RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

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MARK SCHEME FOR THE UNITS

4751 (C1) Introduction to Advanced Mathematics

Section A

1	$x > 6/4$ o.e. isw	2	M1 for $4x > 6$ or for 6/4 o.e. found or for their final ans ft their $4x > k$ or $kx > 6$	2
2	(i) (0, 4) and (6, 0) (ii) −4/6 o.e. or ft their (i) isw	2 2	1 each; allow $x = 0$, $y = 4$ etc; condone $x = 6$, $y = 4$ isw but 0 for (6, 4) with no working 1 for $-\frac{4}{6}x$ or 4/−6 or 4/6 o.e. or ft (accept 0.67 or better) 0 for just rearranging to $y = -\frac{2}{3}x + 4$	4
3	(i) 0 or −3/2 o.e. (ii) $k < -9/8$ o.e. www	2 3	1 each M2 for $3^2\ (-)(-8k) < 0$ o.e. or −9/8 found or M1 for attempted use of $b^2 - 4ac$ (may be in quadratic formula); SC: allow M1 for $9 - 8k < 0$ and M1 ft for $k > 9/8$	5
4	(i) T (ii) E (iii) T (iv) F	3	3 for all correct, 2 for 3 correct. 1 for 2 correct	3
5	$y(x - 2) = (x + 3)$ $xy - 2y = x + 3$ or ft [ft from earlier errors if of comparable difficulty – no ft if there are no xy terms] $xy - x = 2y + 3$ or ft $[x =]\dfrac{2y+3}{y-1}$ o.e. or ft <u>alt method:</u> $y = 1 + \dfrac{5}{x-2}$ $y - 1 = \dfrac{5}{x-2}$ $x - 2 = \dfrac{5}{y-1}$ $x = 2 + \dfrac{5}{y-1}$	M1 M1 M1 M1 M1 M1 M1 M1	for multiplying by $x - 2$; condone missing brackets for expanding bracket and being at stage ready to collect x terms for collecting x and 'other' terms on opposite sides of eqn for factorising and division for either method: award 4 marks only if fully correct	4

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6	(i) 5 www (ii) $8x^{10}y^{13}z^4$ or $2^3x^{10}y^{13}z^4$	2 3	allow 2 for ±5; M1 for $25^{1/2}$ seen or for 1/5 seen or for using $25^{1/2} = 5$ with another error (ie M1 for coping correctly with fraction and negative index or with square root) mark final answer; B2 for 3 elements correct, B1 for 2 elements correct; condone multn signs included, but −1 from total earned if addn signs	5
7	(i) $\frac{5-\sqrt{3}}{22}$ or $\frac{5+(-1)\sqrt{3}}{22}$ or $\frac{5-1\sqrt{3}}{22}$ (ii) 37 − 12√ 7 isw www	2 3	or $a = 5$, $b = -1$, $c = 22$; M1 for attempt to multiply numerator and denominator by $5-\sqrt{3}$ 2 for 37 and 1 for − 12√ 7 or M1 for 3 correct terms from 9 − 6√ 7 − 6√ 7 + 28 or 9 − 3√ 28 − 3√ 28 + 28 or 9 − √ 252 − √ 252 + 28 o.e. eg using 2√ 63 or M2 for 9 − 12√ 7 + 28 or 9 − 6√ 28 + 28 or 9 − 2√ 252 + 28 or 9 − √ 1008 + 28 o.e.; 3 for 37 − √ 1008 but not other equivs	5
8	−2000 www	4	M3 for $10 \times 5^2 \times (-2[x])^3$ o.e. or M2 for two of these elements or M1 for 10 or (5×4×3)/(3×2×1) o.e. used [$^5C_3$ is not sufficient] or for 1 5 10 10 5 1 seen; or B3 for 2000; condone x^3 in ans; equivs: M3 for e.g $5^5 \times 10 \times \left(-\frac{2}{5}[x]\right)^3$ o.e. [5^5 may be outside a bracket for whole expansion of all terms], M2 for two of these elements etc similarly for factor of 2 taken out at start	4
9	$(y-3)(y-4)$ [= 0] $y = 3$ or 4 cao $x = \pm\sqrt{3}$ or ± 2 cao	M1 A1 B2	for factors giving two terms correct or attempt at quadratic formula or completing square or B2 (both roots needed) B1 for 2 roots correct or ft their y (condone √ 3 and √ 4 for B1)	4

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Section B

10	**i**	$(x-3)^2-7$	3	mark final answer; 1 for $a = 3$, 2 for $b = 7$ or M1 for $-3^2 + 2$; bod 3 for $(x - 3) - 7$	3
	ii	(3, −7) or ft from (i)	1+1		2
	iii	sketch of quadratic correct way up and through (0, 2)	G1	accept (0, 2) o.e. seen in this part [eg in table] if 2 not marked as intercept on graph	
		t.p. correct or ft from (ii)	G1	accept 3 and −7 marked on axes level with turning pt., or better; no ft for (0, 2) as min	2
	iv	$x^2 - 6x + 2 = 2x - 14$ o.e.	M1	or their (i) = $2x - 14$	
		$x^2 - 8x + 16$ [= 0]	M1	dep on first M1; condone one error	
		$(x - 4)^2$ [= 0]	M1	or correct use of formula, giving equal roots; allow $(x + 4)^2$ o.e. ft $x^2 + 8x + 16$	
		$x = 4$, $y = -6$	A1	if M0M0M0, allow SC2 for showing (4, −6) is on both graphs (need to go on to show line is tgt to earn more)	
		equal/repeated roots [implies tgt] - must be explicitly stated; condone 'only one root [so tgt]' or 'line meets curve only once, so tgt' or 'line touches curve only once' etc]	A1	or for use of calculus to show grad of line and curve are same when $x = 4$	5

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11	**i**	f(−4) used	M1		
		−128 + 112 + 28 − 12 [= 0]	A1	or B2 for $(x + 4)(2x^2 - x - 3)$ here; or correct division with no remainder	2
	ii	division of f(x) by (x + 4)	M1	as far as $2x^3 + 8x^2$ in working, or two terms of $2x^2 - x - 3$ obtained by inspection etc (may be earned in (i)), or f(−1) = 0 found	
		$2x^2 - x - 3$	A1	$2x^2 - x - 3$ seen implies M1A1	
		$(x + 1)(2x - 3)$	A1		
		[f(x) =] $(x + 4)(x + 1)(2x - 3)$	A1	or B4; allow final A1 ft their factors if M1A1A0 earned	4
	iii	sketch of cubic correct way up	G1	ignore any graph of $y = f(x - 4)$	
		through −12 shown on y axis	G1	or coords stated near graph	
		roots −4, −1, 1.5 or ft shown on x axis	G1	or coords stated near graph if no curve drawn, but intercepts marked on axes, can earn max of G0G1G1	3
	iv	$x(x - 3)(2[x - 4] - 3)$ o.e. or $x(x - 3)(x - 5.5)$ or ft their factors	M1	or $2(x - 4)^3 + 7(x - 4)^2 - 7(x - 4) - 12$ or stating roots are 0, 3 and 5.5 or ft; condone one error eg $2x - 7$ not $2x - 11$	
		correct expansion of one pair of brackets ft from their factors	M1	or for correct expn of $(x - 4)^3$ [allow unsimplified]; or for showing g(0) = g(3) = g(5.5) = 0 in given ans g(x)	
		correct completion to given answer	M1	allow M2 for working backwards from given answer to $x(x - 3)(2x - 11)$ and M1 for full completion with factors or roots	3

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12	**i**	grad AB = $\frac{9-1}{3--1}$ or 2	M1		
		$y - 9 = 2(x - 3)$ or $y - 1 = 2(x + 1)$	M1	ft their *m*, or subst coords of A or B in y = their $m\,x + c$	
		$y = 2x + 3$ o.e.	A1	or B3	3
	ii	mid pt of AB = (1, 5)	M1	condone not stated explicitly, but used in eqn	
		grad perp = −1/grad AB	M1	soi by use eg in eqn	
		$y - 5 = -\frac{1}{2}(x - 1)$ o.e. or ft [no ft for just grad AB used]	M1	ft their grad and/or midpt, but M0 if their midpt not used; allow M1 for $y = -\frac{1}{2}x + c$ and then their midpt subst	
		at least one correct interim step towards given answer $2y + x = 11$, and correct completion NB ans $2y + x = 11$ given	M1	no ft; correct eqn only	
		<u>alt method working back from ans:</u>		mark one method or the other, to benefit of cand, not a mixture	
		$y = \frac{11-x}{2}$ o.e.	M1		
		grad perp = −1/grad AB and showing/stating same as given line	M1	eg stating $-\frac{1}{2} \times 2 = -1$	
		finding intn of their $y = 2x + 3$ and $2y + x = 11$ [= (1, 5)]	M1	or showing that (1, 5) is on $2y + x = 11$, having found (1, 5) first	4
		showing midpt of AB is (1, 5)	M1	[for both methods: for M4 must be fully correct]	
	iii	showing $(-1 - 5)^2 + (1 - 3)^2 = 40$	M1	at least one interim step needed for each mark; M0 for just $6^2 + 2^2 = 40$ with no other evidence such as a first line of working or a diagram; condone marks earned in reverse order	
		showing B to centre = √40 or verifying that (3, 9) fits given circle	M1		2
	iv	$(x - 5)^2 + 3^2 = 40$	M1	for subst $y = 0$ in circle eqn	
		$(x - 5)^2 = 31$	M1	condone slip on rhs; or for rearrangement to zero (condone one error) <u>and</u> attempt at quad. formula [allow M1 M0 for $(x - 5)^2 = 40$ or for $(x - 5)^2 + 3^2 = 0$]	
		$x = 5 \pm \sqrt{31}$ or $\frac{10 \pm \sqrt{124}}{2}$ isw	A1	or $5 \pm \frac{\sqrt{124}}{2}$	3

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4752 (C2) Concepts for Advanced Mathematics

Section A

1	210 c.a.o.	2	1 for π rads = 180° soi	2	
2	(i) 5.4×10^{-3}, 0.0054 or $\frac{27}{5000}$ (ii) 6 www	1 2	 M1 for S = 5.4 / (1 − 0.1)	 3	
3	stretch, parallel to the y axis, sf 3	2	1 for stretch plus one other element correct	2	
4	[$f'(x)$ =] $12 - 3x^2$ their $f'(x) > 0$ or = 0 soi $-2 < x < 2$	B1 M1 A1	 condone $-2 \le x \le 2$ or "between -2 and 2"	 3	
5	(i) grad of chord = $(2^{3.1} - 2^3)/0.1$ o.e. = 5.74 c.a.o. (ii) correct use of A and C where for C, $2.9 < x < 3.1$ answer in range (5.36, 5.74)	M1 A1 M1 A1	 or chord with ends x = 3 ± h, where 0 < h ≤ 0.1 s.c.1 for consistent use of reciprocal of gradient formula in parts (i) and (ii)	 4	
6	[y =] $kx^{3/2}$ [+ c] k = 4 subst of (9, 105) in their eqn with c or $c = -3$	M1 A1 M1 A1	 may appear at any stage must have c; must have attempted integration	 4	18
7	sector area = 28.8 or $\frac{144}{5}$ [cm²] c.a.o. area of triangle = $\frac{1}{2} \times 6^2 \times \sin 1.6$ o.e. their sector – their triangle s.o.i. 10.8 to 10.81 [cm²]	2 M1 M1 A1	M1 for $\frac{1}{2} \times 6^2 \times 1.6$ must both be areas leading to a positive answer	 5	
8	$a + 10d = 1$ *or* 121 = 5.5(2a+10d) $5(2a + 9d) = 120$ o.e. $a = 21$ s.o.i. www and $d = -2$ s.o.i. www 4th term is 15	M1 M1 A1 A1 A1	or 121 = 5.5(a + 1) gets M2 eg $2a + 9d = 24$	 5	
9	$x \log 5 = \log 235$ or $x = \frac{\log 235}{\log 5}$ 3.39	M1 A2	or $x = \log_5 235$ A1 for 3.4 or versions of 3.392…	 3	
10	$2(1 - \cos^2\theta) = \cos\theta + 2$ $-2\cos^2\theta = \cos\theta$ s.o.i. valid attempt at solving their quadratic in $\cos\theta$ $\cos\theta = -\frac{1}{2}$ www θ = 90, 270, 120, 240	M1 A1 DM1 A1 A1	for $1 - \cos^2\theta = \sin^2\theta$ substituted graphic calc method: allow M3 for intersection of $y = 2\sin^2\theta$ and $y = \cos\theta + 2$ and A2 for all four roots. All four answers correct but unsupported scores B2. 120 and 240 only: B1.	 5	18

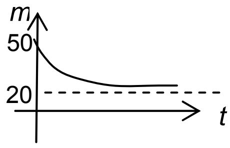
Section B

11	**i**	$(x + 5)(x - 2)(x + 2)$	2	M1 for $a\ (x + 5)(x - 2)(x + 2)$	2
	ii	$[(x + 2)](x^2 + 3x - 10)$ $x^3 + 3x^2 - 10x + 2x^2 + 6x - 20$ o.e.	M1 M1	for correct expansion of one pair of their brackets for clear expansion of correct factors – accept given answer from $(x + 5)(x^2 - 4)$ as first step	2
	iii	$y' = 3x^2 + 10x - 4$ their $3x^2 + 10x - 4 = 0$ s.o.i. x = 0.36… from formula o.e. (−3.7, 12.6)	M2 M1 A1 B1+1	M1 if one error or M1 for substitution of 0.4 if trying to obtain 0, and A1 for correct demonstration of sign change	6
	iv	(-1.8, 12.6)	B1+1	accept (−1.9, 12.6) or f.t.(½ their max x, their max y)	2
12	**i**	Area = (-)0.136 seen [m^2] www Volume = 0.34 [m^3] or ft from their area × 2.5	4 1	M3 for 0.1/2 × (0.14 + 0.16 + 2[0.22 + 0.31 + 0.36 + 0.32]) M2 for one slip; M1 for two slips must be positive	5
	ii	$2x^4 - x^3 - 0.25\ x^2 - 0.15x$ o.e. value at 0.5 [– value at 0] = –0.1375 area of cross section (of trough) or area between curve and x-axis 0.34375 r.o.t. to 3 or more sf [m^3] m^3 seen in (i) or (ii)	M2 M1 A1 E1 B1 U1	M1 for 2 terms correct dep on integral attempted must have neg sign	7
13	**i**	log P = log a + b log t www comparison with $y = mx + c$ intercept = $\log_{10} a$	1 1 1	 must be with correct equation condone omission of base	3
	ii	log t 0 0.78 1.15 1.18 1.20 log P 1.49 1.64 1.75 1.74 1.76 plots f.t. ruled line of best fit	1 1 1 1	accept to 2 or more dp	4
	iii	gradient rounding to 0.22 or 0.23 $a = 10^{1.49}$ s.o.i. $P = 31t^{m}$ allow the form $P = 10^{0.22\log t + 1.49}$	2 1 1	M1 for y step / x-step accept1.47 – 1.50 for intercept accept answers that round to 30 – 32 , their positive m	4
	iv	answer rounds in range 60 to 63	1		1

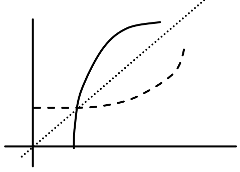
4753 (C3) Methods for Advanced Mathematics

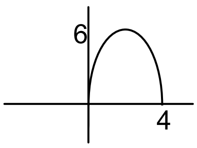
Section A

1 $\lvert 2x-1\rvert \le 3$		
⇒ $-3 \le 2x - 1 \le 3$	M1	$2x - 1 \le 3$ (or =)
⇒ $-2 \le 2x \le 4$	A1	$x \le 2$
⇒ $-1 \le x \le 2$	M1	$2x - 1 \ge -3$ (or =)
or	A1	$x \ge -1$
$(2x-1)^2 \le 9$		
⇒ $4x^2 - 4x - 8 \le 0$	M1	squaring and forming quadratic = 0 (or ≤)
⇒ $(4)(x+1)(x-2) \le 0$	A1	factorising or solving to get $x = -1, 2$
⇒ $-1 \le x \le 2$	A1	$x \ge -1$
	A1	$x \le 2$ (www)
	[4]	
2 Let $u = x$, $dv/dx = e^{3x}$ ⇒ $v = e^{3x}/3$	M1	parts with $u = x$, $dv/dx = e^{3x}$ ⇒ v
⇒ $\int x e^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}.1.dx$	A1	
$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$	A1	$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}$
	B1	$+c$
	[4]	
3 (i) $f(-x) = f(x)$	B1	
Symmetrical about Oy.	B1	
	[2]	
(ii) (A) even	B1	
(B) neither	B1	
(C) odd	B1	
	[3]	
4 Let $u = x^2 + 2$ ⇒ $du = 2x\,dx$		
$\int_1^4 \frac{x}{x^2+2}dx = \int_3^{18} \frac{1/2}{u}du$	M1	$\int \frac{1/2}{u}du$ or $k \ln (x^2 + 1)$
$= \frac{1}{2}\left[\ln u\right]_3^{18}$	A1	½ ln u or ½ ln(x^2 + 2)
= ½ (ln18 – ln 3)	M1	substituting correct limits (u or x)
= ½ ln(18/3)		
= ½ ln 6*	E1	must show working for ln 6
	[4]	
5 $y = x^2 \ln x$		
⇒ $\frac{dy}{dx} = x^2.\frac{1}{x} + 2x\ln x$	M1	product rule
	B1	d/dx (ln x) = 1/x soi
$= x + 2x\ln x$	A1	oe
dy/dx = 0 when x +2$x$ln x = 0		
⇒ $x(1 + 2\ln x) = 0$	M1	their deriv = 0 or attempt to verify
⇒ ln x = – ½	M1	ln x = –½ ⇒ $x = e^{-½}$ or ln (1/√e) = – ½
⇒ $x = e^{-½}$ = 1/√e *	E1	
	[6]	

6(i) Initial mass = 20 + 30 e^0 = 50 grams Long term mass = 20 grams	M1A1 B1 [3]	
(ii) $30 = 20 + 30\,e^{-0.1t}$ ⇒ $e^{-0.1t} = 1/3$ ⇒ $-0.1t = \ln(1/3) = -1.0986\ldots$ ⇒ $t = 11.0$ mins	M1 M1 A1 [3]	 anti-logging correctly 11, 11.0, 10.99, 10.986 (not more than 3 d.p)
(iii) 	B1 B1 [2]	correct shape through (0, 50) – ignore negative values of t $\to 20$ as $t \to \infty$
7 $x^2 + xy + y^2 = 12$ ⇒ $2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ ⇒ $(x+2y)\frac{dy}{dx} = -2x - y$ ⇒ $\frac{dy}{dx} = -\frac{2x+y}{(x+2y)}$	M1 B1 A1 M1 A1 [5]	Implicit differentiation $x\frac{dy}{dx} + y$ correct equation collecting terms in dy/dx and factorising oe cao

Section B

8(i) $y = 1/(1+\cos\pi/3) = 2/3$.	B1 [1]	or 0.67 or better
(ii) $f'(x) = -1(1+\cos x)^{-2}.-\sin x$ $= \dfrac{\sin x}{(1+\cos x)^2}$ When $x = \pi/3$, $f'(x) = \dfrac{\sin(\pi/3)}{(1+\cos(\pi/3))^2}$ $= \dfrac{\sqrt{3}/2}{(1\frac{1}{2})^2} = \dfrac{\sqrt{3}}{2}\times\dfrac{4}{9} = \dfrac{2\sqrt{3}}{9}$	M1 B1 A1 M1 A1 [5]	chain rule or quotient rule d/dx (cos x) = –sin x soi correct expression substituting $x = \pi/3$ oe or 0.38 or better. (0.385, 0.3849)
(iii) deriv = $\dfrac{(1+\cos x)\cos x - \sin x.(-\sin x)}{(1+\cos x)^2}$ $= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ $= \dfrac{\cos x + 1}{(1+\cos x)^2}$ $= \dfrac{1}{1+\cos x}$ * Area $= \int_0^{\pi/3} \dfrac{1}{1+\cos x}dx$ $= \left[\dfrac{\sin x}{1+\cos x}\right]_0^{\pi/3}$ $= \dfrac{\sin\pi/3}{1+\cos\pi/3}(-0)$ $= \dfrac{\sqrt{3}}{2}\times\dfrac{2}{3} = \dfrac{\sqrt{3}}{3}$	M1 A1 M1dep E1 B1 M1 A1 cao [7]	Quotient or product rule – condone uv' – u'v for M1 correct expression $\cos^2 x + \sin^2 x = 1$ used dep M1 www substituting limits or $1/\sqrt{3}$ - must be exact
(iv) $y = 1/(1 + \cos x)$ $x \leftrightarrow y$ $x = 1/(1 + \cos y)$ $\Rightarrow$ $1 + \cos y = 1/x$ $\Rightarrow$ $\cos y = 1/x - 1$ $\Rightarrow$ $y = \arccos(1/x - 1)$ * Domain is $\frac{1}{2} \le x \le 1$ 	M1 A1 E1 B1 B1 [5]	attempt to invert equation www reasonable reflection in $y = x$

9 (i) $y=\sqrt{4-x^2}$ $\Rightarrow \quad y^2 = 4 - x^2$ $\Rightarrow \quad x^2 + y^2 = 4$ which is equation of a circle centre O radius 2 Square root does not give negative values, so this is only a semi-circle.	M1 A1 B1 [3]	squaring $x^2 + y^2 = 4$ + comment (correct) oe, e.g. f is a function and therefore single valued
(ii) (*A*) Grad of OP = b/a $\Rightarrow$ grad of tangent = $-\dfrac{a}{b}$ (*B*) $\mathrm{f}'(x)=\dfrac{1}{2}(4-x^2)^{-1/2}.(-2x)$ $=-\dfrac{x}{\sqrt{4-x^2}}$ $\Rightarrow \quad \mathrm{f}'(a)=-\dfrac{a}{\sqrt{4-a^2}}$ (*C*) $b = \sqrt{(4 - a^2)}$ so $\mathrm{f}'(a)=-\dfrac{a}{b}$ as before	M1 A1 M1 A1 B1 E1 [6]	 chain rule or implicit differentiation oe substituting a into their $\mathrm{f}'(x)$
(iii) Translation through $\begin{pmatrix}2\\0\end{pmatrix}$ followed by stretch scale factor 3 in y-direction 	M1 A1 M1 A1 M1 A1 [6]	Translation in x-direction through $\begin{pmatrix}2\\0\end{pmatrix}$ or 2 to right ('shift', 'move' M1 A0) $\begin{pmatrix}2\\0\end{pmatrix}$ alone is SC1 stretch in y-direction (condone y 'axis') (scale) factor 3 elliptical (or circular) shape through (0, 0) and (4, 0) and (2, 6) (soi) –1 if whole ellipse shown
(iv) $y = 3\mathrm{f}(x - 2)$ $= 3\sqrt{(4 - (x - 2)^2)}$ $= 3\sqrt{(4 - x^2 + 4x - 4)}$ $= 3\sqrt{(4x - x^2)}$ $\Rightarrow \quad y^2 = 9(4x - x^2)$ $\Rightarrow \quad 9x^2 + y^2 = 36x$ *	 M1 A1 E1 [3]	 or substituting $3\sqrt{(4 - (x - 2)^2)}$ oe for y in $9x^2 + y^2$ $4x - x^2$ www

4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{x}{x^2-4}+\frac{2}{x+2}=\frac{x}{(x-2)(x+2)}+\frac{2}{x+2}$ $=\frac{x+2(x-2)}{(x+2)(x-2)}$ $=\frac{3x-4}{(x+2)(x-2)}$	M1 M1 A1 [3]	combining fractions correctly factorising and cancelling (may be $3x^2+2x-8$)
2 $V=\int_0^1 \pi y^2 dx = \int_0^1 \pi(1+e^{2x})dx$ $=\pi\left[x+\frac{1}{2}e^{2x}\right]_0^1$ $=\pi(1+\frac{1}{2}e^2-\frac{1}{2})$ $=\frac{1}{2}\pi(1+e^2)$ *	M1 B1 M1 E1 [4]	must be π x their y^2 in terms of x $\left[x+\frac{1}{2}e^{2x}\right]$ only substituting both x limits in a function of x www
3 $\cos 2\theta = \sin\theta$ $\Rightarrow 1-2\sin^2\theta=\sin\theta$ $\Rightarrow 1-\sin\theta-2\sin^2\theta=0$ $\Rightarrow (1-2\sin\theta)(1+\sin\theta)=0$ $\Rightarrow \sin\theta = ½$ or -1 $\Rightarrow \theta=\pi/6, 5\pi/6, 3\pi/2$	M1 M1 A1 M1 A1 A2,1,0 [7]	$\cos2\theta = 1-2\sin^2\theta$ oe substituted forming quadratic(in one variable) =0 correct quadratic www factorising or solving quadratic ½ , −1 oe www cao penalise extra solutions in the range
4 $\sec\theta = x/2$, $\tan\theta = y/3$ $\sec^2\theta = 1+\tan^2\theta$ $\Rightarrow x^2/4 = 1+y^2/9$ $\Rightarrow x^2/4 - y^2/9 = 1$ * **OR** $x^2/4-y^2/9= 4\sec^2\theta/4-9\tan^2\theta/9$ $=\sec^2\theta-\tan^2\theta=1$	M1 M1 E1 [3]	$\sec^2\theta = 1+\tan^2\theta$ used (oe, e.g. converting to sines and cosines and using $\cos^2\theta + \sin^2\theta = 1$) eliminating θ (or x and y) www
5(i) $dx/du = 2u$, $dy/du = 6u^2$ $\Rightarrow \frac{dy}{dx}=\frac{dy/du}{dx/du}=\frac{6u^2}{2u}$ $= 3u$ **OR** $y=2(x-1)^{3/2}$, $dy/dx=3(x-1)^{1/2}=3u$	B1 M1 A1 [3]	both $2u$ and $6u^2$ B1(y=f(x)), M1 differentiation, A1
(ii) At (5, 16), $u = 2$ $\Rightarrow dy/dx = 6$	M1 A1 [2]	cao

