{\sqrt{LC}}$ and t is time measured from the instant at which the switch was moved from position 1 to position 2. **[5]**

4 Fig. 4 shows a diagram of a circuit containing a resistor with resistance R, a capacitor with capacitance C, an inductor with inductance L, a single-pole two-way switch and a constant DC voltage source, V.

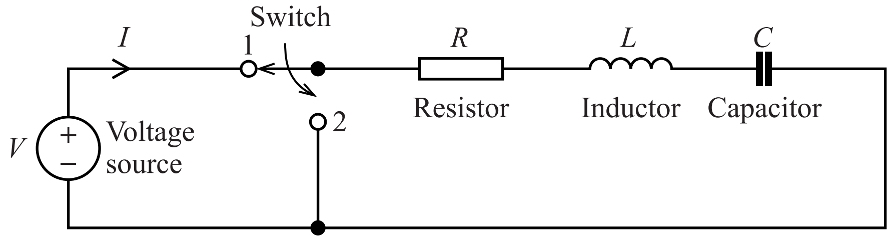

Fig. 4

(a) Kirchhoff's voltage law states that when the switch is in position 1,

$$V_R + V_L + V_C - V = 0$$

where V_R, V_L and V_C are the potential differences across the resistor, the inductor and the capacitor respectively.

In this case show that the following relationship applies.

$$\frac{dI}{dt} = \frac{1}{L}(V - IR - V_C)$$

where I is the current flowing and t is time. **[2]**

(b) For the circuit shown in Fig. 4, $L = 1\,\text{H}$, $R = 10\,\Omega$ and $C = 40\,\text{mF}$.

(i) Write down the second order differential equation that models the current flowing around the circuit. **[2]**

(ii) Use the information in the appendix of the pre-release document to find the general solution to the equation involving two arbitrary constants. **[2]**

(c) Assume that in Fig. 4, V is constant at 20 V and the switch has been in position 2 for a sufficient length of time for no current to be flowing. The switch is then moved to position 1 at time $t = 0$ and left in that position.

Using your solution to part **(b)(ii)** and appropriate initial conditions, determine the formula that relates the subsequent current flow, I, to time, t. **[4]**

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