6(i) $(1+4x^2)^{-\frac{1}{2}} = 1 - \frac{1}{2}.4x^2 + \frac{(-\frac{1}{2}).(-\frac{3}{2})}{2!}(4x^2)^2 + ...$ $= 1 - 2x^2 + 6x^4 + ...$ Valid for $-1 < 4x^2 < 1 \Rightarrow -½ < x < ½$	M1 A1 A1 M1A1 [5]	binomial expansion with $p = -1/2$ $1 - 2x^2$ … $+ 6x^4$
(ii) $\frac{1-x^2}{\sqrt{1+4x^2}} = (1-x^2)(1-2x^2+6x^4+...)$ $= 1 - 2x^2 + 6x^4 - x^2 + 2x^4 + ...$ $= 1 - 3x^2 + 8x^4 + …$	M1 A1 A1 [3]	substitituting their $1-2x^2+6x^4+...$ and expanding ft their expansion (of three terms) cao
7 $\sqrt{3} \sin x - \cos x = R\sin(x - \alpha)$ $= R(\sin x\cos \alpha - \cos x \sin \alpha)$ $\Rightarrow$ $\sqrt{3} = R\cos \alpha$, $1 = R \sin \alpha$ $\Rightarrow$ $R^2 = 3 + 1 = 4 \Rightarrow R = 2$ $\tan \alpha = 1/\sqrt{3}$ $\Rightarrow \alpha = \pi/6$ $\Rightarrow$ $y = 2\sin(x - \pi/6)$ Max when $x - \pi/6 = \pi/2 \Rightarrow x = \pi/6 + \pi/2 = 2\pi/3$ max value $y = 2$ So maximum is $(2\pi/3, 2)$	 M1 B1 M1 A1 B1 B1 [6]	 correct pairs soi $R = 2$ ft cao www cao ft their R SC B1 $(2, 2\pi/3)$ no working

Section B

8(i) At A: $3\times0 + 2\times0 + 20\times(-15) + 300 = 0$ At B: $3\times100 + 2\times0 + 20\times(-30) + 300 = 0$ At C: $3\times0 + 2\times100 + 20\times(-25) + 300 = 0$ So ABC has equation $3x + 2y + 20z + 300 = 0$	M1 A2,1,0 [3]	substituting co-ords into equation of plane… for ABC **OR** using two vectors in the plane form vector product M1A1 then $3x + 2y + 20z = c = -300$ A1 **OR** using vector equation of plane M1,elim both parameters M1, A1
(ii) $\overrightarrow{DE} = \begin{pmatrix}100\\0\\-10\end{pmatrix}$ $\overrightarrow{DF} = \begin{pmatrix}0\\100\\5\end{pmatrix}$	B1B1	
$\begin{pmatrix}100\\0\\-10\end{pmatrix}.\begin{pmatrix}2\\-1\\20\end{pmatrix} = 100\times2 + 0\times-1 + -10\times20 = 200-200 = 0$	B1	need evaluation
$\begin{pmatrix}0\\100\\5\end{pmatrix}.\begin{pmatrix}2\\-1\\20\end{pmatrix} = 0\times2 + 100\times-1 + 5\times20 = -100+100 = 0$	B1	need evaluation
Equation of plane is $2x - y + 20z = c$ At D (say) $c = 20\times-40 = -800$ So equation is $2x - y + 20z + 800 = 0$	M1 A1 [6]	
(iii) Angle is θ, where $\cos\theta = \dfrac{\begin{pmatrix}2\\-1\\20\end{pmatrix}.\begin{pmatrix}3\\2\\20\end{pmatrix}}{\sqrt{2^2+(-1)^2+20^2}\sqrt{3^2+2^2+20^2}} = \dfrac{404}{\sqrt{405}\sqrt{413}}$ $\Rightarrow$ $\theta = 8.95°$	M1 A1 A1 A1cao [4]	formula with correct vectors top bottom (or 0.156 radians)
(iv) RS: $\mathbf{r} = \begin{pmatrix}15\\34\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$	B1	$\begin{pmatrix}15\\34\\0\end{pmatrix} + \dots$
$= \begin{pmatrix}15+3\lambda\\34+2\lambda\\20\lambda\end{pmatrix}$	B1	$\dots + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$
$\Rightarrow$ $3(15+3\lambda) + 2(34+2\lambda) + 20.20\lambda + 300 = 0$ $\Rightarrow$ $45 + 9\lambda + 68 + 4\lambda + 400\lambda + 300 = 0$	M1	solving with plane
$\Rightarrow$ $413 + 413\lambda = 0$ $\Rightarrow$ $\lambda = -1$	A1	$\lambda = -1$
so S is (12, 32, –20)	A1 [5]	cao

9(i) $v = \int 10e^{-\frac{1}{2}t}dt$ $= -20e^{-\frac{1}{2}t} + c$ when $t = 0$, $v = 0$ $\Rightarrow$ $0 = -20 + c$ $\Rightarrow$ $c = 20$ so $v = 20 - 20e^{-\frac{1}{2}t}$	M1 A1 M1 A1 [4]	separate variables and intend to integrate $-20e^{-\frac{1}{2}t}$ finding c cao
(ii) As $t \to \infty$ $e^{-1/2\,t} \to 0$ $\Rightarrow$ $v \to 20$ So long term speed is 20 m s^{-1}	M1 A1 [2]	 ft (for their c>0, found)
(iii) $\frac{1}{(w-4)(w+5)} = \frac{A}{w-4} + \frac{B}{w+5}$ $= \frac{A(w+5)+B(w-4)}{(w-4)(w+5)}$ $\Rightarrow$ $1 \equiv A(w+5) + B(w-4)$ $w = 4$: $1 = 9A$ $\Rightarrow A = 1/9$ $w = -5$: $1 = -9B \Rightarrow B = -1/9$ $\Rightarrow \frac{1}{(w-4)(w+5)} = \frac{1/9}{w-4} - \frac{1/9}{w+5}$ $= \frac{1}{9(w-4)} - \frac{1}{9(w+5)}$	M1 M1 A1 A1 [4]	 cover up, substitution or equating coeffs 1/9 −1/9
(iv) $\frac{dw}{dt} = -\frac{1}{2}(w-4)(w+5)$ $\Rightarrow$ $\int \frac{dw}{(w-4)(w+5)} = \int -\frac{1}{2}dt$ $\Rightarrow \int [\frac{1}{9(w-4)} - \frac{1}{9(w+5)}]dw = \int -\frac{1}{2}dt$ $\Rightarrow \frac{1}{9}\ln(w-4) - \frac{1}{9}\ln(w+5) = -\frac{1}{2}t + c$ $\Rightarrow \frac{1}{9}\ln\frac{w-4}{w+5} = -\frac{1}{2}t + c$ When $t = 0$, $w = 10 \Rightarrow$ $c = \frac{1}{9}\ln\frac{6}{15} = \frac{1}{9}\ln\frac{2}{5}$ $\Rightarrow \ln\frac{w-4}{w+5} = -\frac{9}{2}t + \ln\frac{2}{5}$ $\Rightarrow \frac{w-4}{w+5} = e^{-\frac{9}{2}t + \ln\frac{2}{5}} = \frac{2}{5}e^{-\frac{9}{2}t} = 0.4e^{-4.5t}$ *	 M1 M1 A1ft M1 M1 E1 [6]	 separating variables substituting their partial fractions integrating correctly (condone absence of c) correctly evaluating c (at any stage) combining lns (at any stage) www
(v) As $t \to \infty$ $e^{-4.5\,t} \to 0$ $\Rightarrow$ $w - 4 \to 0$ So long term speed is 4 m s^{-1}.	M1 A1 [2]	

Comprehension

1. (i)

2	1	3
3	**2**	**1**
1	**3**	**2**

B1 cao

(ii)

2	3	**1**
3	**1**	**2**
1	2	**3**

B1 cao

2. Dividing the grid up into four 2 x 2 blocks gives

1	2	3	4
3	1	4	2
2	4	1	3
4	3	2	1

Lines drawn on diagram or reference to 2 x 2 blocks. **M1**

One (or more) block does not contain all 4 of the symbols 1, 2, 3 and 4. oe. **E1**

3.

1	2	3	4
4	**3**	**1**	**2**
2	**1**	**4**	**3**
3	**4**	**2**	**1**

<u>**Many possible answers**</u> **Row 2 correct** **B1**

Rest correct **B1**

4. **Either**

4	2	3	**1**
		2	4
		4	2
2	4	1	**3**

Or

4	2	3	1
		2	**4**
		4	**2**
2	4	1	3

B2

5. In the top row there are 9 ways of allocating a symbol to the left cell, then 8 for the next, 7 for the next and so on down to 1 for the right cell, giving **M1**

9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 9! ways. **E1**

So there must be 9!× the number of ways of completing the rest of the puzzle.

6.

(i)

Block side length, b	**Sudoku,** $s \times s$	M
1	1 × 1	-
2	4 × 4	12
3	9 × 9	**77**
4	16 × 16	**252**
5	**25 × 25**	**621**

25 × 25 **B1**

77, 252 and 621 **B1**

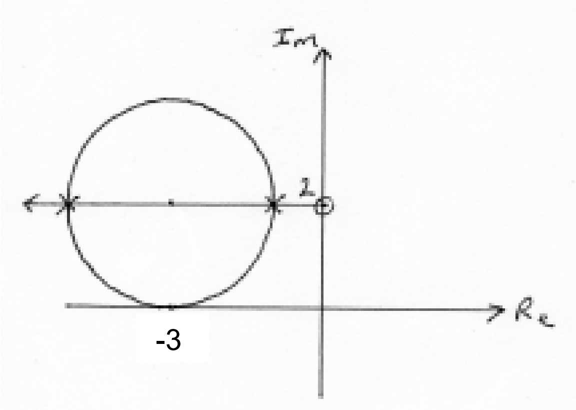
(ii) $M = b^4 - 4$

b^4 **B1**

- 4 **B1**

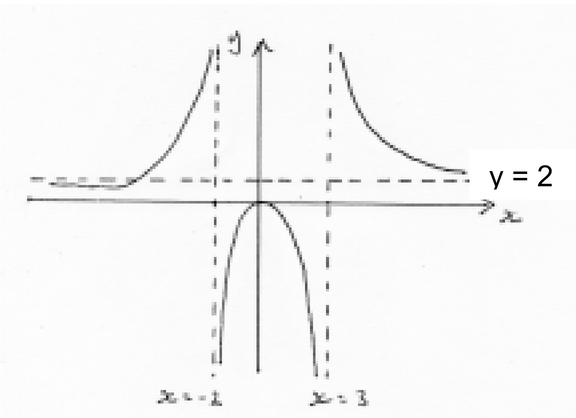
7.		
(i)	There are neither 3s nor 5s among the givens.	**M1**
	So they are interchangeable and therefore there is no unique solution	**E1**
(ii)	The missing symbols form a 3×3 embedded Latin square.	**M1**
	There is not a unique arrangement of the numbers 1, 2 and 3 in this square.	**E1**
		[18]

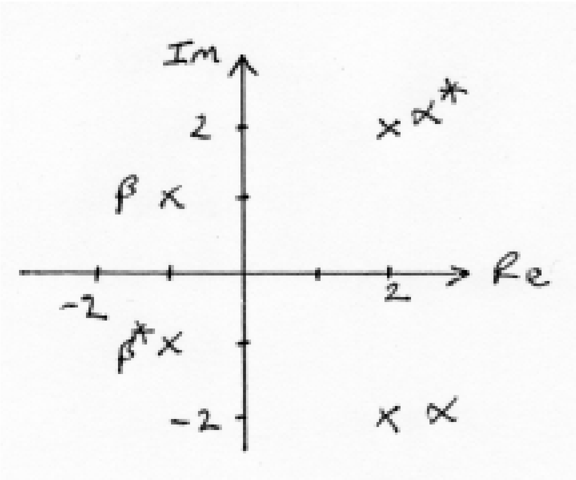
4755 (FP1) Further Concepts for Advanced Mathematics

Qu	Answer	Mark	Comment
Section A			
1(i)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	B1	
1(ii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1	
1(iii)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$	M1 A1 **[4]**	Multiplication, or other valid method (may be implied) c.a.o.
2		B3 B3 B1 **[7]**	Circle, B1; centre $-3+2j$, B1; radius = 2, B1 Line parallel to real axis, B1; through (0, 2), B1; correct half line, B1 Points $-1+2j$ and $-5+2j$ indicated c.a.o.
3	$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow -x-y=x,\ 2x+2y=y$ $\Rightarrow y=-2x$	M1 M1 B1 **[3]**	For $\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$
4	$3x^3-x^2+2\equiv A(x-1)^3+(x^3+Bx^2+Cx+D)$ $\equiv Ax^3-3Ax^2+3Ax-A+x^3+Bx^2+Cx+D$ $\equiv (A+1)x^3+(B-3A)x^2+(3A+C)x+(D-A)$ $\Rightarrow A=2,\ B=5,\ C=-6,\ D=4$	M1 B4 **[5]**	Attempt to compare coefficients One for each correct value

5(i)	$\mathbf{AB}=\begin{pmatrix}7&0&0\\0&7&0\\0&0&7\end{pmatrix}$	B3 **[3]**	Minus 1 each error to minimum of 0
5(ii)	$\mathbf{A}^{-1}=\frac{1}{7}\begin{pmatrix}-1&0&2\\14&-14&7\\-5&7&-4\end{pmatrix}$	M1 A1 **[2]**	Use of B c.a.o.
6	$w=2x\Rightarrow x=\frac{w}{2}$ $\Rightarrow 2\left(\frac{w}{2}\right)^3+\left(\frac{w}{2}\right)^2-3\left(\frac{w}{2}\right)+1=0$ $\Rightarrow w^3+w^2-6w+4=0$	B1 M1 A1 A2 **[5]**	Substitution. For substitution $x=2w$ give B0 but then follow through for a maximum of 3 marks Substitute into cubic Correct substitution Minus 1 for each error (including '= 0' missing), to a minimum of 0 Give full credit for integer multiple of equation
6	**OR** $\alpha+\beta+\gamma=-\frac{1}{2}$ $\alpha\beta+\alpha\gamma+\beta\gamma=-\frac{3}{2}$ $\alpha\beta\gamma=-\frac{1}{2}$ Let new roots be *k*, *l*, *m* then $k+l+m=2(\alpha+\beta+\gamma)=-1=\frac{-B}{A}$ $kl+km+lm=4(\alpha\beta+\alpha\gamma+\beta\gamma)=-6=\frac{C}{A}$ $klm=8\alpha\beta\gamma=-4=\frac{-D}{A}$ $\Rightarrow \omega^3+\omega^2-6\omega+4=0$	B1 M1 A1 A2 **[5]**	All three Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation Sums and products all correct ft their coefficients; minus one for each error (including '= 0' missing), to minimum of 0 Give full credit for integer multiple of equation

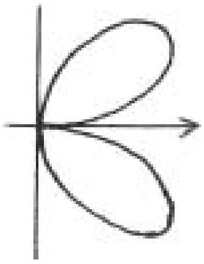
7(i)	$\frac{1}{3r-1}-\frac{1}{3r+2}\equiv\frac{3r+2-(3r-1)}{(3r-1)(3r+2)}$	M1	Attempt at correct method
	$\equiv\frac{3}{(3r-1)(3r+2)}$	A1	Correct, without fudging
		[2]	
7(ii)	$\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}=\frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{3r-1}-\frac{1}{3r+2}\right]$	M1	Attempt to use identity
	$=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+.......+\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]$	A1 M1	Terms in full (at least two) Attempt at cancelling
	$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right]$	A2	A1 if factor of $\frac{1}{3}$ missing,
		[5]	A1 max if answer not in terms of *n*
			Section A Total: 36

Section B			
8(i)	$x = 3$, $x = -2$, $y = 2$	B1 B1 B1 **[3]**	
8(ii)	Large positive *x*, $y \to 2^+$ (e.g. consider $x = 100$) Large negative *x*, $y \to 2^-$ (e.g. consider $x = -100$)	M1 B1 B1 **[3]**	Evidence of method required
8(iii)	Curve Central and RH branches correct Asymptotes correct and labelled LH branch correct, with clear minimum 	B1 B1 B1 **[3]**	
8(iv)	$-2 < x < 3$ $x \neq 0$	B2 B1 **[3]**	B2 max if any inclusive inequalities appear B3 for $-2 < x < 0$ and $0 < x < 3$,

9(i)	$2+2\text{j}$ and $-1-\text{j}$	B2 **[2]**	1 mark for each
9(ii)		B2 **[2]**	1 mark for each correct pair
9(iii)	$(x-2-2\text{j})(x-2+2\text{j})(x+1+\text{j})(x+1-\text{j})$	M1 B2	Attempt to use factor theorem Correct factors, minus 1 each error B1 if only errors are sign errors
	$=(x^2-4x+8)(x^2+2x+2)$	A1	One correct quadratic with real coefficients (may be implied)
	$=x^4+2x^3+2x^2-4x^3-8x^2-8x+8x^2+16x+16$ $=x^4-2x^3+2x^2+8x+16$	M1	Expanding
	$\Rightarrow A=-2,\ B=2,\ C=8,\ D=16$	A2 **[7]**	Minus 1 each error, A1 if only errors are sign errors
	OR		
	$\sum\alpha=2$ $\alpha\beta\gamma\delta=16$ $\sum\alpha\beta=\alpha\alpha^*+\alpha\beta+\alpha\beta^*+\beta\beta^*+\beta\alpha^*+\beta^*\alpha^*$ $\sum\alpha\beta\gamma=\alpha\alpha^*\beta+\alpha\alpha^*\beta^*+\alpha\beta\beta^*+\alpha^*\beta\beta^*$	B1 B1 M1 M1	
	$\sum\alpha\beta=2$, $\sum\alpha\beta\gamma=-8$	A1	Both correct
	$A=-2,\ B=2,\ C=8,\ D=16$	A2 **[7]**	Minus 1 each error, A1 if only errors are sign errors
	OR		
	Attempt to substitute in one root Attempt to substitute in a second root	M1 M1 A1	Both correct
	Equating real and imaginary parts to 0 Attempt to solve simultaneous equations	M1 M1	
	$A=-2,\ B=2,\ C=8,\ D=16$	A2 **[7]**	Minus 1 each error, A1 if only errors are sign errors

Qu	Answer	Mark	Comment
Section B (continued)			
10(i)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	M1	Separation of sums (may be implied)
	$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	B1	One mark for both parts
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+2(2n+1)\right]$	M1	Attempt to factorise (at least two linear algebraic factors)
	$= \frac{1}{12}n(n+1)(3n^2+7n+2)$	A1	Correct
	$= \frac{1}{12}n(n+1)(n+2)(3n+1)$	E1	Complete, convincing argument
		[5]	
10(ii)	$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$		
	n = 1, LHS = RHS = 2	B1	2 must be seen
	Assume true for *n* = *k*	E1	Assuming true for *k*
	$\sum_{r=1}^{k} r^2(r+1) = \frac{1}{12}k(k+1)(k+2)(3k+1)$		
	$\sum_{r=1}^{k+1} r^2(r+1)$		
	$= \frac{1}{12}k(k+1)(k+2)(3k+1)+(k+1)^2(k+2)$	B1	(*k* + 1)th term
	$= \frac{1}{12}(k+1)(k+2)\left[k(3k+1)+12(k+1)\right]$	M1	Attempt to factorise
	$= \frac{1}{12}(k+1)(k+2)(3k^2+13k+12)$	A1	Correct
	$= \frac{1}{12}(k+1)(k+2)(k+3)(3k+4)$	A1	Complete convincing argument
	$= \frac{1}{12}(k+1)((k+1)+1)((k+1)+2)(3(k+1)+1)$		
	But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1.	E1	Dependent on previous A1 and previous E1
	Since it is true for *k* = 1, it is true for *k* = 1, 2, 3 and so true for all positive integers.	E1	Dependent on first B1 and previous E1
		[8]	
			Section B Total: 36
			Total: 72

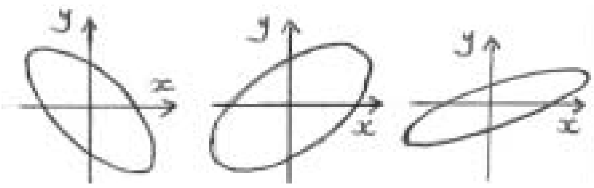
4756 (FP2) Further Methods for Advanced Mathematics

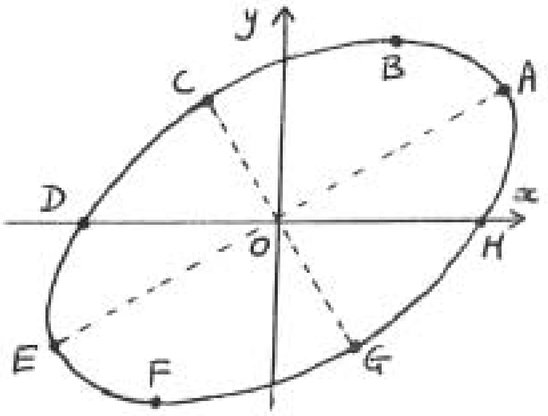
1(a)(i)	$x = r\cos\theta,\ y = r\sin\theta$ $(r^2\cos^2\theta + r^2\sin^2\theta)^2 = 3(r\cos\theta)(r\sin\theta)^2$ $r^4 = 3r^3\cos\theta\sin^2\theta$ $r = 3\cos\theta\sin^2\theta$	M1 A1 A1 ag **3**	(M0 for $x = \cos\theta,\ y = \sin\theta$)
(ii)		B1 B1 B1 **3**	Loop in 1st quadrant Loop in 4th quadrant Fully correct curve *Curve may be drawn using continuous or broken lines in any combination*
(b)	$\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,dx = \left[\frac{1}{\sqrt{3}}\arcsin\frac{\sqrt{3}\,x}{2}\right]_0^1$ $= \frac{1}{\sqrt{3}}\arcsin\frac{\sqrt{3}}{2}$ $= \frac{\pi}{3\sqrt{3}}$	M1 A1A1 M1 A1 **5**	For arcsin For $\frac{1}{\sqrt{3}}$ and $\frac{\sqrt{3}\,x}{2}$ Exact numerical value *Dependent on first M1* (M1A0 for $60/\sqrt{3}$)
	OR Put $\sqrt{3}\,x = 2\sin\theta$ — M1 A1 $\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,dx = \int_0^{\frac{\pi}{3}} \frac{1}{\sqrt{3}}\,d\theta$ — A1 $= \frac{\pi}{3\sqrt{3}}$ — M1A1		Any sine substitution For $\int \frac{1}{\sqrt{3}}\,d\theta$ *M1 dependent on first M1*
(c)(i)	$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \ldots$ $\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \ldots$	B1 B1 **2**	*Accept unsimplified forms*
(ii)	$\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$ $= 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \ldots$	M1 A1 **2**	Obtained from two correct series *Terms need not be added* If M0, then B1 for $2x + \frac{2}{3}x^3 + \frac{2}{5}x^5$

(iii)	$\sum_{r=0}^{\infty} \frac{1}{(2r+1)4^r} = 1 + \frac{1}{3\times 4} + \frac{1}{5\times 4^2} + ...$ $= 2\times\frac{1}{2} + \frac{2}{3}\times(\frac{1}{2})^3 + \frac{2}{5}\times(\frac{1}{2})^5 + ...$ $= \ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \ln 3$	B1 B1 B1 ag **3**	*Terms need not be added* For $x=\frac{1}{2}$ seen or implied Satisfactory completion
2 (i)	$\lvert z \rvert = 8,\quad \arg z = \frac{1}{4}\pi$ $\lvert z^* \rvert = 8,\quad \arg z^* = -\frac{1}{4}\pi$ $\lvert zw \rvert = 8\times 8 = 64$ $\arg(zw) = \frac{1}{4}\pi + \frac{7}{12}\pi = \frac{5}{6}\pi$ $\left\lvert \frac{z}{w} \right\rvert = \frac{8}{8} = 1$ $\arg(\frac{z}{w}) = \frac{1}{4}\pi - \frac{7}{12}\pi = -\frac{1}{3}\pi$	B1B1 B1 ft B1 ft B1 ft B1 ft B1 ft **7**	*Must be given separately* *Remainder may be given in exponential or* $r\,\text{cjs}\,\theta$ *form* (B0 for $\frac{7}{4}\pi$) (B0 if left as $8/8$)
(ii)	$\frac{z}{w} = \cos(-\frac{1}{3}\pi) + \text{j}\sin(-\frac{1}{3}\pi)$ $= \frac{1}{2} - \frac{\sqrt{3}}{2}\text{j}$ $a = \frac{1}{2},\quad b = -\frac{1}{2}\sqrt{3}$	M1 A1 **2**	If M0, then B1B1 for $\frac{1}{2}$ and $-\frac{\sqrt{3}}{2}$
(iii)	$r = \sqrt[3]{8} = 2$ $\theta = \frac{1}{12}\pi$ $\theta = \frac{\pi}{12} + \frac{2k\pi}{3}$ $\theta = -\frac{7}{12}\pi,\quad \frac{3}{4}\pi$	B1 ft B1 M1 A1 **4**	Accept $\sqrt[3]{8}$ Implied by one further correct (ft) value *Ignore values outside the required range*
(iv)	$w^* = 8\text{e}^{-\frac{7}{12}\pi\text{j}}$, so $2\text{e}^{-\frac{7}{12}\pi\text{j}} = \frac{1}{4}w^*$ $k_1 = \frac{1}{4}$ $z^* = 8\text{e}^{-\frac{1}{4}\pi\text{j}} = -8\text{e}^{\frac{3}{4}\pi\text{j}}$ So $2\text{e}^{\frac{3}{4}\pi\text{j}} = -\frac{1}{4}z^*$ $k_2 = -\frac{1}{4}$ $\text{j}w = 8\text{e}^{(\frac{1}{2}\pi+\frac{7}{12}\pi)\text{j}} = 8\text{e}^{\frac{13}{12}\pi\text{j}}$ $= -8\text{e}^{\frac{1}{12}\pi\text{j}}$, so $2\text{e}^{\frac{1}{12}\pi\text{j}} = -\frac{1}{4}\text{j}w$ $k_3 = -\frac{1}{4}$	B1 ft M1 A1 ft M1 A1 ft **5**	Matching *w** to a cube root with argument $-\frac{7}{12}\pi$ and $k_1 = \frac{1}{4}$ or ft ft is $\frac{r}{8}$ Matching *z** to a cube root with argument $\frac{3}{4}\pi$ *May be implied* ft is $-\frac{r}{\lvert z^* \rvert}$ Matching j*w* to a cube root with argument $\frac{1}{12}\pi$ *May be implied* OR M1 for $\arg(\text{j}w) = \frac{1}{2}\pi + \arg w$ *(implied by* $\frac{13}{12}\pi$ *or* $-\frac{11}{12}\pi$*)* ft is $-\frac{r}{8}$

3 (i)	$\mathbf{Q}^{-1} = \frac{1}{k-3}\begin{pmatrix} -1 & k+2 & -1 \\ 1 & 4-3k & k-2 \\ 1 & -5 & 1 \end{pmatrix}$ When $k=4$, $\mathbf{Q}^{-1} = \begin{pmatrix} -1 & 6 & -1 \\ 1 & -8 & 2 \\ 1 & -5 & 1 \end{pmatrix}$	M1 A1 M1 A1 M1 A1 **6**	Evaluation of determinant *(must involve k)* For $(k-3)$ Finding at least four cofactors *(including one involving k)* Six signed cofactors correct *(including one involving k)* Transposing and dividing by det *Dependent on previous M1M1* $\mathbf{Q}^{-1}$ correct (in terms of *k*) and result for $k=4$ stated After 0, SC1 for $\mathbf{Q}^{-1}$ when $k=4$ obtained correctly with some working
(ii)	$\mathbf{P} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ $\mathbf{M} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ $= \begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 6 & -1 \\ 1 & -8 & 2 \\ 1 & -5 & 1 \end{pmatrix}$ $= \begin{pmatrix} 2 & 1 & 12 \\ 1 & 0 & 3 \\ 3 & -1 & 6 \end{pmatrix}\begin{pmatrix} -1 & 6 & -1 \\ 1 & -8 & 2 \\ 1 & -5 & 1 \end{pmatrix}$ $= \begin{pmatrix} 11 & -56 & 12 \\ 2 & -9 & 2 \\ 2 & -4 & 1 \end{pmatrix}$	B1B1 B2 M1 A2 **7**	For B2, order must be consistent Give B1 for $\mathbf{M} = \mathbf{P}^{-1}\mathbf{D}\mathbf{P}$ or $\begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 6 & -1 \\ -1 & 8 & -2 \\ 3 & -15 & 3 \end{pmatrix}$ Good attempt at multiplying two matrices (no more than 3 errors), leaving third matrix in correct position Give A1 for five elements correct Correct **M** implies B2M1A2 5-8 elements correct implies B2M1A1
(iii)	Characteristic equation is $(\lambda-1)(\lambda+1)(\lambda-3)=0$ $\lambda^3 - 3\lambda^2 - \lambda + 3 = 0$ $\mathbf{M}^3 = 3\mathbf{M}^2 + \mathbf{M} - 3\mathbf{I}$ $\mathbf{M}^4 = 3\mathbf{M}^3 + \mathbf{M}^2 - 3\mathbf{M}$ $= 3(3\mathbf{M}^2 + \mathbf{M} - 3\mathbf{I}) + \mathbf{M}^2 - 3\mathbf{M}$ $= 10\mathbf{M}^2 - 9\mathbf{I}$ $a=10,\ b=0,\ c=-9$	B1 M1 A1 M1 A1 **5**	In any correct form *(Condone omission of =0)* **M** satisfies the characteristic equation Correct expanded form *(Condone omission of* **I** *)*

4 (i)	$\cosh^2 x = [\frac{1}{2}(e^x + e^{-x})]^2 = \frac{1}{4}(e^{2x} + 2 + e^{-2x})$ $\sinh^2 x = [\frac{1}{2}(e^x - e^{-x})]^2 = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$ $\cosh^2 x - \sinh^2 x = \frac{1}{4}(2+2) = 1$	B1 B1 B1 ag **3**	For completion
	OR $\cosh x + \sinh x = \frac{1}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x}) = e^x$ B1 $\cosh x - \sinh x = \frac{1}{2}(e^x + e^{-x}) - \frac{1}{2}(e^x - e^{-x}) = e^{-x}$ B1 $\cosh^2 x - \sinh^2 x = e^x \times e^{-x} = 1$ B1		Completion
(ii)	$4(1+\sinh^2 x) + 9\sinh x = 13$ $4\sinh^2 x + 9\sinh x - 9 = 0$ $\sinh x = \frac{3}{4},\ -3$ $x = \ln 2,\ \ln(-3+\sqrt{10})$	M1 M1 A1A1 A1A1 ft **6**	(M0 for $1 - \sinh^2 x$) Obtaining a value for $\sinh x$ Exact logarithmic form *Dep on M1M1* Max A1 if any extra values given
	OR $2e^{4x} + 9e^{3x} - 22e^{2x} - 9e^x + 2 = 0$ $(2e^{2x} - 3e^x - 2)(e^{2x} + 6e^x - 1) = 0$ M1 M1 $e^x = 2,\ -3+\sqrt{10}$ A1A1 $x = \ln 2,\ \ln(-3+\sqrt{10})$ A1A1 ft		Quadratic and / or linear factors Obtaining a value for e^x Ignore extra values *Dependent on M1M1* Max A1 if any extra values given *Just* $x = \ln 2$ *earns M0M1A1A0A0A0*
(iii)	$\frac{dy}{dx} = 8\cosh x \sinh x + 9\cosh x$ $= \cosh x(8\sinh x + 9)$ $= 0$ only when $\sinh x = -\frac{9}{8}$ $\cosh^2 x = 1 + (-\frac{9}{8})^2 = \frac{145}{64}$ $y = 4 \times \frac{145}{64} + 9 \times (-\frac{9}{8}) = -\frac{17}{16}$	B1 B1 M1 A1 **4**	Any correct form *or* $y = (2\sinh x + \frac{9}{4})^2 + \ldots\ (-\frac{17}{16})$ Correctly showing there is only one solution Exact evaluation of *y* or $\cosh^2 x$ or $\cosh 2x$ Give B2 (replacing M1A1) for -1.06 or better
(iv)	$\int_0^{\ln 2} (2 + 2\cosh 2x + 9\sinh x)\, dx$ $= [\ 2x + \sinh 2x + 9\cosh x\ ]_0^{\ln 2}$ $= \left\{ 2\ln 2 + \frac{1}{2}\left(4 - \frac{1}{4}\right) + \frac{9}{2}\left(2 + \frac{1}{2}\right) \right\} - 9$ $= 2\ln 2 + \frac{33}{8}$	M1 A2 M1 A1 ag **5**	Expressing in integrable form Give A1 for two terms correct $\sinh(2\ln 2) = \frac{1}{2}(4 - \frac{1}{4})$ *Must see both terms for M1* *Must also see* $\cosh(\ln 2) = \frac{1}{2}(2 + \frac{1}{2})$ *for A1*

	OR $\int_0^{\ln 2} (e^{2x} + 2 + e^{-2x} + \frac{9}{2}(e^x - e^{-x}))\,dx$ M1 $= \left[\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + \frac{9}{2}e^x + \frac{9}{2}e^{-x} \right]_0^{\ln 2}$ A2 $= \left(2 + 2\ln 2 - \frac{1}{8} + 9 + \frac{9}{4}\right) - \left(\frac{1}{2} - \frac{1}{2} + \frac{9}{2} + \frac{9}{2}\right)$ M1 $= 2\ln 2 + \frac{33}{8}$ A1 ag		Expanded exponential form (M0 if the 2 is omitted) Give A1 for three terms correct $e^{2\ln 2} = 4$ and $e^{-2\ln 2} = \frac{1}{4}$ *both seen* *Must also see* $e^{\ln 2} = 2$ and $e^{-\ln 2} = \frac{1}{2}$ *for A1*
5 (i)	$\lambda = 0.5$ $\lambda = 3$ $\lambda = 5$ 	B1B1B1 **3**	
(ii)	Ellipse	B1 **1**	
(iii)	$y = \sqrt{2}\cos(\theta - \frac{1}{4}\pi)$ Maximum $y = \sqrt{2}$ when $\theta = \frac{1}{4}\pi$	M1 A1 ag **2**	or $\sqrt{2}\sin(\theta + \frac{1}{4}\pi)$
	OR $\frac{dy}{d\theta} = -\sin\theta + \cos\theta = 0$ when $\theta = \frac{1}{4}\pi$ M1 $y = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ A1		
(iv)	$x^2 + y^2 = \lambda^2\cos^2\theta - 2\cos\theta\sin\theta + \frac{1}{\lambda^2}\sin^2\theta$ $+\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta$ $= (\lambda^2 + 1)(1 - \sin^2\theta) + (\frac{1}{\lambda^2} + 1)\sin^2\theta$ $= 1 + \lambda^2 + (\frac{1}{\lambda^2} - \lambda^2)\sin^2\theta$ When $\sin^2\theta = 0$, $x^2 + y^2 = 1 + \lambda^2$ When $\sin^2\theta = 1$, $x^2 + y^2 = 1 + \frac{1}{\lambda^2}$ Since $0 \le \sin^2\theta \le 1$, distance from O, $\sqrt{x^2 + y^2}$, is between $\sqrt{1 + \frac{1}{\lambda^2}}$ and $\sqrt{1 + \lambda^2}$	M1 M1 A1 ag M1 M1 A1 ag **6**	Using $\cos^2\theta = 1 - \sin^2\theta$
(v)	When $\lambda = 1$, $x^2 + y^2 = 2$ Curve is a circle (centre O) with radius $\sqrt{2}$	M1 A1 **2**	

(vi)		B4 **4**	A, E at maximum distance from O C, G at minimum distance from O B, F are stationary points D, H are on the *x*-axis Give ½ mark for each point, then round down Special properties must be clear from diagram, or stated *Max 3 if curve is not the correct shape*

4757 (FP3) Further Applications of Advanced Mathematics

1 (i)	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}6\\8\\5\end{pmatrix}\times\begin{pmatrix}10\\-5\\1\end{pmatrix}=\begin{pmatrix}33\\44\\-110\end{pmatrix}$	B2	*Ignore subsequent working* Give B1 for one element correct SC1 for minus the correct vector
	ABC is $3x+4y-10z=-9+20-20$ $3x+4y-10z+9=0$	M1 A1 **4**	For $3x+4y-10z$ Accept $33x+44y-110z=-99$ etc
(ii)	Distance is $\dfrac{3\times5+4\times4-10\times8+9}{\sqrt{3^2+4^2+10^2}}$	M1 A1 ft	Using distance formula (or other complete method)
	$=(-)\ \dfrac{40}{\sqrt{125}}\quad\left(=\dfrac{8}{\sqrt5}\right)$	A1 **3**	*Condone negative answer* *Accept a.r.t. 3.58*
(iii)	$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{pmatrix}6\\8\\5\end{pmatrix}\times\begin{pmatrix}-2\\4\\5\end{pmatrix}=\begin{pmatrix}20\\-40\\40\end{pmatrix}\quad\left[=20\begin{pmatrix}1\\-2\\2\end{pmatrix}\right]$	M1 A1	Evaluating $\overrightarrow{AB}\times\overrightarrow{CD}$ *or* method for finding end-points of common perp PQ *or* P $(\tfrac{3}{2},11,\tfrac{23}{4})$ & Q $(\tfrac{71}{18},\tfrac{55}{9},\tfrac{383}{36})$ *or* $\overrightarrow{PQ}=(\tfrac{22}{9},-\tfrac{44}{9},\tfrac{44}{9})$
	Distance is $\overrightarrow{AC}\,.\,\hat{\mathbf{n}}=\dfrac{\begin{pmatrix}10\\-5\\1\end{pmatrix}.\begin{pmatrix}1\\-2\\2\end{pmatrix}}{\sqrt{1^2+2^2+2^2}}$	M1	
	$=\dfrac{22}{3}$	A1 **4**	
(iv)	Volume is $\tfrac16(\overrightarrow{AB}\times\overrightarrow{AC})\,.\,\overrightarrow{AD}$	M1 A1	Scalar triple product
	$=\dfrac16\begin{pmatrix}33\\44\\-110\end{pmatrix}.\begin{pmatrix}8\\-1\\6\end{pmatrix}$	M1	
	$=(-)\ \dfrac{220}{3}$	A1 **4**	*Accept a.r.t. 73.3*
(v)	E is $(-3+10\lambda,\ 5-5\lambda,\ 2+\lambda)$ $3(-3+10\lambda)-2(2+\lambda)+5=0$	M1	
	$\lambda=\dfrac27$	A1	
	F is $(-3+8\mu,\ 5-\mu,\ 2+6\mu)$ $3(-3+8\mu)-2(2+6\mu)+5=0$	M1	
	$\mu=\dfrac23$	A1	
	Since $0<\lambda<1$, E is between A and C Since $0<\mu<1$, F is between A and D	B1 **5**	

(vi)	$V_{\text{ABEF}} = \frac{1}{6}(\overrightarrow{\text{AB}} \times \overrightarrow{\text{AE}}) \,.\, \overrightarrow{\text{AF}}$ $= \frac{1}{6}\lambda\mu(\overrightarrow{\text{AB}} \times \overrightarrow{\text{AC}}) \,.\, \overrightarrow{\text{AD}}$ $= \lambda\mu V_{\text{ABCD}}$ $= \frac{4}{21}V_{\text{ABCD}}$ Ratio of volumes is $\frac{4}{21} : \frac{17}{21}$ $= 4 : 17$	M1 A1 M1 A1 ag **4**	($13\frac{61}{63}$) *ft if numerical* Finding ratio of volumes of two parts SC1 for 4 : 17 deduced from $\frac{4}{21}$ without working
2 (i)	$\frac{\partial g}{\partial x} = 6z - 2(x+2y+3z) = -2x-4y$ $\frac{\partial g}{\partial y} = -4(x+2y+3z)$ $\frac{\partial g}{\partial z} = 6x - 6(x+2y+3z) = -12y-18z$	M1 A1 A1 A1 **4**	Partial differentiation *Any correct form, ISW*
(ii)	At P, $\frac{\partial g}{\partial x} = 16$, $\frac{\partial g}{\partial y} = -4$, $\frac{\partial g}{\partial z} = 36$ Normal line is $\mathbf{r} = \begin{pmatrix} 7 \\ -7.5 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$	M1 A1 A1 ft **3**	Evaluating partial derivatives at P All correct *Condone omission of* ' **r =** '
(iii)	$\delta g \approx 16\delta x - 4\delta y + 36\delta z$ If $\overrightarrow{\text{PQ}} = \lambda\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$, $\delta g \approx 16(4\lambda) - 4(-\lambda) + 36(9\lambda)$ $(= 392\lambda)$ $h = \delta g$, so $h \approx 392\lambda$ $\overrightarrow{\text{PQ}} \approx \frac{h}{392}\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$, so $\mathbf{n} = \frac{1}{392}\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$	M1 M1 A1 ft M1 A1 **5**	*Alternative:* *M3 for substituting* $x = 7+4\lambda$, … *into* $g = 125+h$ *and neglecting* λ^2 *A1 ft for linear equation in* λ *and* h *A1 for* **n** *correct*
(iv)	Require $\frac{\partial g}{\partial x} = \frac{\partial g}{\partial y} = 0$ $-2x-4y=0$ and $x+2y+3z=0$ $x+2y=0$ and $z=0$ $g(x, y, z) = 0 - 0^2 = 0 \neq 125$ Hence there is no such point on *S*	M1 M1 M1 A1 **4**	Useful manipulation using both eqns Showing there is no such point on *S* Fully correct proof
(v)	Require $\frac{\partial g}{\partial z} = 0$ and $\frac{\partial g}{\partial y} = 5\frac{\partial g}{\partial x}$ $-4x-8y-12z = 5(-2x-4y)$	M1 M1 M1	Implied by $\frac{\partial g}{\partial x} = \lambda$, $\frac{\partial g}{\partial y} = 5\lambda$ *This M1 can be awarded for* $-2x-4y=1$ and $-4x-8y-12z=5$

	$y=-\frac{3}{2}z$ and $x=5z$ $6(5z)z-(5z)^2=125$ $z=\pm 5$ Points are $(25,\ -7.5,\ 5)$ and $(-25,\ 7.5,\ -5)$	A1 M1 M1 A1 A1 ft **8**	*or* $z=-\frac{2}{3}y$ and $x=-\frac{10}{3}y$ *or* $y=-\frac{3}{10}x$ and $z=\frac{1}{5}x$ *or* $x=-\frac{5}{4}\lambda,\ y=\frac{3}{8}\lambda,\ z=-\frac{1}{4}\lambda$ *or* $x:y:z=10:-3:2$ Substituting into $g(x,y,z)=125$ Obtaining one value of *x, y, z or* λ *Dependent on previous M1* *ft is minus the other point, provided all M marks have been earned*
3 (i)	$\dot{x}^2+\dot{y}^2=(24t^2)^2+(18t-8t^3)^2$ $=576t^4+324t^2-288t^4+64t^6$ $=324t^2+288t^4+64t^6$ $=(18t+8t^3)^2$ Arc length is $\int_0^2 (18t+8t^3)\,dt$ $=\left[\ 9t^2+2t^4\ \right]_0^2$ $=68$	B1 M1 A1 ag M1 A1 A1 **6**	*Note* $\int_0^2 (18+8t^3)\,dt\ =\left[\ 18t+2t^4\ \right]_0^2=68$ *earns M1A0A0*
(ii)	Curved surface area is $\int 2\pi y\,ds$ $=\int_0^2 2\pi(9t^2-2t^4)(18t+8t^3)\,dt$ $=\int_0^2 \pi(324t^3+72t^5-32t^7)\,dt$ $=\pi\left[\ 81t^4+12t^6-4t^8\ \right]_0^2$ $=1040\pi\quad(\approx 3267)$	M1 M1 A1 M1 M1 A1 **6**	Using $ds=(18t+8t^3)\,dt$ Correct integral expression including limits *(may be implied by later work)*
(iii)	$\kappa=\dfrac{\dot{x}\ddot{y}-\ddot{x}\dot{y}}{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}=\dfrac{(24t^2)(18-24t^2)-(48t)(18t-8t^3)}{(18t+8t^3)^3}$ $=\dfrac{48t^2(9-12t^2-18+8t^2)}{8t^3(9+4t^2)^3}=\dfrac{-48t^2(9+4t^2)}{8t^3(9+4t^2)^3}$ $=\dfrac{-6}{t(4t^2+9)^2}$	M1 A1A1 M1 A1 ag **5**	Using formula for κ (or ρ) For numerator and denominator Simplifying the numerator

(iv)	When $t=1,\ x=8,\ y=7,\ \kappa=-\frac{6}{169}$ $\rho=(-)\,\dfrac{169}{6}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\dot{y}}{\dot{x}}=\dfrac{18t-8t^3}{24t^2}=\dfrac{10}{24}$ $\hat{\mathbf{n}}=\begin{pmatrix}5/13\\-12/13\end{pmatrix}$ $\mathbf{c}=\begin{pmatrix}8\\7\end{pmatrix}+\frac{169}{6}\begin{pmatrix}5/13\\-12/13\end{pmatrix}$ Centre of curvature is $(18\frac{5}{6},\ -19)$	B1 M1 M1 A1 M1 A1A1 **7**	Finding gradient (or tangent vector) Finding direction of the normal Correct unit normal (either direction)
4 (i)	*Commutative:* $x*y=y*x$ (for all *x, y*) *Associative:* $(x*y)*z=x*(y*z)$ (for all *x, y, z*)	B1 B2 **3**	Accept e.g. 'Order does not matter' Give B1 for a partial explanation, e.g. 'Position of brackets does not matter'
(ii)	$2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=2xy+x+y+\frac{1}{2}-\frac{1}{2}$ $=2xy+x+y=x*y$	B1 ag **1**	*Intermediate step required*
(iii)(*A*)	If $x,y\in S$ then $x>-\frac{1}{2}$ and $y>-\frac{1}{2}$ $x+\frac{1}{2}>0$ and $y+\frac{1}{2}>0$, so $2(x+\frac{1}{2})(y+\frac{1}{2})>0$ $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}>-\frac{1}{2}$, so $x*y\in S$	M1 A1 A1 **3**	
(*B*)	0 is the identity since $0*x=0+x+0=x$ If $x\in S$ and $x*y=0$ then $2xy+x+y=0$ $y=\dfrac{-x}{2x+1}$ $y+\frac{1}{2}=\dfrac{1}{2(2x+1)}>0$ (since $x>-\frac{1}{2}$) so $y\in S$ *S* is closed and associative; there is an identity; and every element of *S* has an inverse in *S*	B1 B1 M1 A1 M1 A1 **6**	or $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=0$ or $y+\frac{1}{2}=\dfrac{1}{4(x+\frac{1}{2})}$ *Dependent on M1A1M1*
(iv)	If $x*x=0,\ \ 2x^2+x+x=0$ $x=0$ or -1 0 is the identity (and has order 1) -1 is not in *S*	M1 A1 A1 **3**	

(v)	$4*6=48+4+6=58$ $=56+2=7\times8+2$ So $4\circ6=2$	B1 B1 ag **2**	
(vi)	Element: 0, 1, 2, 4, 5, 6 Order: 1, 6, 6, 3, 3, 2	B3 **3**	Give B2 for 4 correct B1 for 2 correct
(vii)	$\{0\}$, G $\{0,\ 6\}$ $\{0,\ 4,\ 5\}$	B1 B1 B1 **3**	*Condone omission of G* If more than 2 non-trivial subgroups are given, deduct 1 mark (from final B1B1) for each non-trivial subgroup in excess of 2

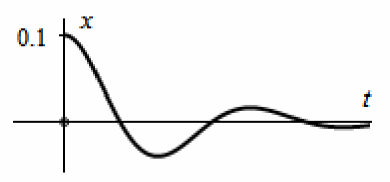
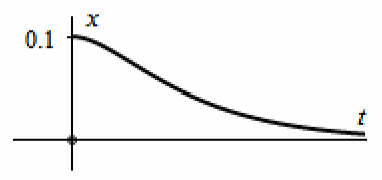
Pre-multiplication by transition matrix

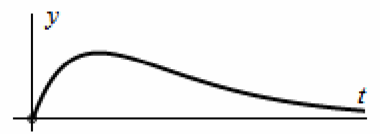
5 (i)	$\mathbf{P}=\begin{pmatrix}0.1 & 0.7 & 0.1\\ 0.4 & 0.2 & 0.6\\ 0.5 & 0.1 & 0.3\end{pmatrix}$	B2	**2**	Give B1 for two columns correct
(ii)	$\mathbf{P}^6\begin{pmatrix}1/3\\ 1/3\\ 1/3\end{pmatrix}=\begin{pmatrix}0.328864\\ 0.381536\\ 0.2896\end{pmatrix}$ P(*B* used on 7th day) $=0.3815$	M1 M1 M1 A1	**4**	Using $\mathbf{P}^6$ (or $\mathbf{P}^7$) For matrix of initial probabilities For evaluating matrix product *Accept 0.381 to 0.382*
(iii)	$0.328864\times0.1+0.381536\times0.2+0.2896\times0.3$ $=0.1961$	M1 M1 A1	**3**	Using diagonal elements from **P** Correct method *Accept a.r.t. 0.196*
(iv)	$\mathbf{P}^3=\begin{pmatrix}0.352 & 0.328 & 0.304\\ 0.364 & 0.404 & 0.372\\ 0.284 & 0.268 & 0.324\end{pmatrix}$ $0.328864\times0.352+0.381536\times0.404+0.2896\times0.324$ $=0.3637$	M1 M1 M1 A1	**4**	For evaluating $\mathbf{P}^3$ Using diagonal elements from $\mathbf{P}^3$ Correct method *Accept a.r.t. 0.364*
(v)	$\mathbf{Q}=\begin{pmatrix}0.3289 & 0.3289 & 0.3289\\ 0.3816 & 0.3816 & 0.3816\\ 0.2895 & 0.2895 & 0.2895\end{pmatrix}$ 0.3289, 0.3816, 0.2895 are the long-run probabilities for the routes *A, B, C*	B1 B1 B1 B1	**4**	*Deduct 1 if not given as a (3×3) matrix* *Deduct 1 if not 4 dp* *Accept 'equilibrium probabilities'*
(vi)	$\begin{pmatrix}0.1 & 0.7 & a\\ 0.4 & 0.2 & b\\ 0.5 & 0.1 & c\end{pmatrix}\begin{pmatrix}0.4\\ 0.2\\ 0.4\end{pmatrix}=\begin{pmatrix}0.4\\ 0.2\\ 0.4\end{pmatrix}$ $0.04+0.14+0.4a=0.4$, so $a=0.55$ $0.16+0.04+0.4b=0.2$, so $b=0$ $0.2+0.02+0.4c=0.4$, so $c=0.45$ After *C*, routes *A, B, C* will be used with probabilities 0.55, 0, 0.45	M1 M1 A2	**4**	Obtaining a value for *a, b* or *c* Give A1 for one correct
(vii)	$0.4\times0.1+0.2\times0.2+0.4\times0.45$ $=0.26$	M1 M1 A1	**3**	Using long-run probs 0.4, 0.2, 0.4 Using diag elements from new matrix

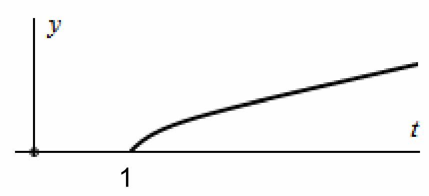
Post-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.3 \end{pmatrix}$	B2	**2**	Give B1 for two rows correct
(ii)	$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \mathbf{P}^6 = \begin{pmatrix} 0.328864 & 0.381536 & 0.2896 \end{pmatrix}$ P(*B* used on 7th day) $= 0.3815$	M1 M1 M1 A1	**4**	Using $\mathbf{P}^6$ (or $\mathbf{P}^7$) For matrix of initial probabilities For evaluating matrix product *Accept 0.381 to 0.382*
(iii)	$0.328864 \times 0.1 + 0.381536 \times 0.2 + 0.2896 \times 0.3$ $= 0.1961$	M1 M1 A1	**3**	Using diagonal elements from **P** Correct method *Accept a.r.t. 0.196*
(iv)	$\mathbf{P}^3 = \begin{pmatrix} 0.352 & 0.364 & 0.284 \\ 0.328 & 0.404 & 0.268 \\ 0.304 & 0.372 & 0.324 \end{pmatrix}$ $0.328864 \times 0.352 + 0.381536 \times 0.404 + 0.2896 \times 0.324$ $= 0.3637$	M1 M1 M1 A1	**4**	For evaluating $\mathbf{P}^3$ Using diagonal elements from $\mathbf{P}^3$ Correct method *Accept a.r.t. 0.364*
(v)	$\mathbf{Q} = \begin{pmatrix} 0.3289 & 0.3816 & 0.2895 \\ 0.3289 & 0.3816 & 0.2895 \\ 0.3289 & 0.3816 & 0.2895 \end{pmatrix}$ 0.3289, 0.3816, 0.2895 are the long-run probabilities for the routes *A, B, C*	B1B1B1 B1	**4**	*Deduct 1 if not given as a (3×3) matrix* *Deduct 1 if not 4 dp* *Accept 'equilibrium probabilities'*
(vi)	$\begin{pmatrix} 0.4 & 0.2 & 0.4 \end{pmatrix} \begin{pmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.1 \\ a & b & c \end{pmatrix} = \begin{pmatrix} 0.4 & 0.2 & 0.4 \end{pmatrix}$ $0.04 + 0.14 + 0.4a = 0.4$, so $a = 0.55$ $0.16 + 0.04 + 0.4b = 0.2$, so $b = 0$ $0.2 + 0.02 + 0.4c = 0.4$, so $c = 0.45$ After *C*, routes *A, B, C* will be used with probabilities 0.55, 0, 0.45	M1 M1 A2	**4**	Obtaining a value for *a, b* or *c* Give A1 for one correct
(vii)	$0.4 \times 0.1 + 0.2 \times 0.2 + 0.4 \times 0.45$ $= 0.26$	M1 M1 A1	**3**	Using long-run probs 0.4, 0.2, 0.4 Using diag elements from new matrix

4758 Differential Equations

1 (i)	$2\ddot{x} = 2g - 8(x + 0.25g) - 2kv$	M1	N2L equation with all forces using given expressions for tension and resistance	
	Weight positive as down, tension negative as up.	B1		
	Resistance negative as opposes motion.	B1		
	$\Rightarrow \ddot{x} + k\dot{x} + 4x = 0$	E1	Must follow correct N2L equation	
				4
(ii)	$x = A\cos 2t + B\sin 2t$	B1		
	$t = 0, x = 0.1 \Rightarrow A = 0.1$	M1	Find the coefficient of cos	
	$\dot{x} = -2A\sin 2t + 2B\cos 2t$ so $t = 0, \dot{x} = 0 \Rightarrow B = 0$	M1	Find the coefficient of sin	
	$x = 0.1\cos 2t$	A1	cao	
				4
(iii)	$\alpha^2 + 2\alpha + 4 = 0$	M1	Auxiliary equation	
	$\alpha = -1 \pm \sqrt{3}\,j$	A1		
		M1	CF for complex roots	
	$x = e^{-t}\left(C\cos\sqrt{3}\,t + D\sin\sqrt{3}\,t\right)$	F1	CF for their roots	
	$t = 0, x = 0.1 \Rightarrow C = 0.1$	M1	Condition on x	
	$\dot{x} = -e^{-t}\left(C\cos\sqrt{3}\,t + D\sin\sqrt{3}\,t\right)$ $+e^{-t}\left(-\sqrt{3}\,C\sin\sqrt{3}\,t + \sqrt{3}\,D\cos\sqrt{3}\,t\right)$	M1	Differentiate (product rule)	
	$0 = -C + \sqrt{3}\,D$	M1	Condition on $\dot{x}$	
	$D = \frac{0.1}{\sqrt{3}}$			
	$x = 0.1e^{-t}\left(\cos\sqrt{3}\,t + \frac{1}{\sqrt{3}}\sin\sqrt{3}\,t\right)$	A1	cao	
		B1 B1 B1	Curve through (0,0.1) with zero gradient Oscillating Asymptote x = 0	
				11
(iv)	$k^2 - 4\cdot 1\cdot 4 > 0$	M1	Use of discriminant	
		A1	Correct inequality	
	(As *k* is positive) $k > 4$	A1	Accept $k < -4$ in addition (but not *k* > –4)	
		B1 B1	Curve through (0,0.1) Decays without oscillating (at most one intercept with positive *t* axis)	
				5

2 (i)	$x = Ae^{-2t}$	M1	Any valid method	
	$t=0, x=8 \Rightarrow A=8$	M1	Condition on x	
	$x = 8e^{-2t}$	A1		
				3
(ii)	$\dot{y} + y = 16e^{-2t}$	M1	Substitute for x	
	$\alpha + 1 = 0 \Rightarrow \alpha = -1$	M1	Auxiliary equation	
	CF $y = Be^{-t}$	A1		
	PI $y = ae^{-2t}$	B1		
	$-2ae^{-2t} + ae^{-2t} = 16e^{-2t}$	M1	Differentiate and substitute	
	$a = -16$	A1	cao	
	GS $y = -16e^{-2t} + Be^{-t}$	F1	Their PI + CF (with one arbitrary constant)	
	$t=0, y=0 \Rightarrow B=16$	M1	Condition on y	
	$y = 16\left(e^{-t} - e^{-2t}\right)$	F1	Follow a non-trivial GS	
	Alternative mark scheme for first 7 marks:			
		M1	*Substitute for x*	
	$I = e^{t}$	*M1*	*Attempt integrating factor*	
		A1	*IF correct*	
	$d(ye^{t})/dt = 16e^{-t}$	*B1*		
		M1	*Integrate*	
	$ye^{t} = -16e^{-t} + B$	*A1*	*cao*	
	$y = -16e^{-2t} + Be^{-t}$	*F1*	*Divide by their I (must divide constant)*	
				9
(iii)	$y = 16e^{-t}\left(1 - e^{-t}\right)$	M1	Or equivalent (NB $e^{-t} > e^{-2t}$ needs justifying)	
	$16e^{-t} > 0$ and $t > 0 \Rightarrow e^{-t} < 1$ hence $y > 0$	E1	Complete argument	
		B1	Starts at origin	
		B1	General shape consistent with their solution and $y > 0$	
		B1	Tends to zero	
				5
(iv)	$\frac{d}{dt}(x+y+z) = (-2x) + (2x - y) + (y) = 0$	M1	Consider sum of DE's	
	$\Rightarrow x + y + z = c$	E1		
	Hence initial conditions $\Rightarrow x + y + z = 8$	E1		
	$z = 8 - x - y$	M1	Substitute for x and y and find z	
	$z = 8\left(1 - 2e^{-t} + e^{-2t}\right) = 8\left(1 - e^{-t}\right)^2$	E1	Convincingly shown (x, y must be correct)	
				5
(v)	$0.99 \times 8 = 8\left(1 - e^{-t}\right)^2$	B1	Correct equation (any form)	
	$t = -0.690638$ or 5.29581			
	99% is Z after 5.30 hours	B1	Accept value in [5.29, 5.3]	
				2

3 (i)	$\dot{y}+\frac{k}{t}y=1$	M1	Divide by *t* (condone LHS only)	
	$I=\exp\left(\int \frac{k}{t}\,\mathrm{d}t\right)=\exp(k\ln t)=t^k$	M1	Attempt integrating factor	
		A1	Integrating factor	
	$t^k\dot{y}+kt^{k-1}y=t^k$	F1	Multiply DE by their *I*	
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(yt^k\right)=t^k$	M1	LHS	
	$yt^k=\int t^k\,\mathrm{d}t$	M1	Integrate	
	$=\frac{1}{k+1}t^{k+1}+A$	A1	cao (including constant)	
	$y=\frac{1}{k+1}t+At^{-k}$	F1	Divide by their *I* (must divide constant)	
	$t=1, y=0 \Rightarrow 0=\frac{1}{k+1}+A \Rightarrow A=-\frac{1}{k+1}$	M1	Use condition	
	$y=\frac{1}{k+1}\left(t-t^{-k}\right)$	F1	Follow a non-trivial GS	10
(ii)	$y=\frac{1}{3}\left(t-t^{-2}\right)$ 	B1 B1 B1	Shape consistent with their solution for $t \geq 1$ Passes through (1, 0) Behaviour for large *t*	3
(iii)	$yt^{-1}=\int t^{-1}\,\mathrm{d}t$	M1	Follow their (i)	
	$=\ln t+B$	A1	cao	
	$y=t(\ln t+B)$	F1	Divide by their *I* (must divide constant)	
	$t=1, y=0 \Rightarrow B=0 \Rightarrow y=t\ln t$	A1	cao	4
(iv)	$\frac{\mathrm{d}y}{\mathrm{d}t}=1+t^{-1}\sin y$	M1	Rearrange DE (may be implied)	
	t / *y* / d*y*/d*t* 1 / 0 / 1 1.1 / 0.1 / 1.0908 1.2 / 0.2091	M1 A1 A1	Use algorithm *y*(1.1) *y*(1.2)	4
(v)	0.2138 as smaller step size	B1	Must give reason	
	Decreasing step length has increased estimate. Assuming this estimate is more accurate, decreasing step length further will increase estimate further, so true value likely to be greater.	M1	Identify effect of decreasing step length	
	Hence underestimates.	A1	Convincing argument	
	Alternative mark scheme for last 2 marks: *dy/dt seems to be increasing, hence Euler's method*	*M1*	*Identify derivative increasing*	
	will underestimate true value + sketch (or explanation).	*A1*	*Convincing argument*	3

4 (i)	$\ddot{x} = 4\dot{x} - 6\dot{y} - 9\cos t$	M1	Differentiate first equation	
	$= 4\dot{x} - 6(3x - 5y - 7\sin t) - 9\cos t$	M1	Substitute for $\dot{y}$	
	$y = \frac{1}{6}(4x - \dot{x} - 9\sin t)$	M1	y in terms of $x, \dot{x}$	
	$\ddot{x} = 4\dot{x} - 18x + 5(4x - \dot{x} - 9\sin t) + 42\sin t - 9\cos t$	M1	Substitute for *y*	
	$\ddot{x} + \dot{x} - 2x = -3\sin t - 9\cos t$	E1	LHS	
		E1	RHS	
				6
(ii)	$\alpha^2 + \alpha - 2 = 0$	M1	Auxiliary equation	
	$\alpha = 1$ or -2	A1		
	CF $x = Ae^t + Be^{-2t}$	F1	CF for their roots	
	PI $x = a\cos t + b\sin t$	B1	PI of this form	
	$(-ac - bs) + (-as + bc) - 2(ac + bs) = -3s - 9c$	M1	Differentiate twice and substitute	
	$-a + b - 2a = -9$	M1	Compare coefficients (2 equations)	
	$-b - a - 2b = -3$	M1	Solve (2 equations)	
	$\Rightarrow a = 3, b = 0$	A1		
	$x = 3\cos t + Ae^t + Be^{-2t}$	F1	Their PI + CF (with two arbitrary constants)	
				9
(iii)	$y = \frac{1}{6}(4x - \dot{x} - 9\sin t)$	M1	y in terms of $x, \dot{x}$	
	$= \frac{1}{6}(12\cos t + 4Ae^t + 4Be^{-2t} + 3\sin t - Ae^t + 2Be^{-2t} - 9\sin t)$	M1	Differentiate *x* and substitute	
	$y = 2\cos t - \sin t + \frac{1}{2}Ae^t + Be^{-2t}$	A1	Constants must correspond with those in *x*	
				3
(iv)	*x* bounded $\Rightarrow A = 0$	M1	Identify coefficient of exponentially growing term must be zero	
	$\Rightarrow y$ bounded	E1	Complete argument	
				2
(v)	$t = 0, y = 0 \Rightarrow 0 = B + 2 \Rightarrow B = -2$	M1	Condition on *y*	
	$x = 3\cos t - 2e^{-2t}$, $y = 2\cos t - \sin t - 2e^{-2t}$	F1	Follow their (non-trivial) general solutions	
	$x = 3\cos t$	A1	cao	
	$y = 2\cos t - \sin t$	A1	cao	
				4

4761 Mechanics 1

Q 1		mark	comment	sub
(i)	N2L ↑ $1000 - 100 \times 9.8 = 100a$ *a* = 0.2 so 0.2 m s^{-2} upwards	M1 B1 A1	N2L. Accept *F* = *mga* and no weight Weight correct (including sign). Allow if seen. Accept ± 0.2. Ignore units and direction	3
(ii)	$T_{BA} - 980 = 100 \times 0.8$ so tension is 1060 N	M1 A1	N2L. *F* = *ma*. Weight present, no extras. Accept sign errors.	2
(iii)	$T_{BA} \cos 30 = 1060$ $T_{BA} = 1223.98...$ so 1220 N (3 s. f.)	M1 A1 A1	Attempt to resolve their (ii). Do not award for **their** 1060 resolved unless all forces present and all resolutions needed are attempted. If start again allow no weight. Allow $\sin \leftrightarrow \cos$. No extra forces. Condone sign errors FT **their** 1060 only cao	3
		8		

Q 2		mark	comment	sub
(i)		B1	Sketch. O, **i**, **j** and **r** (only require correct quadrant.) Vectors must have arrows. Need not label **r**.	1
(ii)	$\sqrt{4^2 + (-5)^2}$ = $\sqrt{41}$ or 6.4031… so 6.40 (3 s. f.) Need $180 - \arctan\left(\frac{4}{5}\right)$ 141.340 so 141°	M1 A1 M1 A1	Accept $\sqrt{4^2 - 5^2}$ Or equivalent. Award for $\arctan\left(\pm\frac{4}{5}\right)$ or $\arctan\left(\pm\frac{5}{4}\right)$ or equivalent seen without 180 or 90. cao	4
(iii)	12**i** – 15**j** or $\begin{pmatrix} 12 \\ -15 \end{pmatrix}$	B1	Do not award for magnitude given as the answer. Penalise spurious notation by 1 mark at most once in paper	1
		6		

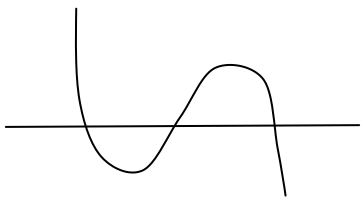
Q 3		mark	comment	sub
			Penalise spurious notation by 1 mark at most once in paper	
(i)	**F** = $5\begin{pmatrix}-1\\2\end{pmatrix}=\begin{pmatrix}-5\\10\end{pmatrix}$ so $\begin{pmatrix}-5\\10\end{pmatrix}$ N	M1	Use of N2L in vector form	
		A1	Ignore units. [Award 2 for answer seen] [SC1 for $\sqrt{125}$ or equiv seen]	2
(ii)	**s** = $\begin{pmatrix}-2\\3\end{pmatrix}+4\begin{pmatrix}4\\5\end{pmatrix}+\frac{1}{2}\times4^2\times\begin{pmatrix}-1\\2\end{pmatrix}$	M1	Use of $\mathbf{s}=t\mathbf{u}+0.5t^2\mathbf{a}$ or integration of **a**. Allow $\mathbf{s}_0$ omitted. If integrated need to consider **v** when t = 0	
		A1	Correctly evaluated; accept $\mathbf{s}_0$ omitted.	
	$\mathbf{s}=\begin{pmatrix}6\\39\end{pmatrix}$ so $\begin{pmatrix}6\\39\end{pmatrix}$ m	B1	Correctly adding $\mathbf{s}_0$ to a vector (FT). Ignore units. [NB $\begin{pmatrix}8\\36\end{pmatrix}$ seen scores M1 A1]	3
		5		

Q 4		mark	comment	sub
(i)	The distance travelled by P is $0.5\times0.5\times t^2$	B1		
	The distance travelled by Q is 10t	B1	Accept 10t + 125 if used correctly below.	2
(ii)	Meet when $0.25t^2=125+10t$	M1	Allow **their** wrong expressions for P and Q distances Allow ± 125 or 125 omitted	
		F1	Award for **their** expressions as long as one is quadratic and one linear. Must have 125 with correct sign.	
	so $t^2-40t-500=0$			
	Solving	M1	Accept any method that yields (smaller) + ve root of their 3 term quadratic	
	t = 50 (or -10)	A1	cao Allow –ve root not mentioned	
	Distance is $0.25\times50^2=625$ m	A1	cao [SC2 400 m seen]	5
		7		

Q 5		mark	comment	sub
	either Overall, N2L → 135 – 9 = (5 +4)*a* *a* = 14 so 14 m s $^{-2}$ For A, N2L → $T - 9 = 4 \times 14$ so 65 N **or** 135 – *T* = 5*a* *T* – 9 = 4*a* Solving *T* = 65 so 65 N	 M1 A1 M1 A1 M1 A1 M1 A1	 Use of N2L. Allow *F* = *mga* but no extra forces. Allow 9 omitted. N2L on A or B with correct mass. *F* = *ma*. All relevant forces and no extras. cao * 1 equation in *T* and *a*. Allow sign errors. Allow *F* = *mga* Both equations correct and consistent Dependent on M* solving for *T*. cao.	4
		4		

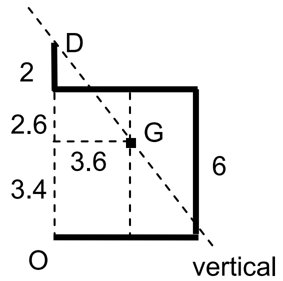
Q 6		mark	comment	sub
(i)	$40 \times 0.6t - 5t^2$ = $24t - 5t^2$	M1 A1	Use of $s = ut + 0.5at^2$ with $a = \pm 9.8,\ \pm 10$. Accept 40 or 40×0.8 for '*u*'. Any form	2
(ii)	**either** Need zero vertical distance so $24t - 5t^2 = 0$ so *t* = 0 or *t* = 4.8 **or** Time to highest point, *T* 0 = 40 × 0.6 – 10*T* so *T* = 2.4 and time of flight is 4.8 range is $40 \times 0.8 \times 4.8$ = 153.6 so 154 m (3 s. f.)	 M1 A1 M1 A1 M1 A1	 Equate **their** *y* to zero. With fresh start must have correct *y*. Accept no reference to *t* = 0 and the other root in any form. FT **their** *y* if gives *t* > 0 Allow use of *u* = 40 and 40 × 0.8. Award even if half range found. May be awarded for doubling half range later. Horiz cpt. Accept 0.6 instead of 0.8 only if consistent with expression in (i). FT **their** *t*. cao [NB Use of half range or half time to get 76.8… (g = 10) or 78.36… (g = 9.8) scores 2] [If range formula used: M1 sensible attempt at substitution; allow sin2α wrong B1 sin2α correct A1 all correct A1 cao]	4
		6		

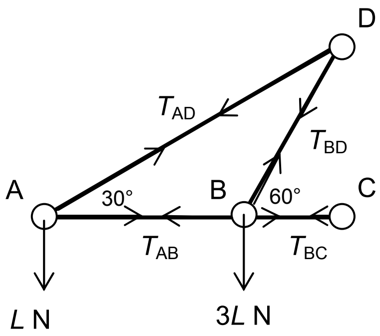
Q 7		mark	comment	sub
(i)	Continuous string: smooth ring: light string	E1 E1	One reason Another reason	2
(ii)	Resolve $\leftarrow$: $60\cos\alpha - 60\cos\beta = 0$ (so $\cos\alpha = \cos\beta$) and so $\alpha = \beta$	M1 E1	[(ii) and (iii) may be argued using Lami or triangle of forces] Resolution and an equation or equivalent. Accept $s \leftrightarrow c$. Accept a *correct* equation seen without method stated. Accept the use of '*T*' instead of '60'. Shown. Must have stated method (allow $\rightarrow$ seen).	2
(iii)	Resolve $\uparrow$ $2 \times 60 \times \sin\alpha - 8g = 0$ so α = 40.7933… so 40.8° (3 s. f.)	M1 B1 B1 A1 A1	Resolution and an equation. Accept $s \leftrightarrow c$. Do not award for resolution that cannot give solution (e.g. horizontal) Both strings used (accept use of half weight), seen in an equation $\sin\alpha$ or equivalent seen in an equation All correct	5
(iv)	Resolve $\rightarrow$ $10 + T_{QC}\cos 25 - T_{PC}\cos 45 = 0$ Resolve $\uparrow$ $T_{PC}\sin 45 + T_{QC}\sin 25 - 8g = 0$ Solving $T_{CQ} = 51.4701...$ so 51.5 N (3 s. f.) $T_{CP} = 80.1120...$ so 80.1 N (3 s. f.)	M1 M1 A1 M1 A1 M1 A1 F1	Recognise strings have different tensions. Resolution and an equation. Accept $s \leftrightarrow c$. No extra forces. All forces present. Allow sign errors. Correct. Any form. Resolution and an equation. Accept $s \leftrightarrow c$. No extra forces. All forces present. Allow sign errors. Correct. Any form. * A method that leads to at least one solution of a pair of simultaneous equations. cao either tension other tension. Allow FT only if M1* awarded [Scale drawing: 1st M1 then A1, A1 for answers correct to 2 s.f.]	8
		17		

Q 8		mark	comment	sub
(i)	10	B1		1
(ii)	$v = 36 + 6t - 6t^2$	M1 A1	Attempt at differentiation	2
(iii)	$a = 6 - 12t$	M1 F1	Attempt at differentiation	2
(iv)	Take a = 0 so $t = 0.5$ and v = 37.5 so 37.5 m s $^{-1}$	M1 A1 A1	Allow table if maximum indicated or implied FT **their** *a* cao Accept no justification given that this is maximum	3
(v)	**either** Solving $36 + 6t - 6t^2 = 0$ so t = -2 or t = 3 **or** Sub the values in the expression for v Both shown to be zero A quadratic so the only roots **then** $x(-2) = -34$ $x(3) = 91$	 M1 B1 E1 M1 E1 B1 B1 B1	 A method for two roots using **their** *v* Factorization or formula or … of **their** expression Shown Allow just 1 substitution shown Both shown Must be a clear argument cao cao	5
(vi)	$\lvert x(3) - x(0)\rvert + \lvert x(4) - x(3)\rvert$ = $\lvert 91 - 10\rvert + \lvert 74 - 91\rvert$ = 98 so 98 m	M1 A1 A1	Considering two parts Either correct cao [SC 1 for $s(4) - s(0) = 64$]	3
(vii)	At the SP of v $x(-2) = -34$ i.e. < 0 and $x(3) = 91$ i.e. > 0 Also $x(-4) = 42$ > 0 and $x(6) = -98 < 0$  so three times	M1 B1 B1	Or any other valid argument e.g find all the zeros, sketch, consider sign changes. Must have some working. If only a sketch, must have correct shape. Doing appropriate calculations e.g. find all 3 zeros; sketch cubic reasonably (showing 3 roots); sign changes in range 3 times seen	3
		19		

4762 Mechanics 2

Q 1		mark	comment	sub
(a)(i)	In **i** direction: $6u - 12 = 18$	M1	Use of I-M	
	so u = 5 i.e. 5**i** m s^{-1}	E1	Accept $6u - 12 = 18$ as total working. Accept 5 instead of 5**i**.	
	either			
	In **i** direction: $0.5v + 12 = 0.5 \times 11$	M1	Use of I-M	
		B1	Use of + 12**i** or equivalent	
	$v = -13$ so -13**i** m s^{-1}	A1	Accept direction indicated by any means	
	or			
	6 × 5 + 0.5 v = 6 × 3 + 0.5 × 11	M1	PCLM	
	v = – 13	A1	Allow only sign errors	
	so – 13**i** m s^{-1}	A1	Accept direction indicated by any means	5
(ii)	Using NEL: $\dfrac{11-3}{-13-5} = -e$	M1	Use of NEL. Condone sign errors but not reciprocal expression	
		F1	FT only **their** -13 (even if +ve)	
	$e = 4/9$ $(0.\dot{4})$	F1	FT only **their** -13 and only if –ve (allow 1 s.f. accuracy)	3
(iii)	In **i** direction: $-2 \times 7 = 0.5v - 0.5 \times 11$	M1	Use of **I** = **F**t	
		M1	Use of **I** = m(**v** –**u**)	
	v = -17 so -17 **i** m s^{-1}	A1	For ± 17	
		A1	cao. Direction (indicated by any means)	
	or			
	– 2 **i** = 0.5 a	M1	Use of **F** = m**a**	
	so a = – 4 **i** m s^{-2}	A1	For ± 4	
	v = 11**i** – 4**i** × 7	M1	Use of **uvas**t	
	v = -17 so -17 **i** m s^{-1}	A1	cao. Direction (indicated by any means)	4
(b)	u **i** + ev **j**	B1	For u	
		B1	For ev	
	$\tan\alpha = \dfrac{v}{u}$, $\tan\beta = \dfrac{ev}{u}$	M1	Use of tan. Accept reciprocal argument. Accept use of **their** components	
		B1	Both correct. Ignore signs.	
	$\tan\beta = e\left(\dfrac{v}{u}\right) = e\tan\alpha$	E1	Shown. Accept signs not clearly dealt with.	5
		17		

Q 2		mark	comment	sub
(i)	$(2+3\times 6)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 6\begin{pmatrix}3\\0\end{pmatrix}+6\begin{pmatrix}6\\3\end{pmatrix}+6\begin{pmatrix}3\\6\end{pmatrix}+2\begin{pmatrix}0\\7\end{pmatrix}$ $20\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}18+36+18\\18+36+14\end{pmatrix}=\begin{pmatrix}72\\68\end{pmatrix}$ $\bar{x}=3.6$ $\bar{y}=3.4$	M1 B1 B1 B1 E1 A1	Method for c.m. Total mass correct For any of the 1st 3 RHS terms For the 4th RHS term cao [If separate cpts, award the 2nd B1 for 2 *x*- terms correct and 3rd B1 for 2×7 in *y* term]	6
(ii)	 $\arctan\left(\frac{3.6}{2+(6-3.4)}\right)=\arctan\left(\frac{3.6}{4.6}\right)$ so 38.047… so 38.0° (3 s. f.)	B1 B1 M1 B1 A1	Diagram showing G vertically below D 3.6 and **their** 3.4 correctly placed (may be implied) Use of arctan on **their** lengths. Allow reciprocal of argument. Some attempt to calculate correct lengths needed 2 + (6 – **their** 3.4) seen cao	5
(iii)	moments about D $5\times 3.6 = 6\times T_{BP}$ so tension in BP is 3 N Resolve vert: $3+T_{DQ}=5$ so tension in DQ is 2 N	M1 F1 M1 F1	moments about D. No extra forces FT **their** values if calc 2nd Resolve vertically or moments about B. FT **their** values if calc 2nd	4
(iv)	We require *x*-cpt of c.m. to be zero **either** $(20+L)\bar{x}=20\times 3.6-\frac{1}{2}L^2$ **or** $2\times 6\times(0.5\times 6)+6\times 6-0.5\times L^2=0$ *L* = 12	M1 B1 A1 A1	A method to achieve this with all cpts For the $0.5\times L^2$ All correct	4
		19		

Q 3		mark	comment	sub
(a) (i)		B1 B1	Internal forces all present and labelled All forces correct with labels and arrows (Allow the internal forces set as tensions, thrusts or a mixture)	2
(ii)	A ↑ T_{AD} sin 30 – L = 0 so T_{AD} = 2L so 2L N (T) A → T_{AB} + T_{AD} cos 30 = 0 so $T_{AB} = -\sqrt{3}L$ so $\sqrt{3}L$ N (C) B ↑ T_{BD} sin 60 – 3L = 0 so $T_{BD} = 2\sqrt{3}L$ so $2\sqrt{3}L$ N (T) B → T_{BC} + T_{BD} cos 60 –T_{AB} = 0 so $T_{BC} = -2\sqrt{3}L$ so $2\sqrt{3}L$ N (C)	M1 A1 M1 F1 M1 A1 M1 F1 E1	Equilibrium equation at a pin-joint attempted 1st ans. Accept + or –. Second equation attempted 2nd ans. FT any previous answer(s) used. Third equation attempted 3rd ans. FT any previous answer(s) used. Fourth equation attempted 4th ans. FT any previous answer(s) used. All T/C consistent [SC 1 all T/C correct WWW]	9
(b)	Leg QR with frictional force F ← moments c.w. about R $U \times 2l \sin 60 - Wl \cos 60 = 0$ Horiz equilibrium for QR $F = U$ Hence $\frac{1}{2}W = \sqrt{3}F$ and so $F = \frac{\sqrt{3}}{6}W$	M1 A1 A1 M1 E1 M1 E1	Accept only 1 leg considered (and without comment) Suitable moments equation. Allow 1 force omitted a.c. moments c.w. moments A second correct equation for horizontal or vertical equilibrium to eliminate a force (U or reaction at foot) [Award if correct moments equation containing only W and F] * This second equation explicitly derived Correct use of 2nd equation with the moments equation Shown. CWO but do not penalise * again.	7
		18		

Q 4		mark	comment	sub
(a) (i)	Tension is perp to the motion of the sphere (so WD, $Fd\cos\theta$ = 0)	E1		1
(ii)	Distance dropped is $2-2\cos 40$ = 0.467911.. GPE is *mgh* so $0.15\times 9.8\times 0.467911... = 0.687829...$ J	M1 E1 M1 B1	Attempt at distance with resolution used. Accept $\sin\leftrightarrow\cos$ Accept seeing $2-2\cos 40$ Any reasonable accuracy	4
(iii)	$0.5\times 0.15\times v^2 = 0.687829...$ so *v* = 3.02837… so 3.03 m s^{-1} (3 s. f.)	M1 F1	Using KE + GPE constant FT **their** GPE	2
(iv)	$\frac{1}{2}\times 0.15\left(v^2-2.5^2\right)$ $= 0.687829... - 0.6\times\frac{40}{360}\times 2\pi\times 2$ *v* = 2.06178… so 2.06 m s^{-1} (3 s. f.)	M1 B1 M1 A1 A1	Use of W-E equation (allow 1 KE term or GPE term omitted) KE terms correct WD against friction WD against friction correct (allow sign error) cao	5
(b)	N2L down slope: $3g\sin 30 - F = 3\times\frac{1}{8}g$ so $F=\frac{9g}{8}$ (= 11.025) $R = 3g\times\frac{\sqrt{3}}{2}$ (= 25.4611…) $\mu = \frac{F}{R} = \frac{\sqrt{3}}{4}$ (= 0.43301…)	M1 A1 A1 B1 M1 E1	Must have attempt at weight component Allow sign errors. Use of $F=\mu R$ Must be worked precisely	6
		18		

4763 Mechanics 3

1(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*(Deduct 1 mark if kg, m, s are consistently used instead of M, L, T)*
(ii)	$[\lambda] = \dfrac{[\text{Force}]}{[v^2]} = \dfrac{\mathrm{M\,L\,T^{-2}}}{(\mathrm{L\,T^{-1}})^2}$ $= \mathrm{M\,L^{-1}}$	M1 A1 cao **2**	
(iii)	$\left[\dfrac{U^2}{2g}\right] = \dfrac{(\mathrm{L\,T^{-1}})^2}{\mathrm{L\,T^{-2}}} = \mathrm{L}$ $\left[\dfrac{\lambda U^4}{4mg^2}\right] = \dfrac{(\mathrm{M\,L^{-1}})(\mathrm{L\,T^{-1}})^4}{\mathrm{M}(\mathrm{L\,T^{-2}})^2}$ $= \dfrac{\mathrm{M\,L^3\,T^{-4}}}{\mathrm{M\,L^2\,T^{-4}}} = \mathrm{L}$ [*H*] = L ; all 3 terms have the same dimensions	B1 cao M1 A1 cao E1 **4**	*(Condone constants left in)* *Dependent on B1M1A1*
(iv)	$(\mathrm{M\,L^{-1}})^2\,(\mathrm{L\,T^{-1}})^\alpha\,\mathrm{M}^\beta\,(\mathrm{L\,T^{-2}})^\gamma = \mathrm{L}$ $\beta = -2$ $-2+\alpha+\gamma = 1$ $-\alpha - 2\gamma = 0$ $\alpha = 6$ $\gamma = -3$	B1 cao M1 A1 A1 cao A1 cao **5**	At least one equation in α, γ One equation correct

(b)	EE is $\frac{1}{2}\times\frac{2060}{24}\times 6^2 \quad (=1545)$	B1	
	(PE gained) = (EE lost) + (KE lost)	M1	Equation involving PE, EE and KE Can be awarded from start to point where string becomes slack *or* any complete method (e.g. SHM) for finding v^2 at natural length If B0, give A1 for $v^2 = 88.2$ correctly obtained
	$50\times 9.8\times h = 1545 + \frac{1}{2}\times 50\times 12^2$ $490h = 1545 + 3600$ $h = 10.5$	F1	*or* $0 = 88.2 - 2\times 9.8\times s \quad (s = 4.5)$
	OA $= 30 - h = 19.5$ m	A1 **4**	*Notes* $\frac{1}{2}\times\frac{2060}{24}\times 6$ *used as EE can earn B0M1F1A0* $\frac{2060}{24}\times 6$ *used as EE gets B0M0*

2 (i)	$T\cos\alpha = mg$ $3.92\cos\alpha = 0.3\times 9.8$ $\cos\alpha = 0.75$ Angle is $41.4°$ (0.723 rad)	M1 A1 **2**	Resolving vertically *(Condone sin / cos mix for M marks throughout this question)*
(ii)	$T\sin\alpha = m\dfrac{v^2}{r}$ $3.92\sin\alpha = 0.3\times\dfrac{v^2}{4.2\sin\alpha}$ Speed is $4.9\ \mathrm{m\,s^{-1}}$	M1 B1 A1 A1 **4**	Force and acceleration towards centre (condone $v^2/4.2$ or $4.2\omega^2$) For radius is $4.2\sin\alpha$ $(=2.778)$ Not awarded for equation in ω unless $v=(4.2\sin\alpha)\omega$ also appears
(iii)	$T - mg\cos\theta = m\dfrac{v^2}{a}$ $T - 0.3\times 9.8\times\cos 60° = 0.3\times\dfrac{8.4^2}{4.2}$ Tension is 6.51 N	M1 A1 A1 **3**	Forces and acceleration towards O
(iv)	$\frac{1}{2}mv^2 - mg\times 4.2\cos\theta = \frac{1}{2}m\times 8.4^2 - mg\times 4.2\cos 60°$ $v^2 - 82.32\cos\theta = 70.56 - 41.16$ $v^2 = 29.4 + 82.32\cos\theta$	M1 M1 A1 E1 **4**	For $(-)mg\times 4.2\cos\theta$ in PE Equation involving $\frac{1}{2}mv^2$ and PE
(v)	$(T) - mg\cos\theta = m\dfrac{v^2}{a}$ $(T) - m\times 9.8\cos\theta = m\times\dfrac{29.4+82.32\cos\theta}{4.2}$ String becomes slack when $T=0$ $-9.8\cos\theta = 7 + 19.6\cos\theta$ $\cos\theta = -\dfrac{7}{29.4}$ $\theta = 104°$ (1.81 rad)	M1 M1 A1 M1 A1 **5**	Force and acceleration towards O Substituting for v^2 *Dependent on first M1* *No marks for* $v=0 \Rightarrow \theta = 111°$

3 (i)	$T_{PB} = 35(x-3.2) \quad [=35x-112]$ $T_{BQ} = 5(6.5-x-1.8)$ $= 5(4.7-x) \quad [=23.5-5x]$	B1 M1 A1 **3**	Finding extension of BQ
(ii)	$T_{BQ} + mg - T_{PB} = m\dfrac{d^2x}{dt^2}$ $5(4.7-x)+2.5\times 9.8-35(x-3.2) = 2.5\dfrac{d^2x}{dt^2}$ $160-40x = 2.5\dfrac{d^2x}{dt^2}$ $\dfrac{d^2x}{dt^2} = 64-16x$	M1 A2 E1 **4**	Equation of motion (condone one missing force) Give A1 for three terms correct
(iii)	At the centre, $\dfrac{d^2x}{dt^2} = 0$ $x = 4$	M1 A1 **2**	
(iv)	$\omega^2 = 16$ Period is $\dfrac{2\pi}{\sqrt{16}} = \frac{1}{2}\pi = 1.57$ s	M1 A1 **2**	Seen or implied *(Allow M1 for $\omega = 16$)* Accept $\frac{1}{2}\pi$
(v)	Amplitude $A = 4.4-4 = 0.4$ m Maximum speed is $A\omega$ $= 0.4\times 4 = 1.6 \text{ m s}^{-1}$	B1 ft M1 A1 cao **3**	ft is $\lvert 4.4-(\text{iii}) \rvert$
(vi)	$x = 4+0.4\cos 4t$ $v = (-)1.6\sin 4t$ When $v = 0.9$, $\sin 4t = -\dfrac{0.9}{1.6}$ $4t = \pi + 0.5974$ Time is 0.935 s	M1 A1 M1 A1 cao **4**	For $v = C\sin\omega t$ or $C\cos\omega t$ *This M1A1 can be earned in (v)* Fully correct method for finding the required time e.g. $\frac{1}{4}\arcsin\dfrac{0.9}{1.6} + \frac{1}{2}\text{period}$
	OR $0.9^2 = 16(0.4^2 - y^2)$ $y = -0.3307$ M1 $y = 0.4\cos 4t$ A1 $\cos 4t = -\dfrac{0.3307}{0.4}$ $4t = \pi + 0.5974$ M1 Time is 0.935 s A1 cao		Using $v^2 = \omega^2(A^2 - y^2)$ *and* $y = A\cos\omega t$ or $A\sin\omega t$ For $y = (\pm)0.331$ *and* $y = 0.4\cos 4t$

4 (a)(i)	$V = \int \pi x^2 \, dy = \int_0^8 \pi (4 - \frac{1}{2} y) \, dy$ $= \pi \left[4y - \frac{1}{4} y^2 \right]_0^8 = 16\pi$ $V \bar{y} = \int \pi y x^2 \, dy$ $= \int_0^8 \pi y (4 - \frac{1}{2} y) \, dy$ $= \pi \left[2y^2 - \frac{1}{6} y^3 \right]_0^8 = \frac{128}{3} \pi$ $\bar{y} = \frac{\frac{128}{3}\pi}{16\pi}$ $= \frac{8}{3} \quad (\approx 2.67)$	M1 A1 M1 A1 A1 M1 A1 **7**	*π may be omitted throughout* Limits not required for M marks throughout this question *Dependent on M1M1*
(ii)	CM is vertically above lower corner $\tan\theta = \frac{2}{\bar{y}} = \frac{2}{8/3} \quad (= \frac{3}{4})$ $\theta = 36.9° \quad (= 0.6435 \text{ rad})$	M1 M1 A1 A1 **4**	Trig in a triangle including θ *Dependent on previous M1* Correct expression for $\tan\theta$ or $\tan(90-\theta)$ *Notes* $\tan\theta = \frac{2}{\text{cand's } \bar{y}}$ *implies M1M1A1* $\tan\theta = \frac{\text{cand's } \bar{y}}{2}$ *implies M1M1* $\tan\theta = \frac{1}{\text{cand's } \bar{y}}$ *without further evidence is M0M0*

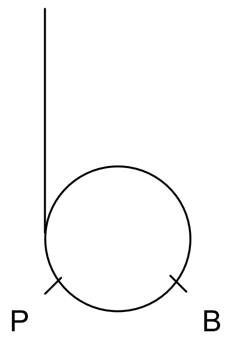
(b)	$A = \int_{-2}^{2} (8 - 2x^2)\,dx$	M1	*May use* $0 \le x \le 2$ *throughout* or $(2)\int_0^8 \sqrt{4 - \frac{1}{2}y}\,dy$
	$= \left[\ 8x - \frac{2}{3}x^3\ \right]_{-2}^{2} = \dfrac{64}{3}$	A1	
	$A\bar{y} = \int_{-2}^{2} \frac{1}{2}(8 - 2x^2)^2\,dx$	M1	or $(2)\int_0^8 y\sqrt{4 - \frac{1}{2}y}\,dy$ *(M0 if ½ is omitted)*
	$= \left[\ 32x - \frac{16}{3}x^3 + \frac{2}{5}x^5\ \right]_{-2}^{2}$	M1	For $32x - \frac{16}{3}x^3 + \frac{2}{5}x^5$ *Allow one error* or $-\frac{8}{3}y(4 - \frac{1}{2}y)^{\frac{3}{2}} - \frac{32}{15}(4 - \frac{1}{2}y)^{\frac{5}{2}}$ or $-\frac{64}{3}(4 - \frac{1}{2}y)^{\frac{3}{2}} + \frac{16}{5}(4 - \frac{1}{2}y)^{\frac{5}{2}}$
	$= \dfrac{1024}{15}$	A1	
	$\bar{y} = \dfrac{1024/15}{64/3}$	M1	*Dependent on first two M1's*
	$= \dfrac{16}{5} = 3.2$	A1	
		7	

4764 Mechanics 4

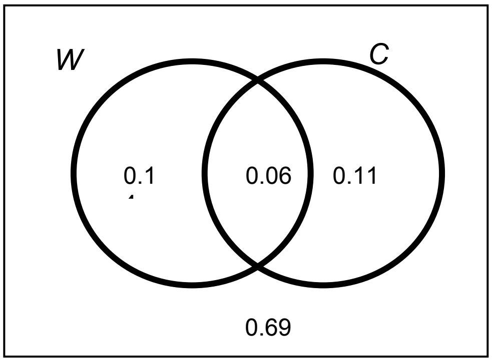
1(i)	If δm is change in mass over time δt PCLM $mv = (m+\delta m)(v+\delta v) + \lvert\delta m\rvert(v-u)$ [N.B. $\delta m < 0$]	M1	Change in momentum over time δt	
	$(m+\delta m)\frac{\delta v}{\delta t} + u\frac{\delta m}{\delta t} = 0 \Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t} = -u\frac{\mathrm{d}m}{\mathrm{d}t}$	M1 A1	Rearrange to produce DE Accept sign error	
	$\frac{\mathrm{d}m}{\mathrm{d}t} = -k \Rightarrow m = m_0 - kt$	M1	Find m in terms of t	
	$\Rightarrow (m_0 - kt)\frac{\mathrm{d}v}{\mathrm{d}t} = uk$	E1	Convincingly shown	
				5
(ii)	$v = \int \frac{uk}{m_0 - kt}\mathrm{d}t$	M1	Separate and integrate	
	$= -u\ln(m_0 - kt) + c$	A1	cao (allow no constant)	
	$t = 0, v = 0 \Rightarrow c = u\ln m_0$	M1	Use initial condition	
	$v = u\ln\left(\frac{m_0}{m_0 - kt}\right)$	A1	All correct	
				4
(iii)	$m = \frac{1}{3}m_0 \Rightarrow m_0 - kt = \frac{1}{3}m_0$	M1 A1	Find expression for mass or time Or $t = 2m_0/3k$	
	$\Rightarrow v = u\ln 3$	A1		
				3

2(i)	$P = Fv$	M1	Used, not just quoted	
	$= mv\frac{\mathrm{d}v}{\mathrm{d}x}v$	M1	Use N2L and expression for acceleration	
	$\Rightarrow mv^2\frac{\mathrm{d}v}{\mathrm{d}x} = m\left(k^2 - v^2\right)$	A1	Correct DE	
	$\Rightarrow \frac{v^2}{k^2 - v^2}\frac{\mathrm{d}v}{\mathrm{d}x} = 1$	M1	Rearrange	
	$\Rightarrow \left(\frac{k^2}{k^2 - v^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}x} = 1$	E1	Convincingly shown	
	$\int\left(\frac{k^2}{k^2 - v^2} - 1\right)\mathrm{d}v = \int \mathrm{d}x$	M1	Separate and integrate	
	$\frac{1}{2}k\ln\left(\frac{k+v}{k-v}\right) - v = x + c$	A1	LHS	
	$x = 0, v = 0 \Rightarrow c = 0$	M1	Use condition	
	$x = \frac{1}{2}k\ln\left(\frac{k+v}{k-v}\right) - v$	A1	cao	
				9
(ii)	Terminal velocity when acceleration zero $\Rightarrow v = k$	M1 A1		
	$v = 0.9k \Rightarrow x = \frac{1}{2}k\ln\left(\frac{1.9}{0.1}\right) - 0.9k = \left(\frac{1}{2}\ln 19 - 0.9\right)k \approx$ 0.572k	F1	Follow their solution to (i)	
				3

3(i)	$M = \int_0^a k(a+r)2\pi r\,\mathrm{d}r$	M1 M1	Use circular elements (for *M* or *I*) Integral for mass	
	$= 2k\pi\left[\frac{1}{2}ar^2 + \frac{1}{3}r^3\right]_0^a$	M1 A1	Integrate (for *M* or *I*) For […]	
	$= \frac{5}{3}k\pi a^3$	E1		
	$I = \int_0^a k(a+r)2\pi r \cdot r^2\,\mathrm{d}r$	M1	Integral for *I*	
	$= 2k\pi\left[\frac{1}{4}ar^4 + \frac{1}{5}r^5\right]_0^a$	A1	For […]	
	$= \frac{9}{10}k\pi a^5$	A1	cao	
	$= \frac{27}{50}Ma^2$	E1	Complete argument (including mass)	9
(ii)	$I = 13.5$ $0.625 \times 50 = I\omega$ $\Rightarrow \omega \approx 2.31$	B1 M1 M1 A1	Seen or used (here or later) Use angular momentum Use moment of impulse cao	4
(iii)	$\ddot{\theta} = \frac{30 - 2.31}{20} \approx 1.38$	M1	Find angular acceleration	
	Couple $= I\ddot{\theta}$ ≈ 18.7	M1 F1	Use equation of motion Follow their ω and *I*	3
(iv)	$I\ddot{\theta} = -3\dot{\theta}$	B1	Allow sign error and follow their *I* (but not *M*)	
	$I\frac{\mathrm{d}\dot{\theta}}{\mathrm{d}t} = -3\dot{\theta}$	M1	Set up DE for $\dot{\theta}$ (first order)	
	$\int \frac{\mathrm{d}\dot{\theta}}{\dot{\theta}} = \int -\frac{3}{I}\,\mathrm{d}t$	M1	Separate and integrate	
	$\ln\lvert\dot{\theta}\rvert = -\frac{t}{4.5} + c$	B1	$\ln(\text{multiple of } \dot{\theta})$ seen	
	$\dot{\theta} = A\mathrm{e}^{-t/4.5}$	M1	Rearrange, dealing properly with constant	
	$t = 0, \dot{\theta} = 30 \Rightarrow A = 30$	M1	Use condition on $\dot{\theta}$	
	$\dot{\theta} = 30\mathrm{e}^{-t/4.5}$	A1		7
(v)	Model predicts $\dot{\theta}$ never zero in finite time.	B1		1

4(i)	$V=\frac{1}{2}\left(\frac{mg}{10a}\right)(a\theta)^2+mga\cos\theta$ (relative to centre of pulley)	M1 B1 M1 A1	EPE term Extension $=a\theta$ GPE relative to any zero level (± constant)	
	$\frac{dV}{d\theta}=\frac{1}{2}\left(\frac{mg}{10a}\right)\cdot 2a^2\theta-mga\sin\theta$	M1	Differentiate	
	$\frac{dV}{d\theta}=mga\left(\frac{1}{10}\theta-\sin\theta\right)$	E1		6
(ii)	$\theta=0\Rightarrow\frac{dV}{d\theta}=mga\left(\frac{1}{10}(0)-\sin 0\right)=0$	M1	Consider value of $\frac{dV}{d\theta}$	
	hence equilibrium	E1		
	$\frac{d^2V}{d\theta^2}=mga\left(\frac{1}{10}-\cos\theta\right)$	M1 A1	Differentiate again	
	$V''(0)=-0.9mga<0$	M1	Consider sign of V''	
	hence unstable	E1	V'' must be correct	6
(iii)	If the pulley is smooth, then the tension in the string is constant. Hence the EPE term is valid.	B1 B1		2
(iv)	Equilibrium positions at $\theta=2.8$, $\theta=7.1$ and $\theta=8.4$	B1 B1	One correct All three correct, no extras Accept answers in [2.7,3.0), [7,7.2], [8.3,8.5]	
	From graph, $V''(2.8)=mga\mathrm{f}'(2.8)>0$	M1	Consider sign of V'' or f'	
	hence stable at $\theta=2.8$ $V''(7.1)=mga\mathrm{f}'(7.1)<0\Rightarrow$ unstable at $\theta=7.1$ $V''(8.4)=mga\mathrm{f}'(8.4)>0\Rightarrow$ stable at $\theta=8.4$	A1 A1 A1	Accept no reference to V'' for one conclusion but other two must relate to sign of V'', not just f'.	6
(v)		B1 B1	P in approximately correct place B in approximately correct place	2
(vi)	If $\theta<0$ then expression for EPE not valid hence not necessarily an equilibrium position.	M1 A1		2

4766 Statistics 1

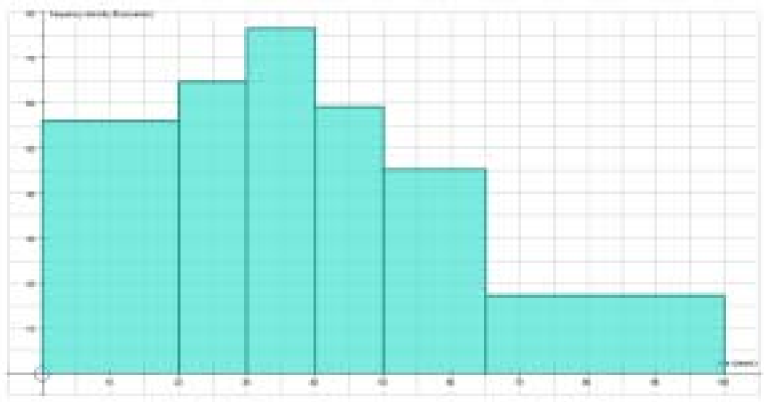
Q1 (i)	Mean = 7.35 (or better) Standard deviation: 3.69 – 3.70 (awfw) Allow s^2 = 13.62 to 13.68 Allow rmsd = 3.64 – 3.66 (awfw) **After B0, B0 scored** then if at least 4 correct mid-points seen or used.{**1.5, 4, 6, 8.5, 15**} Attempt of their mean = $\frac{\sum fx}{\mathbf{44}}$, with $301 \le fx \le 346$ and fx strictly from mid-points not class widths or top/lower boundaries.	B2cao $\sum fx$ = 323.5 B2cao $\sum fx^2$ = 2964.25 (B1) for variance s.o.i.o (B1) for rmsd (B1) mid-points (B1) $6.84 \le \text{mean} \le 7.86$	**4**
(ii)	Upper limit = 7.35 + 2 × 3.69 = 14.73 or 'their sensible mean' + 2 × 'their sensible s.d.' So there could be one or more outliers	M1 (with s.d. < mean) E1**dep** on B2, B2 earned and comment	**2**
		TOTAL	**6**
Q2 (i)	P(W) × P(C) = 0.20 × 0.17 = 0.034 P($W \cap C$) = 0.06 (given in the question) Not equal so not independent (Allow 0.20 × 0.17 ≠ 0.06 or ≠ p ($W \cap C$) so not independent).	M1 for multiplying or 0.034 seen A1 (numerical justification needed)	**2**
(ii)	 The last two G marks are independent of the labels	G1 for two **overlapping** circles labelled G1 for 0.06 and either 0.14 or 0.11 in the **correct places** G1 for all 4 **correct** probs in the **correct** places **(including the 0.69)** NB No credit for Karnaugh maps here	**3**
(iii)	P(W|C) = $\frac{P(W \cap C)}{P(C)} = \frac{0.06}{0.17} = \frac{6}{17} = 0.353$ (awrt 0.35)	M1 for 0.06 / 0.17 A1 cao	**2**

(iv)	Children are more likely than adults to be able to speak Welsh or 'proportionally more children speak Welsh than adults' Do not accept: 'more Welsh children speak Welsh than adults'	E1FT Once the correct idea is seen, apply ISW	**1**
		TOTAL	**8**
Q3 (i)	(*A*) $0.5 + 0.35 + \boldsymbol{p} + \boldsymbol{q} = 1$ so $\boldsymbol{p} + \boldsymbol{q} = 0.15$ (*B*) $0 \times 0.5 + 1 \times 0.35 + 2\boldsymbol{p} + 3\boldsymbol{q} = 0.67$ so $2\boldsymbol{p} + 3\boldsymbol{q} = 0.32$ (*C*) from above $2\boldsymbol{p} + 2\boldsymbol{q} = 0.30$ so $\boldsymbol{q} = 0.02, \boldsymbol{p} = 0.13$	B1 p + q in a correct equation before they reach p + q =0.15 B1 2p + 3q in a correct equation before they reach 2p + 3q = 0.32 (B1) for any 1 correct answer B2 for both correct answers	**1** **1** **2**
(ii)	E(X^2) = $0 \times 0.5 + 1 \times 0.35 + 4 \times 0.13 + 9 \times 0.02 = 1.05$ Var(X) = 'their 1.05' – 0.67^2 = 0.6011 (awrt 0.6) (M1, M1 can be earned with their p^+ and q^+ but not A mark)	M1 $\Sigma x^2 p$ (at least 2 non zero terms correct) M1dep for (– 0.67^2), provided Var(X) > 0 A1 cao (No n or n-1 divisors)	**3**
		TOTAL	**7**
Q4 (i)	$X \sim$ B(8, 0.05) (*A*) $\text{P}(\boldsymbol{X} = 0) = 0.95^8 = 0.6634$ 0.663 or better *Or* using tables $\text{P}(\boldsymbol{X} = 0) = 0.6634$ (*B*) $\text{P}(\boldsymbol{X} = 1) = \begin{pmatrix} 8 \\ 1 \end{pmatrix} \times 0.05 \times 0.95^7 = 0.2793$ $\text{P}(\boldsymbol{X} > 1) = 1 - (0.6634 + 0.2793) = 0.0573$ *Or* using tables P(X > 1) = 1 – 0.9428 = 0.0572	M1 0.95^8 A1 CAO Or B2 (tables) M1 for P(X = 1)(allow 0.28 or better) M1 for 1 – P($X \le$ 1) must have both probabilities A1cao (0.0572 – 0.0573) M1 for P($X \le$ 1) 0.9428 M1 for 1 – P($X \le$ 1) A1 cao (must end in…2)	**2** **3**
(ii)	Expected number of days = 250 × 0.0572 = 14.3 awrt	M1 for 250 x prob(B) A1 FT but no rounding at end	**2**
		TOTAL	**7**

Q5 (i)	Let p = probability of remembering or naming all items (for population) (whilst listening to music.) H_0: $p = 0.35$ H_1: $p > 0.35$ H_1 has this form since the student believes that the probability will be increased/ improved/ got better /gone up.	B1 for definition of p B1 for H_0 B1 for H_1 E1dep on p>0.35 in H_0 In words not just because p > 0.35	**4**
(ii)	Let $X \sim B(15, 0.35)$ ***Either***: $P(X \geq 8) = 1 - 0.8868 = 0.1132 > 5\%$ Or $0.8868 < 95\%$ So not enough evidence to reject H_0 (Accept H_o) Conclude that there is not enough evidence to indicate that the probability of remembering all of the items is improved / improved/ got better /gone up. (when listening to music.) -- ***Or***: Critical region for the test is {9,10,11,12,13,14,15} 8 does not lie in the critical region. So not enough evidence to reject H_0 Conclude that there is not enough evidence to indicate that the probability of remembering all of the items is improved / improved/ got better /gone up. (when listening to music.) -- ***Or***: The smallest critical region that 8 could fall into is {8, 9, 10, 11, 12, 13, 14, and 15}. The size of this region is 0.1132 0.1132 > 5% So not enough evidence to reject H_0 Conclude that there is not enough evidence to indicate that the probability of remembering all of the items is improved (when listening to music)	***Either:*** M1 for probability (0.1132) M1**dep** for comparison A1**dep** E1**dep on all previous marks** for conclusion in **context** ---------------------------- ***Or***: M1 for correct CR(no omissions or additions) M1**dep** for 8 does not lie in CR A1**dep** E1**dep on all previous marks** for conclusion in **context** ---------------------------- **Or**: M1 for CR{8,9,…15}**and** size = 0.1132 M1 **dep** for comparison A1**dep** E1**dep on all previous marks** for conclusion in **context**	**4**
		TOTAL	**8**

	Section B		
Q6 (i)	(*A*) P(both rest of UK) = 0.20×0.20 = 0.04	M1 for multiplying A1cao	**2**
	(*B*) *Either:* **All 5 case** P(at least one England) = (0.79 x 0.20) + (0.79 x 0.01) + (0.20 x 0.79) + (0.01 x 0.79) + (0.79 x 0.79) = 0.158 + 0.0079 + 0.158 + 0.0079 + 0.6241 = 0.9559 *Or* P(at least one England) = 1 – P(neither England) = 1 – (0.21×0.21) = 1 – 0.0441 = 0.9559 or listing all = 1 –{ (0.2 x 0.2) + (0.2 x 0.01) + (0.01 x 0.20) + (0.01x 0.01)} = 1 – (**) = 1 – { 0.04 + 0.002 + 0.002 + 0.0001) = 1 – 0.0441 = 0.9559 Or: **All 3 case** P(at least one England) = = $0.79 \times 0.21 + 0.21 \times 0.79 + 0.79^2$ = 0.1659 + 0.1659 + 0.6241 = 0.9559	M1 for any correct term (3case or 5case) M1 for correct sum of all 3 (or of all 5) with **no extras** A1cao (condone 0.96 www) *Or* M1 for 0.21×0.21 or for (**) fully enumerated or 0.0441 seen M1**dep** for 1 – (1st part) A1cao See above for 3 case	**3**
	(*C*)*Either* 0.79 x 0.79 + 0.79 x 0.2 + 0.2 x 0.79 + 0.2 x 0.2 = 0.9801 *Or* 0.99×0.99 = 0.9801 Or 1 – { 0.79 x 0.01 + 0.2 x 0.01 + 0.01 x 0.79 + 0.01 x 0.02 + 0.01^2 } = 1 – 0.0199 = 0.9801	M1 for sight of all 4 correct terms summed A1 cao (condone 0.98 www) *or* M1 for 0.99 x 0.99 A1cao ***Or*** M1 for everything 1 – {…..} A1cao	**2**
(ii)	P(both the rest of the UK | neither overseas) $= \frac{\text{P(the rest of the UK } and \text{ neither overseas)}}{\text{P(neither overseas)}}$ $= \frac{0.04}{0.9801} = 0.0408$ {Watch for: $\frac{answer(A)}{answer(C)}$ as evidence of method (p <1)}	M1 for numerator of 0.04 or 'their answer to (i)(A)' M1 for denominator of 0.9801 or 'their answer to (i) (C)' A1 FT (0 < p < 1) 0.041 at least	**3**

(iii)	(*A*) Probability = $1 - 0.79^5$ = $1 - 0.3077$ = 0.6923 (accept awrt 0.69) see additional notes for alternative solution	M1 for 0.79^5 or 0.3077... M1 for $1 - 0.79^5$ dep A1 CAO	
	(*B*) $1 - 0.79^n > 0.9$ **EITHER:** $1 - 0.79^n > 0.9$ or $0.79^n < 0.1$ (condone = and $\geq$ throughout) but not reverse inequality $n > \frac{\log 0.1}{\log 0.79}$, so $n > 9.768\ldots$ Minimum $n = 10$ Accept $n \geq 10$	M1 for equation/inequality in n (accept either statement opposite) M1(indep) for process of using logs i.e. $\frac{\log a}{\log b}$ A1 CAO	3
	-- **OR** (using trial and improvement): Trial with 0.79^9 or 0.79^{10} $1 - 0.79^9 = 0.8801$ (< 0.9) or $0.79^9 = 0.1198$ (> 0.1) $1 - 0.79^{10} = 0.9053$ (> 0.9) or $0.79^{10} = 0.09468$ (< 0.1) Minimum $n = 10$ Accept $n \geq 10$ -- NOTE: $n = 10$ unsupported scores SC1 only	---------------------------- M1(indep) for sight of 0.8801 **or** 0.1198 M1(indep) for sight of 0.9053 **or** 0.09468 A1 dep **on both M's** cao ----------------------------	3
		TOTAL	**16**

Q7 (i)	Positive	B1	**1**
(ii)	Number of people = 20×33 (000) + 5×58 (000) = 660 (000) + 290 (000) = 950 000	M1 first term M1(indep) second term A1 cao NB answer of 950 scores M2A0	**3**
(iii)	(*A*) *a* = 1810 + 340 = 2150 (*B*) Median = age of 1 385 (000th) person or 1385.5 (000) Age 30, cf = 1 240 (000); age 40, cf = 1 810 (000) Estimate median = (30) + $\frac{145}{570} \times 10$ Median = 32.5 years (32.54...) If no working shown then 32.54 or better is needed to gain the M1A1. If 32.5 seen with **no previous working** allow SC1	M1 for sum A1 cao 2150 or 2150 thousand but not 215000 B1 for 1 385 (000) or 1385.5 M1 for attempt to interpolate $\frac{145k}{570k} \times 10$ (2.54 or better suggests this) A1 cao min 1dp	**2** **3**
(iv)	Frequency densities: 56, 65, 77, 59, 45, 17 *(accept 45.33 and 17.43 for 45 and 17)* 	B1 for any one correct B1 for all correct (soi by listing or from histogram) **Note: all G marks below *dep* on attempt at frequency density, NOT frequency** G1 Linear scales on both axes (no inequalities) G1 Heights FT their listed fds or all must be correct. **Also widths. All blocks joined** G1 Appropriate label for vertical scale eg 'Frequency density (thousands)', 'frequency (thousands) per 10 years', 'thousands of people per 10 years'. (allow key). **OR f.d.**	**5**

(v)	Any two suitable comments such as: Outer London has a greater proportion (or %) of people under 20 (or almost equal proportion) The modal group in Inner London is 20-30 but in Outer London it is 30-40 Outer London has a greater proportion (14%) of aged 65+ **<u>All</u>** populations in **<u>each</u>** age group are higher in Outer London Outer London has a more evenly spread distribution or balanced distribution (ages) o.e.	E1 E1	**2**
(vi)	Mean increase ↑ median unchanged (-) midrange increase ↑ standard deviation increase ↑ interquartile range unchanged. (-)	Any one correct B1 Any two correct B2 Any three correct B3 All **five** correct B4	**4**
		TOTAL	**20**

4767 Statistics 2

Question 1

(i)	**EITHER:** $S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x\Sigma y = 880.1 - \frac{1}{48} \times 781.3 \times 57.8$ $= -60.72$ $S_{xx} = \Sigma x^2 - \frac{1}{n}(\Sigma x)^2 = 14055 - \frac{1}{48} \times 781.3^2 = 1337.7$ $S_{yy} = \Sigma y^2 - \frac{1}{n}(\Sigma y)^2 = 106.3 - \frac{1}{48} \times 57.8^2 = 36.70$ $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-60.72}{\sqrt{1337.7 \times 36.70}} = -0.274$ **OR:** cov $(x,y) = \frac{\sum xy}{n} - \bar{x}\bar{y} = 880.1/48 - 16.28 \times 1.204$ $= -1.265$ rmsd$(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(1337.7/48)} = \sqrt{27.87} = 5.279$ rmsd$(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(36.70/48)} = \sqrt{0.7646} = 0.8744$ $r = \frac{\text{cov(x,y)}}{rmsd(x)rmsd(y)} = \frac{-1.265}{5.279 \times 0.8744} = -0.274$	M1 for method for S_{xy} M1 for method for at least one of S_{xx} or S_{yy} A1 for at least one of S_{xy}, S_{xx}, S_{yy} correct M1 for structure of r A1 CAO (–0.27 to –0.28) M1 for method for cov (x,y) M1 for method for at least one msd A1 for at least one of cov/msd correct M1 for structure of r A1 CAO (–0.27 to –0.28)	**5**
(ii)	H_0: $\rho = 0$ H_1: $\rho < 0$ (one-tailed test) where ρ is the population correlation coefficient For n = 48, 5% critical value = 0.2403 Since $\| -0.274 \| > 0.2403$ we can reject H_0: There is sufficient evidence at the 5% level to suggest that there is negative correlation between education spending and population growth.	B1 for H_0, H_1 in symbols B1 for defining ρ B1FT for critical value M1 for sensible comparison leading to a conclusion A1 for result (FT r<0) E1 FT for conclusion in words	**6**
(iii)	Underlying distribution must be bivariate Normal. If the distribution is bivariate Normal then the scatter diagram will have an elliptical shape.	B1 CAO for bivariate Normal B1 indep for elliptical shape	**2**
(iv)	• Correlation does not imply causation • There could be a third factor • increased growth could cause lower spending. Allow any sensible alternatives, including example of a possible third factor.	E1 E1 E1	**3**
(v)	Advantage – less effort or cost Disadvantage – the test is less sensitive (ie is less likely to detect any correlation which may exist)	E1 E1	**2**
			18

Question 2

(i)	(*A*) P(*X* = 2) = $e^{-0.37}\dfrac{0.37^2}{2!}$ = 0.0473 (*B*) P(*X* > 2) = $1-\left(e^{-0.37}\dfrac{0.37^2}{2!}+e^{-0.37}\dfrac{0.37^1}{1!}+e^{-0.37}\dfrac{0.37^0}{0!}\right)$ = 1 – (0.0473 + 0.2556 + 0.6907) = 0.0064	M1 A1 (2 s.f.) M1 for P(*X* = 1) and P(*X* = 0) M1 for complete method A1 ***NB Answer given***	**5**
(ii)	P(At most one day more than 2) = $\binom{30}{1}\times 0.9936^{29}\times 0.0064 + 0.9936^{30}$ = = 0.1594 + 0.8248 = 0.9842	M1 for coefficient M1 for $0.9936^{29}\times 0.0064$ M1 for 0.993630 A1 CAO (min 2sf)	**4**
(iii)	$\lambda = 0.37\times 10 = 3.7$ P(*X* > 8) = 1 – 0.9863 = 0.0137	B1 for mean (SOI) M1 for probability A1 CAO	**3**
(iv)	Mean no. per 1000ml = 200 × 0.37 = 74 Using Normal approx. to the Poisson, *X* ~ N(74, 74) P(*X* > 90) = $P\left(Z > \dfrac{90.5-74}{\sqrt{74}}\right)$ = P(*Z* >1.918) = 1 – Φ(1.918) = 1 – 0.9724 = 0.0276	B1 for Normal approx. with correct parameters (SOI) B1 for continuity corr. M1 for probability using correct tail A1 CAO (min 2 s.f.), (but FT wrong or omitted CC)	**4**
(v)	P(questionable) = 0.0064 × 0.0137 ×0.0276 = 2.42×10^{-6}	M1 A1 CAO	**2**
			18

Question 3

(i)	$X \sim N(27500, 4000^2)$ $P(X > 25000) = P\left(Z > \frac{25000-27500}{4000}\right)$ $= P(Z > -0.625)$ $= \Phi(0.625) = 0.7340$ (3 s.f.)	M1 for standardising A1 for -0.625 M1 *dep* for correct tail A1CAO (must include use of differences)	**4**
(ii)	P(7 of 10 last more than 25000) $= \binom{10}{7} \times 0.7340^7 \times 0.2660^3 = 0.2592$	M1 for coefficient M1 for $0.7340^7 \times 0.2660^3$ A1 FT (min 2sf)	**3**
(iii)	From tables $\Phi^{-1}(0.99) = 2.326$ $\frac{k-27500}{4000} = -2.326$ $x = 27500 - 2.326 \times 4000 = 18200$	B1 for 2.326 seen M1 for equation in *k* and negative z-value A1 CAO for awrt 18200	**3**
(iv)	H_0: $\mu = 27500$; H_1: $\mu > 27500$ Where μ denotes the mean lifetime of the new tyres.	B1 for use of 27500 B1 for both correct B1 for definition of μ	**3**
(v)	Test statistic = $\frac{28630-27500}{4000/\sqrt{15}} = \frac{1130}{1032.8}$ = 1.094 5% level 1 tailed critical value of z = 1.645 1.094 < 1.645 so not significant. There is not sufficient evidence to reject H_0 There is insufficient evidence to conclude that the new tyres last longer.	M1 must include √ 15 A1 FT B1 for 1.645 M1 *dep* for a sensible comparison leading to a conclusion A1 for conclusion in words in context	**5**
			18

Question 4

(i)	H_0: no association between location and species. H_1: some association between location and species.	B1 for both	**1**
(ii)	Expected frequency = 38/160 × 42 = 9.975 Contribution = $(3 - 9.975)^2$ / 9.975 = 4.8773	M1 A1 M1 for valid attempt at $(O\text{-}E)^2/E$ A1 ***NB Answer given***	**4**
(iii)	Refer to χ_4^2 Critical value at 5% level = 9.488 Test statistic X^2 = 32.85 Result is significant There appears to be some association between location and species NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final E1	B1 for 4 deg of f(seen) B1 CAO for cv M1 Sensible comparison, using 32.85, leading to a conclusion A1 for correct conclusion (FT their c.v.) E1 conclusion in context	**5**
(iv)	• Limpets appear to be distributed as expected throughout all locations. • Mussels are much more frequent in exposed locations and much less in pools than expected. • Other shellfish are less frequent in exposed locations and more frequent in pools than expected.	E1 E1, E1 E1, E1	**5**
(v)	$\frac{24}{53} \times \frac{32}{65} \times \frac{16}{42} = 0.0849$	M1 for one fraction M1 for product of all 3 A1 CAO	**3**
			18

4768 Statistics 3

Q1	f(x) = k(20 – x) 0 ≤ x ≤ 20			
(a) (i)	$\int_0^{20} k(20-x)\mathrm{d}x = \left[k\left(20x - \frac{x^2}{2}\right)\right]_0^{20} = k \times 200 = 1$ $\therefore k = \frac{1}{200}$ Straight line graph with negative gradient, in the first quadrant. Intercept correctly labelled (20, 0), with nothing extending beyond these points. Sarah is more likely to have only a short time to wait for the bus.	M1 A1 G1 G1 E1	Integral of f(x), including limits (which may appear later), set equal to 1. Accept a geometrical approach using the area of a triangle. C.a.o.	 5
(ii)	Cdf $\mathrm{F}(x) = \int_0^x \mathrm{f}(t)\mathrm{d}t$ $= \frac{1}{200}\left(20x - \frac{x^2}{2}\right)$ $= \frac{x}{10} - \frac{x^2}{400}$ P(X > 10) = 1 – F(10) = 1 – (1 – ¼) = ¼	M1 A1 M1 A1	Definition of cdf, including limits (or use of "+c" and attempt to evaluate it), possibly implied later. Some valid method must be seen. Or equivalent expression; condone absence of domain [0, 20]. Correct use of c's cdf. f.t. c's cdf. Accept geometrical method, e.g area = ½(20 – 10)f(10), or similarity.	 4
(iii)	Median time, m, is given by F(m) = ½. $\therefore \frac{m}{10} - \frac{m^2}{400} = \frac{1}{2}$ $\therefore m^2 - 40m + 200 = 0$ $\therefore m = 5.86$	M1 M1 A1	Definition of median used, leading to the formation of a quadratic equation. Rearrange and attempt to solve the quadratic equation. Other solution is 34.14; no explicit reference to/rejection of it is required.	 3

(b) (i)	A simple random sample is one where every sample of the required size has an equal chance of being chosen.	E2	S.C. Allow E1 for “Every member of the population has an equal chance of being chosen independently of every other member”.	2
(ii)	Identify clusters which are capable of representing the population as a whole. Choose a random sample of clusters. Randomly sample or enumerate within the chosen clusters.	E1 E1 E1		3
(iii)	A random sample of the school population might involve having to interview single or small numbers of pupils from a large number of schools across the entire country. Therefore it would be more practical to use a cluster sample.	E1 E1	For “practical” accept e.g. convenient / efficient / economical.	2
				19

Q2	$A \sim \mathrm{N}(100,\ \sigma = 1.9)$ $B \sim \mathrm{N}(50,\ \sigma = 1.3)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	P(A < 103) = $\mathrm{P}\left(Z < \frac{103-100}{1.9} = 1.5789\right)$ = 0·9429	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	3
(ii)	$A_1 + A_2 + A_3 \sim \mathrm{N}(300,$ $\sigma^2 = 1.9^2 + 1.9^2 + 1.9^2 = 10.83\ )$ P(this > 306) = $\mathrm{P}\left(Z > \frac{306-300}{3\cdot291} = 1\cdot823\right) = 1 - 0\cdot9658 = 0.0342$	B1 B1 A1	Mean. Variance. Accept sd (= 3.291). c.a.o.	3
(iii)	$A + B \sim \mathrm{N}(150,$ $\sigma^2 = 1.9^2 + 1.3^2 = 5.3\ )$ P(this > 147) = $\mathrm{P}\left(Z > \frac{147-150}{2\cdot302} = -1.303\right)$ = 0·9037	B1 B1 A1	Mean. Variance. Accept sd (= 2.302). c.a.o.	3
(iv)	$B_1 + B_2 - A \sim N(0,$ $1\cdot3^2 + 1\cdot3^2 + 1\cdot9^2 = 6\cdot99)$ P(−3< this < 3) $= \mathrm{P}\left(\frac{-3-0}{2.644} < Z < \frac{3-0}{2.644}\right) = \mathrm{P}(-1\cdot135 < Z < 1\cdot135)$ = 2 × 0.8718 – 1 = 0.7436	B1 B1 M1 A1 A1	Mean. Or $A - (B_1 + B_2)$. Variance. Accept sd (= 2.644). Formulation of requirement ... … two sided. c.a.o.	5
(v)	Given $\bar{x} = 302.3 \quad s_{n-1} = 3.7$ CI is given by $302.3 \pm 1\cdot96 \times \frac{3.7}{\sqrt{100}}$ = 302·3 ± 0·7252 = (301·57(48), 303·02(52)) The batch appears not to be as specified since 300 is outside the confidence interval.	M1 B1 A1 E1	Correct use of 302.3 and $3.7/\sqrt{100}$. For 1·96 c.a.o. Must be expressed as an interval.	4
				18

Q3				
(a) (i)	H$_0$: $\mu_D = 0$ (or $\mu_I = \mu_{II}$) H$_1$: $\mu_D \neq 0$ (or $\mu_{II} \neq \mu_I$) where μ_D is "mean for II – mean for I" Normality of <u>differences</u> is required.	B1 B1 B1	Both. Hypotheses in words only must include "population". For adequate verbal definition. Allow absence of "population" if correct notation μ is used, but do NOT allow "$\overline{X}_I = \overline{X}_{II}$" or similar unless $\overline{X}$ is clearly and explicitly stated to be a <u>population</u> mean.	3
(ii)	<u>MUST</u> be PAIRED COMPARISON t test. Differences are: 10.0, 26.8, 42.7, 2.4, –14.9, –2.0, 16.3, 11.5 $\bar{d} = 11.6$ $s_{n-1} = 17.707$ Test statistic is $\dfrac{11.6 - 0}{\frac{17.707}{\sqrt{8}}}$ = 1.852(92). Refer to t_7. Double-tailed 5% point is 2.365. Not significant. Seems there is no difference between the mean yields of the two types of plant.	B1 M1 A1 M1 A1 A1 A1	s_n = 16.563 but do <u>NOT</u> allow this here or in construction of test statistic, but FT from there. Allow c's $\bar{d}$ and/or s_{n-1}. Allow alternative: 0 + (c's 2.365) $\times \frac{17.707}{\sqrt{8}}$ (= 14.806) for subsequent comparison with $\bar{d}$. (Or $\bar{d}$ – (c's 2.365) $\times \frac{17.707}{\sqrt{8}}$ (=–3.206) for comparison with 0.) c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{d}$ scores M1A0, but ft. No ft from here if wrong. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. Special case: (t_8 and 2.306) can score 1 of these last 2 marks if either form of conclusion is given.	7

(b)	<table><tr><td>Diff</td><td>−5</td><td>4</td><td>−14</td><td>−3</td><td>6</td><td>1</td><td>−11</td><td>−8</td><td>−7</td><td>−9</td></tr><tr><td>Rank of |diff|</td><td>4</td><td>3</td><td>10</td><td>2</td><td>5</td><td>1</td><td>9</td><td>7</td><td>6</td><td>8</td></tr></table>			
		M1	For differences. ZERO in this section if differences not used.	
		M1	For ranks.	
		A1	FT from here if ranks wrong	
	$W_+ = 1 + 3 + 5 = 9$ (or $W_- = 2+4+6+7+8+9+10 = 46$)	B1		
	Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 10$.	M1	No ft from here if wrong.	
	Lower (or upper if 46 used) double-tailed 5% point is 8 (or 47 if 46 used).	A1	i.e. a 2-tail test. No ft from here if wrong.	
	Result is not significant.	A1	ft only c's test statistic.	
	No evidence to suggest the tasters differ on the whole.	A1	ft only c's test statistic.	8
				18

Q4				
(a) (i)	$\bar{x} = \frac{310}{100} = 3.1$	B1		
	$s^2 = \frac{1288 - 100 \times 3.1^2}{99} = \frac{327}{99} = 3.303$	B1		
	Evidence could support Poisson since the variance is fairly close to the mean.	E1		3

(ii)

f_o	6	16	19	18	17	14	6	4	0
f_e	4.50	13.97	21.65	22.37	17.33	10.75	5.55	2.46	1.42
Merged	22 18.47						10 9.43		

		M1 A1 A1	Calculation of expected frequencies. Last cell correct. All others correct, but ft if wrong.	
		M1	Combining cells. (Condone if not combined as fully as shown above, but require top two cells combined as a minimum.)	
	X^2 = 0.6747 + 0.3244 + 0.8537 + 0.0063 + 0.9826 + 0.0345	M1	Calculation of X^2.	
	= 2.876(2)	A1	(Condone wrong last cell.) Depends on both of the preceding M marks.	
	Refer to χ_4^2. e.g. Upper 10% point is 7.779.	M1	Allow correct df (= cells – 2) from wrongly grouped or ungrouped table, and FT. Otherwise, no FT if wrong.	
	Not significant.	A1	ft only c's test statistic.	
	Suggests Poisson model does fit …	A1	ft only c's test statistic.	
	… at any reasonable level of significance.	A1	Or other sensible comment.	10
(b)	CI is given by 1.465 ±	M1	If <u>both</u> 1.465 and $0.3288/\sqrt{10}$ are correct.	
	2·262	B1 B1	**If t_9 used.** 95% 2-tail point for c's t distribution (Independent of previous mark).	
	$\times \frac{0.3288}{\sqrt{10}}$			
	= 1.465 ± 0.2352= (1.2298, 1.7002)	A1	c.a.o. Must be expressed as an interval.	4
				17

4769 Statistics 4

Q1				
(i)	$L = \frac{e^{-\theta}\theta^{x_1}}{x_1!} \cdots \frac{e^{-\theta}\theta^{x_n}}{x_n!} \left[= \frac{e^{-n\theta}\theta^{\sum x_i}}{x_1! x_2! \cdots x_n!} \right]$	M1 A1	product form fully correct	
	ln L = const − $n\theta$ + $\sum x_i \ln\theta$	M1 A1		
	$\frac{d \ln L}{d\theta} = -n + \frac{\sum x_i}{\theta} = 0$	M1 A1		
	$\Rightarrow \hat{\theta} = \frac{\sum x_i}{n} (= \bar{x})$	A1	CAO	
	Check this is a maximum	M1		
	e.g. $\frac{d^2 \ln L}{d\theta^2} = -\frac{\sum x_i}{\theta^2} < 0$	A1		9
(ii)	$\lambda = P(X = 0) = e^{-\theta}$	B1		1
(iii)	We have $R \sim B(n, e^{-\theta})$,	M1		
	so $E(R) = ne^{-\theta}$	B1		
	$Var(R) = ne^{-\theta}(1 - e^{-\theta})$	B1		
	$\tilde{\lambda} = \frac{R}{n}$	M1		
	$\therefore E(\tilde{\lambda}) = e^{-\theta}$ i.e. unbiased	A1 A1		
	$Var(\tilde{\lambda}) = \frac{e^{-\theta}(1 - e^{-\theta})}{n}$	A1	BEWARE PRINTED ANSWER	7

(iv)	Relative efficiency of $\tilde{\lambda}$ wrt ML est $= \dfrac{\text{Var(ML Est)}}{\text{Var}(\tilde{\lambda})}$ $= \dfrac{\theta e^{-2\theta}}{n} \cdot \dfrac{n}{e^{-\theta}(1-e^{-\theta})} = \dfrac{\theta}{e^{\theta}-1}$ Eg:- Expression is $\dfrac{\theta}{\theta + \dfrac{\theta^2}{2!} + \ldots}$ always < 1 and this is $\approx$ 1 if θ is small $\approx$ 0 if θ is large	M1 M1 A1 M1 E1 E1 E1	any attempt to compare variances if correct BEWARE PRINTED ANSWER Allow statement that $\dfrac{\theta}{e^{\theta}-1} \to 0 \text{ as } \theta \to \infty$	7

Q2				
(i)	$\mathrm{P}(X=x)=q^{x-1}p$	B1	FT into pgf only	
	Pgf $\mathrm{G}(t)=\mathrm{E}(t^X)=\sum_{x=1}^{\infty} pt^x q^{x-1}$	M1		
	$=pt(1+qt+q^2t^2+\ldots)$	A1		
	$=\underline{\underline{pt(1-qt)^{-1}}}$	A1	BEWARE PRINTED ANSWER [consideration of |qt| < 1 not required]	
	$\mu=\mathrm{G}'(1)\quad \sigma^2=\mathrm{G}''(1)+\mu-\mu^2$	M1	for attempt to find G'(*t*) and/or G"(*t*)	
	$G'(t)=pt(-1)(1-qt)^{-2}(-q)+p(1-qt)^{-1}$ $=pqt(1-qt)^{-2}+p(1-qt)^{-1}$	A1		
	$\therefore \mathrm{G}'(1)=pq(1-q)^{-2}+p(1-q)^{-1}=\frac{q}{p}+1=\underline{\underline{\frac{1}{p}}}$	A1	BEWARE PRINTED ANSWER	
	$\mathrm{G}''(t)=pqt(-2)(1-qt)^{-3}(-q)+pq(1-qt)^{-2}+$ $p(-1)(1-qt)^{-2}(-q)$	A1		
	$\therefore \mathrm{G}''(1)=2pq^2(1-q)^{-3}+pq(1-q)^{-2}+pq(1-q)^{-2}$ $=\frac{2q^2}{p^2}+\frac{2q}{p}$	A1		
	$\therefore \sigma^2=\frac{2q^2}{p^2}+\frac{2q}{p}+\frac{1}{p}-\frac{1}{p^2}=\frac{2q^2+2pq+p-1}{p^2}$	M1	For inserting their values	
	$=\frac{q}{p^2}(2q+2p-1)=\underline{\underline{\frac{q}{p^2}}}$	A1	BEWARE PRINTED ANSWER	
				11

(ii)	X_1=number of trials to first success X_2= " " " " " next " . . . . X_n= " " " " " nth " $\therefore Y=X_1+X_2+\dots X_n$ = total no of trials to the nth success $\therefore$ pgf of $Y=(\text{pgf of } X)^n = \underline{\underline{p^n t^n (1-qt)^{-n}}}$ $\mu_Y = n\mu_X = \underline{\underline{\dfrac{n}{p}}}$ $\sigma_Y^2 = n\sigma_X^2 = \underline{\underline{\dfrac{nq}{p^2}}}$	E1 E1 1 1 1		5
(iii)	N(candidate's μ_Y, candidate's σ_Y^2)	1		1
(iv)	Y = no of tickets to be sold ~ random variable as in (ii) with n = 140 and p = 0.8 ~ Approx N($\frac{140}{0.8}=175,\ \frac{140\times0.2}{(0.8)^2}=43.75$) $\mathrm{P}(Y\ge160)\approx\mathrm{P}(\mathrm{N}(175,43.75)>159\frac{1}{2})$ = P(N(0,1)>-2.343) = 0.9905 For any sensible discussion <u>in context</u> (eg groups of passengers ⇒ not indep.)	E1 1 M1 A1 A1 E1 E1	Do not award if cty corr absent or wrong, but FT if 160 used → -2.268, 0.9884 CAO	7
Q3	X = amount of salt ~ N($\mu[750], \sigma^2[20^2]$) Sample of n=9			
(i)	Type I error: rejecting null hypothesis … … when it is true. Type II error: accepting null hypothesis … … when it is false. OC: P (accepting null hypothesis … … as a function of the parameter under investigation)	B1 B1 B1 B1 B1 B1	Allow B1 for P(rej H_0 when true) Allow B1 for P(acc H_0 when false) [P(type II error | the true value of the parameter) scores B1+B1]	6
(ii)	Reject if $\bar{x}<735$ or $\bar{x}>765$ $\alpha=\mathrm{P}(\bar{X}<735 \text{ or } \bar{X}>765 \mid \bar{X}\sim\mathrm{N}(750,\frac{20^2}{9}))$ $=\mathrm{P}(Z<\frac{(735-750)3}{20}=-2.25$ $\text{or } Z>\frac{(765-750)3}{20}=2.25)$ = 2(1-0.9878) = 2 × 0.0122 = 0.0244 This is the probability of rejecting good output and unnecessarily re-calibrating the machine – seems small [but not very small?]	M1 A1 A1 A1 E1 E1	Might be implicit CAO Accept any sensible comments	6

(iii)	Accept if $735 < \bar{x} < 765$, and now $\mu = 725$. $\beta = P(735 < \bar{X} < 765 \mid \bar{X} \sim N(725, 20^2/9))$ = P(1.5 < Z< 6) = 1 – 0.9332 = $\underline{\underline{0.0668}}$ This is the probability of accepting output and carrying on when in fact μ has slipped to 725 – small[-ish?]	M1 A1 A1 A1 E1 E1	might be implicit CAO If upper limit 765 not considered, maximum 2 of these 4 marks. If Φ(6) not considered, maximum 3 out of 4. accept sensible comments	6
(iv)	OC = $P(\ 735 < \bar{X} < 765 \mid \bar{X} \sim N(\mu, 20^2/9)\ )$ = $\Phi\left(\frac{(765-\mu)3}{20}\right) - \Phi\left(\frac{(735-\mu)3}{20}\right)$ " Φ – Φ" μ=720:Φ(6.75) – Φ(2.25)=1– 0.9878 =0.0122 730: 5.25 0.75 =1– 0.7734 =0.2266 740: 3.75 −0.75 =1– (1–0.7734)= 0.7734 750: similarly or by write-down from part (ii) [FT] : 0.9756 760, 770, 780 by symmetry [FT]: 0.7734, 0.2266, 0.0122	M1 M1 A1 1 1 1	 both correct if any two correct	6
Q4				
(i)	$x_{ij} = \mu + \alpha_i + e_{ij}$ μ = population … .. grand mean for whole experiment α_i = population … .. mean by which *i* th treatment differs from μ e_{ij} are experimental errors… ~ ind N $(0, \sigma^2)$	1 1 1 1 1 1 3	Allow "uncorrelated" 1 for ind N; 1 for 0; 1 for σ^2.	9
(ii)	Totals are 240, 246, 254, 264, 196 each from sample of size 5 Grand total 936 "Correction factor" CF = $\frac{936^2}{20}$ = 43804.8 Total SS = 44544 - CF = 739.2			

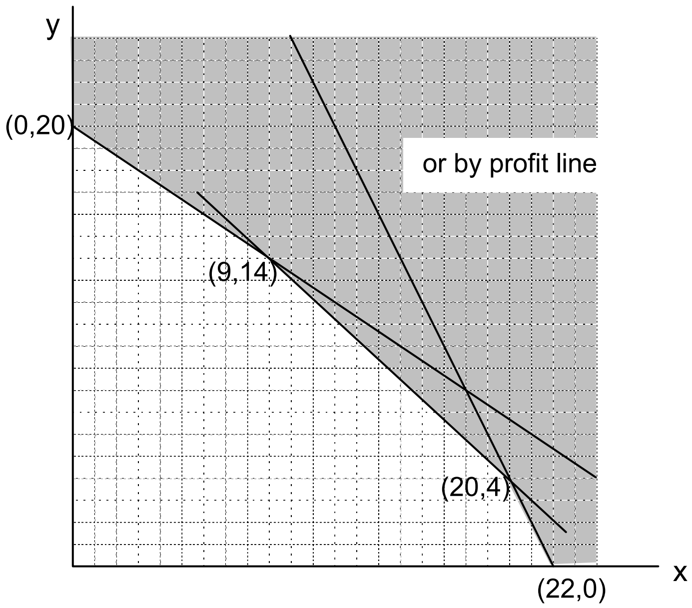
	Between contractors SS = $\frac{240^2}{5} + \ldots + \frac{196^2}{5} - \text{CF}$ = 44209.6 – CF = 404.8 Residual SS (by subtraction) = 739.2 – 404.8 = 334.4	M1 M1 A1	For correct methods for any two, if each calculated SS is correct.	
	(ANOVA table below)	M1 M1 1 A1 1	CAO	
	Refer to $F_{3,16}$ Upper 5% point is 3.24 Significant Seems performances of contractors are not all the same	1 1 1 1	NO FT IF WRONG NO FT IF WRONG	12
(iii)	Randomised blocks Description	B1 E1 E1	Take the subject areas as "blocks", ensure each contractor is used at least once in each block	3

Source of Variation	SS	df	MS	MS ratio
Between Contractors	404.8	3	$134.9\dot{3}$	6.456
Residual	334.4	16	20.9	
Total	739.2	19		

4771 Decision Mathematics 1

Solutions

1.

<table>
<tr><td>

Objective has maximum value of 24 at (20,4)</td><td>M1 A1 third line
B1 shading
B1 (0,20) and (22,0)
B1 (9,14)
B1 (20,4)

M1 A1 solution

or M1 A1 B1,
B1 scale (implied OK),
B1 profit line, B1 (20,4)
M1 A1 (20,4) A1 (24)</td></tr>
</table>

2.

(i)

	X	Y
5, 14, 153, 6, 24, 2, 14, 15	5, 14, 153	5, 2
5, 14, 6, 24,14, 15	5, 14, 24	5
14, 6, 14, 15,	14, 15	14, 6
14, 14		

M1

Answer = 14 — A1
Comparisons = 30 — A1

(ii)

	X	Y
5, 14, 153, 6, 24, 2, 14	5, 14, 153	5, 2
5, 14, 6, 24,14	5, 14, 24	5
14, 6, 14	14	14, 6
14		

M1

Answer = 14 — A1
Comparisons = 24 — A1

(iii) Median — B1

(iv) Time taken approximately proportional to square of length of list (or twice length takes four times the time, or equivalent). — B1

3.

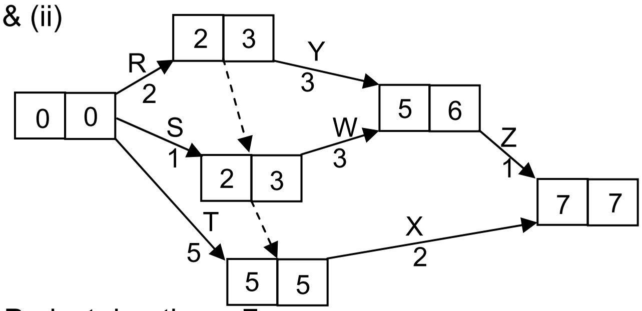
(i)	$T_1 \to T_2$ $T_1 \to T_3 \to T_2$ $T_1 \to T_3$ $T_1 \to T_2 \to T_3$ $T_1 \to T_2 \to T_3 \to T_4$ $T_1 \to T_3 \to T_4$	M1 A1
(ii)	$T_4 \to T_3 \to T_2 \to T_1$ $T_4 \to T_3 \to T_1$ $T_4 \to T_3 \to T_1 \to T_2$ $T_4 \to T_3 \to T_2$ $T_4 \to T_3$	M1 A1
(iii)	22	M1 allow for 23 A1
(iv)	11	M1 halving (not 11.5) A1

4.

(i) e.g. 00–09→1 10–39→2 40–79→3 80–89→4 90–99→5	M1 A1 proportions OK A1 efficient
(ii) e.g. 00–15→1 16–47→2 48–55→3 56–79→4 80–87→5 88–95→6 96, 97, 98, 99 reject	M1 some rejected A2 proportions OK (–1 each error) A1 efficient
(iii) & (iv)	M1 A2 (–1 each error) M1 simulation A1 time intervals A1 passengers A1 time to wait
(v) 0.8 more runs	B1 B1

Sim. no.	Cars arriving after Joe – time interval | number of passengers										Time to 15 passengers (minutes)
1	3	2	2	1	1	2	2	2	3	1	6
2	3	1	2	2	1	4	1	2	5	1	6
3	5	1	2	2	2	1	3	4	2	2	12
4	4	6	3	2	4	1	1	2	2	3	4
5	5	1	4	1	3	2	5	4	2	2	17
6	4	4	4	2	5	3	1	4	1	4	8
7	4	1	4	2	3	1	5	4	1	3	16
8	2	2	2	2	2	4	3	5	1	2	6
9	1	1	1	1	1	1	1	1	1	2	5
10	2	4	3	2	2	6	2	5	2	1	5

5.

(a)(i) Activity D. Depends on A and B in project 1, but on A, B and C in project 2.	M1 A1 A1
(ii) Project 1: Duration is 5 for x<3, thence x+2. Project 2: Duration is 5 for x<2, thence x+3	B1 "5" B1 B1 beyond 5
(b) (i) & (ii) 	M1 activity-on-arc A1 single start and single end A2 precedences (–1 each error) M1 A1 forward pass M1 A1 backward pass
Project duration – 7 Critical activities – T, X	B1 B1

6.

(i)

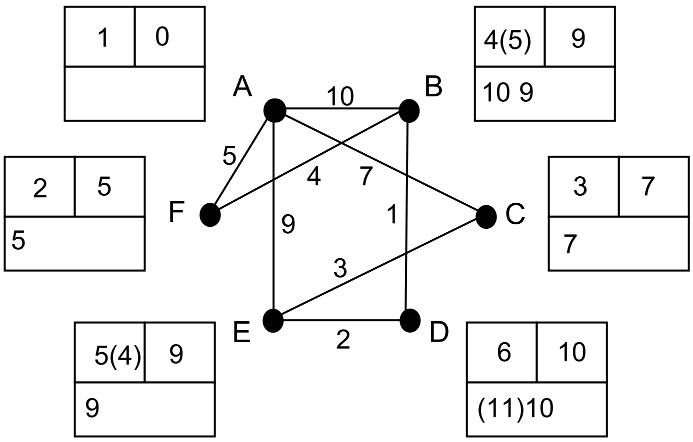
Order of inclusion	1	3	6	4	5	2
	A	B	C	D	E	F
A	–	10	7	–	9	5
B	10	–	–	1	–	(4)
C	7	–	–	–	(3)	–
D	–	(1)	–	–	2	–
E	9	–	3	(2)	–	–
F	(5)	4	–	–	–	–

M1
A1 select
A1 delete
A1 order

Arcs: AF, FB, BD, DE, EC — B1

Length: 15 — B1

(ii) & (iii)



B1 arcs
B1 lengths

M1 Dijkstra
A1 working values
A1 order of labelling
A1 labels

Arcs: AF, FB, BD, AC, AE
Length: 26

M1
A1

(iv) Cubic — B1

n applications of Dijkstra, which is quadratic — B1

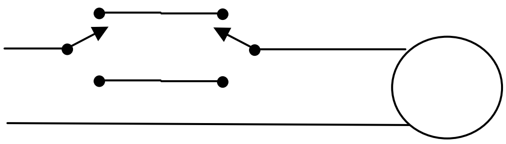
4772 Decision Mathematics 2

1.

(a)(i) "not fail" → "succeed" — B1
"disagree less" → "agree more" — B1

(ii) e.g. "I don't entirely agree with you". — M1 same meaning; A1 simpler

(b) e.g.



M1 2 switches + light in a circuit
A4 one for each correct setting

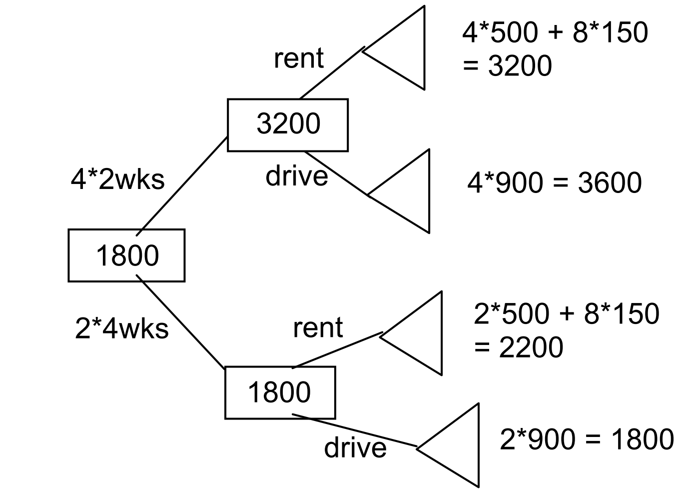
(c)

((a	∧	b)	∨	(~a	∧	~b))	⇔	((~a	∨	b)	∧	(a	∨	~b))
1	1	1	**1**	0	0	0	**1**	0	1	1	**1**	1	1	0
1	0	0	**0**	0	0	1	**1**	0	0	0	**0**	1	1	1
0	0	1	**0**	1	0	0	**1**	1	1	1	**0**	0	0	0
0	0	0	**1**	1	1	1	**1**	1	1	0	**1**	0	1	1

M1 4 lines
A1 a's and b's
A1 negations
A1 level 1 and's
A1 level 1 or's
A1 level 2
A1 result

2.

(i) e.g. (Decisions could be in other order.)

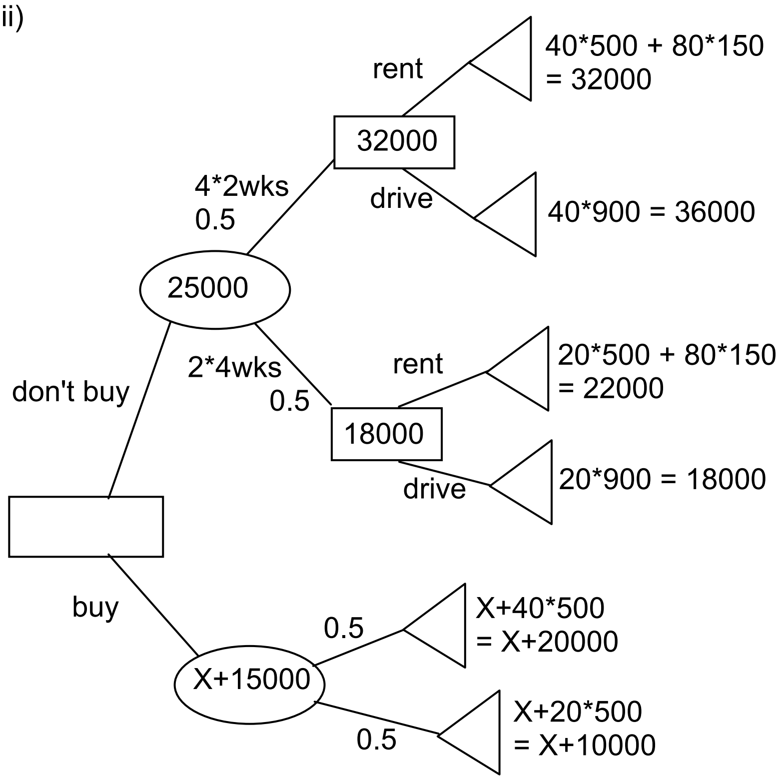


M1 4*2/2*4
M1 rent/drive
A1

M1 costs
A1

Drive down for 2 lots of 4 weeks — B1 advice

(ii)



B1 buy/don't buy

M1 don't buy analysis
A1 costings

M1 chance node
A1 buy analysis

M1 buy costings
A1

B1

Jane could save money if she spent less than £10000 on a car — B1

(iii) EMV – expected monetary value – probabilistically weighted cash values — B1
Utility measure is an alternative.

3.

(a) (i)

	1	**2**	**3**	**4**
1	∞	14	11	24
2	14	∞	15	∞
3	11	15	∞	12
4	24	∞	12	∞

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

M1 sca Floyd
A1 distance
A1 route

	1	**2**	**3**	**4**
1	∞	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

A1

	1	**2**	**3**	**4**
1	28	14	11	24
2	14	28	15	38
3	11	15	22	12
4	24	38	12	48

	1	**2**	**3**	**4**
1	2	2	3	4
2	1	1	3	1
3	1	2	1	4
4	1	1	3	1

A1

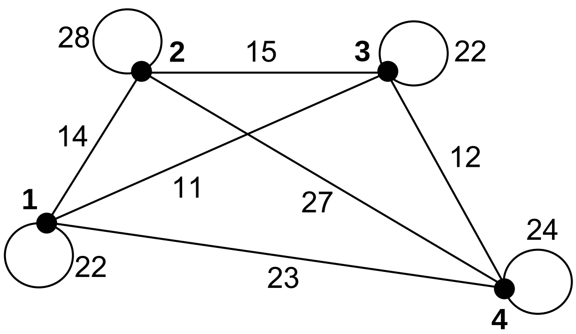
	1	**2**	**3**	**4**
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	**2**	**3**	**4**
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3

A1

	1	**2**	**3**	**4**
1	22	14	11	23
2	14	28	15	27
3	11	15	22	12
4	23	27	12	24

	1	**2**	**3**	**4**
1	3	2	3	3
2	1	1	3	3
3	1	2	1	4
4	3	3	3	3



B1 loops
B1 rest

(ii) **1 3 4 2 1**
64
⇒ **1 3 4 3 2 1**

M1 A1
B1
B1

(iii) 27 + 11 + 14 = 52 TSP solution has length between 52 and 64	M1 A1 M1 A1
(b) e.g. **1 3 1 2 3 4 1** length = 87 One repeated arc → Eulerian	M1 A1 A1 B1

4.

(i) Let a be the number of tonnes of A produced … — M1 A1

Max a+b+c — B1

st 3a+2b+5c<60 — B1

5a+6b+2c<50 — B1

(ii) e.g.

P	a	b	c	s_1	s_2	RHS
1	–1	–1	–1	0	0	0
0	3	2	5	1	0	60
0	5	6	2	0	1	50
1	–0.4	–0.6	0	0.2	0	12
0	0.6	0.4	1	0.2	0	12
0	3.8	5.2	0	–0.4	1	26
1	>0	0	0	>0	>0	15
0		0	1			10
0	19/26	1	0	–2/26	5/26	5

M1 initial tableau
A1

M1 pivot
A1

M1
A1

Make 5 tonnes of B and 10 tonnes of C — B1 interpretation

(iii) & (iv) e.g.

A	P	a	b	c	s_1	s_2	s_3	art	RHS
1	0	1	0	0	0	0	–1	0	8
0	1	–1	–1	–1	0	0	0	0	0
0	0	3	2	5	1	0	0	0	60
0	0	5	6	2	0	1	0	0	50
0	0	1	0	0	0	0	–1	1	8
1	0	0	0	0	0	0	0	–1	0
0	1	0	–1	–1	0	0	–1	1	8
0	0	0	2	5	1	0	3	–3	36
0	0	0	6	2	0	1	5	–5	10
0	0	1	0	0	0	0	–1	1	8
	1	0	2	0	0	0.5	1.5		13
	0	0	–13	0	1	–2.5	–4.5		11
	0	0	3	1	0	0.5	2.5		5
	0	1	0	0	0	0	–1		8

B1 new constraint

M1 surplus +
A1 artificial

B1 new objective

M1
A1

B1

Make 8 tonnes of A and 5 tonnes of C — B1 interpretation

4773 Decision Mathematics Computation

1.

(i) XA + XB +XE + XF >= 1 Indicator variables correspond to matrix column A (or row A) entries which are less than or equal to 5. Ensures that at least one such indicator is 1. (ii) Min XA+XB+XC+XD+XE+XF st XA+XB+XE+XF >= 1 XA+XB+XE+XF >= 1 XC+XF >= 1 XD+XE >= 1 XA+XB+XD+XE+XF >= 1 XA+XB+XC+XE+XF >= 1 (iii) 2 centres, at F&D or E&C or E&F (iv) e.g. add XF=0 to force solution E and C (v) Three solutions are F & D, E & C, E & F. (vi) Problem is unimodular (or convincing argument). In the interests of efficiency (and parsimony).	M1 A1 ">" OK B1 indicator vars B1 <= 5 B1 B1 M1 A3 (–1 each error/ omission) allow (correct) reduced set of inequalities M1 A1 A1 M1 A1 B1 B1 B1

2.

(i) e.g. (candidates should show formulae)	
α = 0.01 · 10 · P(birth) · P(death) · rand · rand · birth · death	B1 handling parameters
β = 0.04 · 9 · 0.1 · 0.4 · 0.4261 · 0.3537 · 0 · 1	
8 · 0.09 · 0.36 · 0.257 · 0.1405 · 0 · 1	B1 births
8 · 0.08 · 0.32 · 0.8854 · 0.8632 · 0 · 0	B1 deaths
	B1 use of "rand"
	B1 use of "if"
	B1 updating population
(ii) e.g. 1 1 3 2 0 2 2 3 3 0 – 0.2	M1 A1 B1
(iii) e.g.	
β: 0.01 0.02 0.03 0.04 0.05 0.06 0.07	M1
prob extinction: 0 0 0.1 0.2 0.5 0.7 0.8	A1 decent range
	A1 reasonable outcomes
(iv) Addition of another rand + another if + extra add-on	B1
	M1 A1
	B1
(v) e.g.	
β: 0.01 0.02 0.03 0.04 0.05 0.06 0.07	M1
prob. extinction: 0 0 0 0 0 0.2 0.3	A1

3.

(a) max AB+AE	B1	objective
st AB-BC-BD+CB+DB=0		
BC+DC-CB-CD-CG=0	M1	flow balance
BD+ED+CD-DB-DC-DE=0	A2	constraints
AE-ED-EF+DE+FE=0		
EF-FE-FG=0		
AB<8		
AE<3		
BC<5	M1	capacity
CB<5	A1	constraints
BD<7		
DB<7		
CD<1		
DC<1		
DE<2		
ED<2		
EF<4		
FE<4		
CG<8		
FG<6		
END		
OBJECTIVE FUNCTION VALUE		
	M1	run
1) 10.00000		
VARIABLE VALUE REDUCED COST		
AB 7.000000 0.000000		
AE 3.000000 0.000000		
BC 5.000000 0.000000		
BD 2.000000 0.000000	A1	results
CB 0.000000 1.000000		
DB 0.000000 0.000000		
DC 1.000000 0.000000		
CD 0.000000 1.000000		
CG 6.000000 0.000000		
ED 0.000000 0.000000		
DE 1.000000 0.000000		
EF 4.000000 0.000000		
FE 0.000000 1.000000		
FG 4.000000 0.000000		
Max flow of 10 with flows of 7 from A to B, … etc.	B1	interpretation
(b)		
min 8X12+3X15+8X21+5X23+7X24+5X32+X34+8X37+7X42	M1	objective
+X43+2X45+3X51+2X54+4X56+4X65+6X67+8X73+6X76	A1	
st X12+X15=10		
X21+X23+X24=10	M1	supply
X32+X34+X37=10		
X42+X43+X45=10	A1	constraints

`` X51+X54+X56=10 X65+X67=10 X73+X76=10 X21+X51=10 X12+X32+X42=10 X23+X43+X73=10 X24+X34+X54=10 X15+X45+X65=10 X56+X76=10 X37+X67=10 END ```	M1 demand A1 constraints
OBJECTIVE FUNCTION VALUE 1) 310.0000	M1 run
VARIABLE VALUE REDUCED COST X12 0.000000 0.000000 X15 10.000000 0.000000 X21 0.000000 0.000000 X23 10.000000 0.000000 X24 0.000000 0.000000 X32 0.000000 0.000000 X34 10.000000 0.000000 X37 0.000000 6.000000 X42 10.000000 0.000000 X43 0.000000 0.000000 X45 0.000000 0.000000 X51 10.000000 0.000000 X54 0.000000 0.000000 X56 0.000000 6.000000 X65 0.000000 0.000000 X67 10.000000 0.000000 X73 0.000000 0.000000 X76 10.000000 0.000000	A1 results B1 interpretation
Cost = 310 by sending 10 from W1 to S5, … etc.	

4.

(a) Auxiliary equation:	
$2\lambda^2 - 3\lambda + 1 = 0$	M1 A1
$(2\lambda - 1)(\lambda - 1) = 0$	M1
$\lambda = 1$ or ½	A1
$u_n = A + B(½)^n$	B1 B1
$5 = A + B$	B1
$3 = A + ½B$	B1
$u_n = 1 + 4(½)^n$	M1 A1
$u_2 = 2$, $u_3 = 1.5$, $u_{10} = 1.003906$	B1 B1
$u_{1000000} \approx 1$	B1
(b)(i) & (ii)	
0 5	
1 3	
2 4.5	
3 8.75	M1
4 13.625	A1
5 16.6875	
6 16.40625	
7 12.92188	
8 7.976563	
9 4.042969	A1 3.087891
10 3.087891	
11 5.588867	
12 10.29541	
13 14.85425	
14 16.98596	
15 15.62469	
16 11.45108	
17 6.551926	
18 3.376808	A1 6.893122
19 3.513287	
20 6.893122	
(iii) Limited wrt to (very) long-term	B1

4776 Numerical Methods

1

x	3	3.5	root = (3 x (-0.8) - 3.5 x 0.5) / (-0.8 - 0.5)	**[M1A1A1]**
f(x)	0.5	-0.8	= .192308 (3.192, 3.19)	**[A1]**

(-) mpe is 3.5 - 3.192308 = 0.307602 (0.308, 0.31) **[M1A1]**

[TOTAL 6]

2

1	2				
3	1	-1			
5	5	4	5		
7	k	k-5	k-9	k-14	
9	2	2-k	7-2k	16-3k	**[M1A1A1A1]**

16-3k = k-14 hence k = 7.5 **[M1A1]**

[TOTAL 6]

3

h	f(2+h)	f(2-h)	f '(2)		**derivatives**
0.2	.494507	.867869	.566594		**[M1A1A1A1]**
0.1	.323418	.010586	.564163	-0.00243	**differences**
0.05	.241636	.085281	.563555	-0.00061	**[M1A1]**

differences reducing by a factor 4 so next estimate about 1.56340. **[M1]**
1.563 secure to 3 dp. **[B1]**

[TOTAL 8]

4

$f(x) = x^3-25$ $f'(x) = 3x^2$ **[M1A1A1]**
$x_{r+1} = x_r - (x_r^3-25)/3x_r^2$ (a.g.)

r	0	1	2	3	
x_r	4	3.1875	.945197	2.92417	**[M1A1]**
diffs		-0.8125	-0.2423	-0.02103	**[B1]**
ratios			.298219	.086783	**[B1]**

differences reducing at an increasing rate *(hence faster than first order)* **[E1]**

[TOTAL 8]

---

5 (i) 0.001 369 352 (accept 0.001 369 4) **[B1]**

(ii) sin 86^o = 0.997 564 sin 85^o = 0.996 195 **[B1B1]**
sin 86^o - sin 86^o = 0.001 369 **[A1]**

(iii) 2 x 0.0784591 x 0.008 726 54 **[M1]**
= 0.00136935 **[A1]**

(iv) Rounding has different effects in the two expressions *(may be implied)* **[E1]**
First method involves subtraction of nearly equal numbers and so loses accuracy **[E1]**

[TOTAL 8]

6 (i)

h	M	T		
2	2.763547	**2.425240**		
1	**2.677635**	**2.594393**	mid-point:	**[M1A1A1]**
0.5	**2.656743**	**2.636014**	trapezium:	**[M1A1A1]**

[subtotal 7]

(ii)

M:		diffs		
	2.763547			
	2.677635	-0.08591		
	2.656743	-0.02089	reducing by a factor 4 *(may be implied)*	**[M1A1E1]**

Differences in T reduce by a factor 4, too **[B1]**

[subtotal 4]

(iii)

M	T	S			
2.763547	2.425240	2.650778			**[M1]**
2.677635	2.594393	2.649888	-0.00089033	S values:	**[A1A1]**
2.656743	2.636014	2.649833	-0.000054333	diffs	**[A1]**

Differences in S reducing fast e.g by a factor of (about) 16 **[E1]**
How this leads to an answer, e.g:
Next difference about -0.0000034 and/or next answer about 2.649830 **[E1]**
Accept 2.6498 or 2.64983 **[A1]**

[subtotal 7]

[TOTAL 18]

---

7 (i) Eg: graph of x^2 and 4 + 1/x for x > 0 showing single intersection **[G2]**
Change of sign to find interval (2,3) - i.e. a = 2 **[B1]**

r	0	1	2	3	4	5	
x_r	2.5	2.097618	2.115829	2.114859	2.11491	2.114907	**[M1A1A1]**

2.1149 secure to 4 dp **[A1]**

[subtotal 7]

(ii) The iteration gives positive values only. **[E1]**

r	0	1	2	3	4	5	
x_r	-2	-1.87083	-1.86158	-1.86087	-1.86081	-1.86081	**[M1A1A1]**

-1.8608 secure to 4 dp **[A1]**

[subtotal 5]

(iii) Eg

r	0	1	2	3	4
x_r	-0.5	-1.41421	-1.81463	-1.85713	-1.86052

not converging to required root (converging to previous root) **[M1A1]**

Eg $x_{r+1} = 1 / (x_r^2 - 4)$ **[M1]**

r	0	1	2	3	4	5
x_r	-0.5	-0.26667	-0.25452	-0.25412	-0.2541	-0.2541

[M1A1]

-0.2541 secure to 4 dp **[A1]**

[subtotal 6]

[TOTAL 18]

4777 Numerical Computation

1 (i) Eg: e_{r+1} is approximately ke_r **[E2]**

Uses $y_0 = \alpha + e_0$, $y_1 = \alpha + ke_0$, $y_2 = \alpha + k^2e_0$ or equivalent **[M1A1]**

Convincing algebra to eliminate k hence given result **[A1A1]**

[subtotal 6]

(ii) Convincing re-arrangment **[A1]**

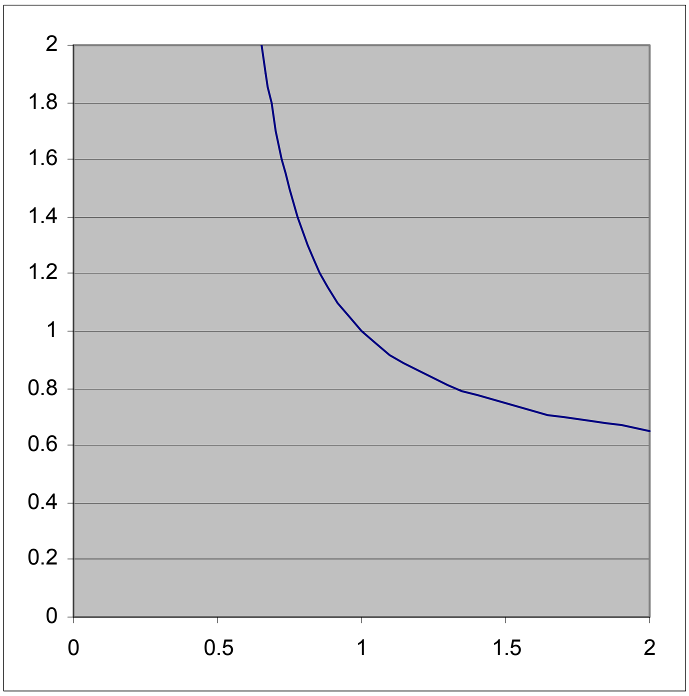
y_0	y_1	y_2	extrap (new y_o)	new y_1	new y_2	extrap		
							once	**[M1A1]**
1	0.908662	0.917409	0.916644	0.916648	0.916647	0.916647	*twice*	**[M1A1]**
					4 or 5 sf looks secure			**[A1]**

[subtotal 6]

(iii)

x	y_0	y_1	y_2	extrap (new y_o)	new y_1	new y_2	extrap	
1.1	1	0.908662	0.917409	0.916644	0.916648	0.916647	0.916647	
1.2	0.916647	0.845937	0.858962	0.856936	0.85695	0.856947	0.856948	**set up**
1.3	0.856948	0.799744	0.815042	0.811814	0.81184	0.811833	0.811835	**SS**
1.4	0.811835	0.763904	0.780556	0.776263	0.776302	0.776288	0.776292	**[M2A2]**
1.5	0.776292	0.734953	0.752555	0.747298	0.747351	0.747329	0.747335	
1.6	0.747335	0.7108	0.729213	0.723043	0.72311	0.723076	0.723087	**values**
1.7	0.723087	0.690112	0.70934	0.702258	0.70234	0.702292	0.70231	**[A3]**
1.8	0.70231	0.671996	0.692131	0.684095	0.684194	0.684128	0.684155	
1.9	0.684155	0.655831	0.677026	0.667954	0.668075	0.667985	0.668023	
2	0.668023	0.641175	0.663627	0.653402	0.65355	0.653427	0.653483	
					3 or 4 sf looks secure			**[A1]**

x	y
0.653483	2
0.668023	1.9
0.684155	1.8
0.70231	1.7
0.723087	1.6
0.747335	1.5
0.776292	1.4
0.811835	1.3
0.856948	1.2
0.916647	1.1
1	1
1.1	0.916647
1.2	0.856948
1.3	0.811835
1.4	0.776292
1.5	0.747335
1.6	0.723087
1.7	0.70231
1.8	0.684155
1.9	0.668023
2	0.653483



organise data [M1A1]

graph G2

Sub Total 12

TOTAL 24

2 (i) $T_n - I = A_2h^2 + A_4h^4 + A_6h^6 + \ldots$

$T_{2n} - I = A_2(h/2)^2 + A_4(h/2)^4 + A_6(h/2)^6 + \ldots$ **[M1A1]**

$4(T_{2n} - I) - (T_n - I) = b_4h^4 + b_6h^6 + \ldots$ **[M1]**

$4T_{2n} - T_n - 3I = b_4h^4 + b_6h^6 + \ldots$ **[A1]**

$(4T_{2n} - T_n)/3 - I = B_4h^4 + B_6h^6 + \ldots$ **[A1]**

($T_n^* = (4T_{2n} - T_n)/3$ has error of order h^4 as given)

$T_n^{**} = (16T_{2n}^* - T_n^*)/15$ has error of order h^6 **[B1]**

[subtotal 6]

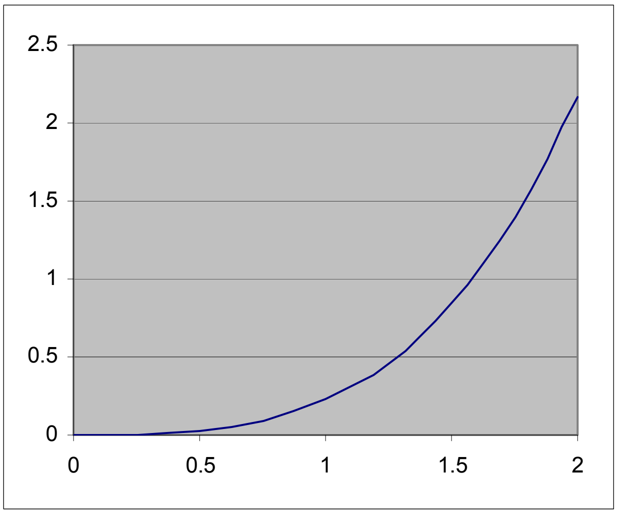
(ii)

x	f(x)	T	T*	T**	(T***)		
0	0						
2	3.523188	3.523188					
1	0.731059	2.492653	2.149141				
0.5	0.155615					f:	[A1]
1.5	1.839543	2.243905	2.160989	2.161779			
0.25	0.035136					*T:*	[M1A2]
0.75	0.382038						
1.25	1.214531					*T*:*	[M1A1]
1.75	2.609105	2.182155	2.161572	2.161611	2.161608		
0.125	0.0083					*T**:*	[M1A1]
0.375	0.083344						
0.625	0.254435					*answer:*	[A1]
0.875	0.540367						
1.125	0.955439						
1.375	1.509072						
1.625	2.206199						
1.875	3.048173	2.166744	2.161606	2.161609	2.161609		

[subtotal 9]

(iii)

k	I
0	0
0.25	0.002847
0.5	0.024686
0.75	0.089495
1	0.225935
1.25	0.466242
1.5	0.845007
1.75	1.398068
2	2.161609



modify SS **[M2]**

values of I **[A2]**

graph **[G2]**

[subtotal 6]

(iv)

k	I
1.57	0.980739
1.58	1.001291
1.579	0.999223

accept 1.57 or 1.58 (or in between)

evidence of t&e: **[M2]**

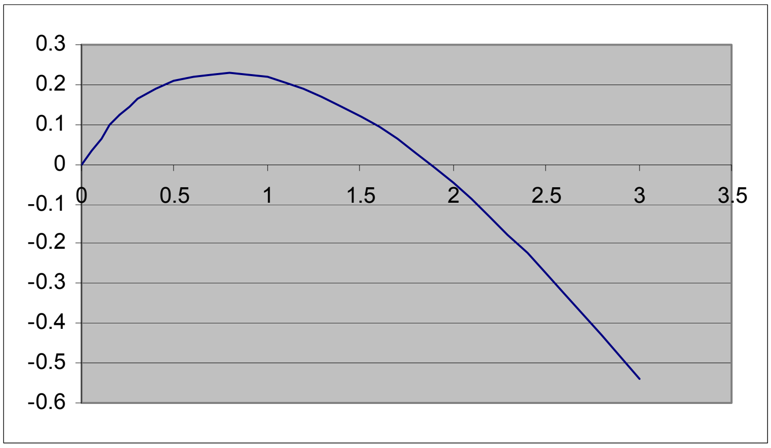
result: **[A1]**

[subtotal 3]

[TOTAL 24]

3 (i)

h	x	y	k 1	k 2	k 3	k 4	
0.2	0	0	0.2	0.110557	0.121189	0.086653	
0.2	0.2	0.125024	0.085978	0.063177	0.064854	0.046393	
0.2	0.4	0.189763	0.046408	0.031125	0.032033	0.018694	
0.2	0.6	0.221666	0.018708	0.007021	0.007628	-0.00291	**setup**
0.2	0.8	0.229182	-0.0029	-0.01239	-0.01194	-0.02066	**[M3]**
0.2	1	0.217146	-0.02065	-0.02863	-0.02828	-0.0357	
0.2	1.2	0.188783	-0.03569	-0.04256	-0.04228	-0.04872	**values**
0.2	1.4	0.146433	-0.04871	-0.05472	-0.05449	-0.06015	**[A3]**
0.2	1.6	0.091887	-0.06015	-0.06547	-0.06527	-0.07031	
0.2	1.8	0.026567	-0.0703	-0.07506	-0.07488	-0.07941	
0.2	2	-0.04836	-0.0794	-0.08369	-0.08353	-0.08762	
0.2	2.2	-0.13194	-0.08761	-0.0915	-0.09136	-0.09507	
0.2	2.4	-0.22334	-0.09507	-0.0986	-0.09849	-0.10187	
0.2	2.6	-0.32186	-0.10187	-0.1051	-0.105	-0.1081	
0.2	2.8	-0.42689	-0.1081	-0.11107	-0.11097	-0.11382	
0.2	3	-0.53789	-0.11382	-0.11656	-0.11647	-0.1191	



[G2]

Maximum about (0.8, 0.23) root about 1.8 **[A1A1A1]**

[subtotal11]

(ii) Eg: **[M2]**

h = 0.01 gives (p, q) as (0.77, 0.22743) hence (0.77, 0.23) **[A1A1]**

h = 0.01 gives root as between 1.87 and 1.88 accept either **[A1]**

[subtotal5]

(iii) Eg:

s	h	x	y	k 1	k 2	k 3	k 4	
1	0.01	0	0	0.01	0.009	0.009025	0.008621	
1	0.01	0.01	0.009112	0.008618	0.008314	0.008319	0.008065	
1	0.01	0.02	0.017437	0.008065	0.007844	0.007847	0.007649	**Mods**
1	0.01	0.03	0.025286	0.007649	0.007468	0.00747	0.007303	**[M3]**
1	0.01	0.04	0.032757	0.007303	0.007147	0.007148	0.007002	**t & e**
								[M3]

s = 0.715, h = 0.01 gives root closest to x = 1 accept 0.71 to 0.72 **[A2]**

[subtotal8]

[TOTAL 24]

4 (i)

$Q = \Sigma (y - a - bx - cx^2)^2$ **[M1]**

dQ/da = 0 gives $\Sigma y = na + b \Sigma x + c \Sigma x^2$ as given **[M1A1]**

other equations:

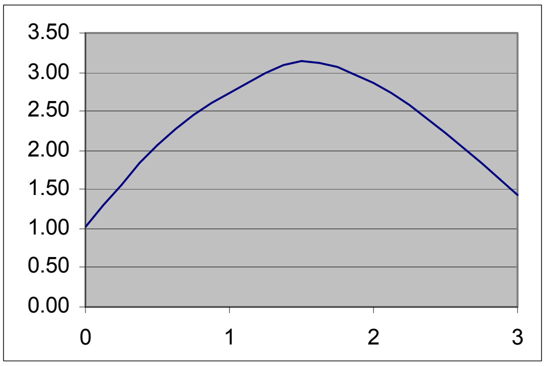
$\Sigma xy = a \Sigma x + b \Sigma x^2 + c \Sigma x^3$ **[B1]**

$\Sigma x^2 y = a \Sigma x^2 + b \Sigma x^3 + c \Sigma x^4$ **[B1]**

[subtotal 5]

(ii)

X	Y
0	1.02
0.5	2.08
1	2.73
1.5	3.14
2	2.87
2.5	2.22
3	1.43



[G2]

roughly parabolic (quadratic) in shape **[E1]**

[subtotal 3]

(iii)

x	y	xy	x^2y	x^2	x^3	x^4	
0	1.02	0	0	0	0	0	
0.5	2.08	1.04	0.52	0.25	0.125	0.0625	
1	2.73	2.73	2.73	1	1	1	
1.5	3.14	4.71	7.065	2.25	3.375	5.0625	
2	2.87	5.74	11.48	4	8	16	
2.5	2.22	5.55	13.875	6.25	15.625	39.0625	
3	1.43	4.29	12.87	9	27	81	**[M2]**
10.5	**15.49**	**24.06**	**48.54**	**22.75**	**55.125**	**142.1875**	**[A2]**

normal equations:

7	10.5	22.75	15.49		**form equations**
10.5	22.75	55.125	24.06		**[M1A1]**
22.75	55.125	142.1875	48.54	a= 1.017619	**solution**
	-6.46154	-21	0.554615	b= 2.562143	**[M2A2]**
	-2.69231	-10.5	1.656923		
		-1.75	1.425833	c= -0.81476	

x	y	y fitted	residual	res^2	
0	1.02	1.017619	0.002381	5.67E-06	
0.5	2.08	2.095	-0.015	0.000225	
1	2.73	2.765	-0.035	0.001225	**y fitted**
1.5	3.14	3.027619	0.112381	0.012629	**[M1A1]**
2	2.87	2.882857	-0.01286	0.000165	
2.5	2.22	2.330714	-0.11071	0.012258	**residuals**
3	1.43	1.37119	0.05881	0.003459	**[M1A1]**
			-3.6E-15	**0.029967**	

residual sum is zero (except for rounding errors) as it should be **[E1]**

residual sum of squares is 0.029967 **[A1]**

[subtotal 16]

[TOTAL 24]

Grade Thresholds

Advanced GCE MEI Mathematics 3895 7895
June 2008 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	61	53	45	37	30	0
4752	Raw	72	55	48	41	34	28	0
4753	Raw	72	59	52	46	40	33	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	75	67	59	51	43	0
4755	Raw	72	60	51	42	34	26	0
4756	Raw	72	57	51	45	39	33	0
4757	Raw	72	50	44	38	33	28	0
4758	Raw	72	58	50	42	34	26	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	57	48	39	30	22	0
4762	Raw	72	56	48	40	33	26	0
4763	Raw	72	53	45	37	29	21	0
4764	Raw	72	55	47	40	33	26	0
4766	Raw	72	53	45	38	31	24	0
4767	Raw	72	57	49	41	33	26	0
4768	Raw	72	56	49	42	35	28	0
4769	Raw	72	57	49	41	33	25	0
4771	Raw	72	58	51	44	37	31	0
4772	Raw	72	51	44	37	31	25	0
4773	Raw	72	51	44	37	30	24	0
4776	Raw	72	57	49	41	34	26	0
4776/02	Raw	18	14	12	10	8	7	0
4777	Raw	72	54	46	39	32	25	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	42.5	63.7	79.2	90.7	97.5	100	9600
7896	58.0	78.2	89.2	95.3	98.7	100	1539
7897	73.5	85.3	88.2	100	100	100	34
7898	27.8	52.8	61.1	77.8	91.7	100	36
3895	30.5	46.0	60.6	73.6	83.7	100	12767
3896	49.7	68.6	81.4	90.0	95.2	100	2039
3897	82.1	88.5	92.3	97.4	100	100	78
3898	47.8	52.2	69.6	87.0	95.7	100	23

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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