OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS 4723

Core Mathematics 3

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Find

(i) $\int 6e^{2x+1}\,dx$,

(ii) $\int 10(2x+1)^{-1}\,dx$. **[5]**

2 The curve $y = \ln x$ is transformed by:

a reflection in the x-axis,
followed by a stretch with scale factor 3 parallel to the y-axis,
followed by a translation in the positive y-direction by $\ln 4$.

Find the equation of the resulting curve, giving your answer in the form $y = \ln\big(f(x)\big)$. **[4]**

3 **(a)** Given that $7\sin 2\alpha = 3\sin\alpha$, where $0° < \alpha < 90°$, find the exact value of $\cos\alpha$. **[3]**

(b) Given that $3\cos 2\beta + 19\cos\beta + 13 = 0$, where $90° < \beta < 180°$, find the exact value of $\sec\beta$. **[5]**

4 **(i)** Show by means of suitable sketch graphs that the equation

$$(x-2)^4 = x + 16$$

has exactly 2 real roots. **[3]**

(ii) State the value of the smaller root. **[1]**

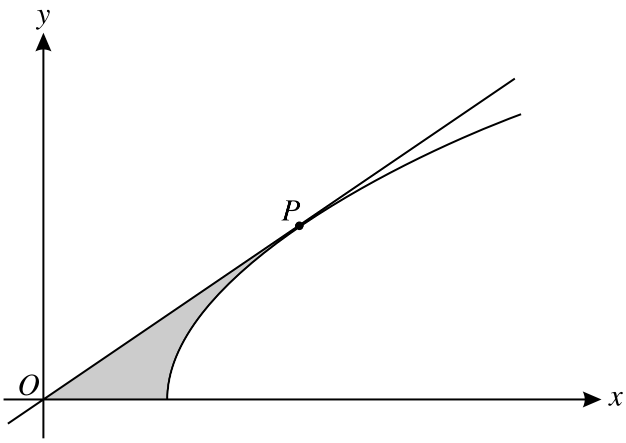
(iii) Use the iterative formula

$$x_{n+1} = 2 + \sqrt[4]{x_n + 16},$$

with a suitable starting value, to find the larger root correct to 3 decimal places. **[4]**

5 The equation of a curve is $y = x^2\ln(4x-3)$. Find the exact value of $\dfrac{d^2y}{dx^2}$ at the point on the curve for which $x = 2$. **[8]**

6



The diagram shows the curve with equation $y = \sqrt{3x - 5}$. The tangent to the curve at the point P passes through the origin. The shaded region is bounded by the curve, the x-axis and the line OP. Show that the x-coordinate of P is $\frac{10}{3}$ and hence find the exact area of the shaded region. **[9]**

7 The functions f, g and h are defined for all real values of x by

$$\mathrm{f}(x) = |x|, \qquad \mathrm{g}(x) = 3x + 5 \qquad \text{and} \qquad \mathrm{h}(x) = \mathrm{gg}(x).$$

(i) Solve the equation $\mathrm{g}(x + 2) = \mathrm{f}(-12)$. **[3]**

(ii) Find $\mathrm{h}^{-1}(x)$. **[3]**

(iii) Determine the values of x for which

$$x + \mathrm{f}(x) = 0.$$ **[2]**

8 An experiment involves two substances, Substance 1 and Substance 2, whose masses are changing. The mass, M_1 grams, of Substance 1 at time t hours is given by

$$M_1 = 400\mathrm{e}^{-0.014t}.$$

The mass, M_2 grams, of Substance 2 is increasing exponentially and the mass at certain times is shown in the following table.

t (hours)	0	10	20
M_2 (grams)	75	120	192

A critical stage in the experiment is reached at time T hours when the masses of the two substances are equal.

(i) Find the rate at which the mass of Substance 1 is decreasing when $t = 10$, giving your answer in grams per hour correct to 2 significant figures. **[3]**

(ii) Show that T is the root of an equation of the form $\mathrm{e}^{kt} = c$, where the values of the constants k and c are to be stated. **[5]**

(iii) Hence find the value of T correct to 3 significant figures. **[2]**

[Question 9 is printed overleaf.]

9 **(i)** Prove that $\dfrac{\sin(\theta - \alpha) + 3\sin\theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3\cos\theta + \cos(\theta + \alpha)} \equiv \tan\theta$ for all values of α. **[5]**

(ii) Find the exact value of $\dfrac{4\sin 149^\circ + 12\sin 150^\circ + 4\sin 151^\circ}{3\cos 149^\circ + 9\cos 150^\circ + 3\cos 151^\circ}$. **[3]**

(iii) It is given that k is a positive constant. Solve, for $0^\circ < \theta < 60^\circ$ and in terms of k, the equation

$$\frac{\sin(6\theta - 15^\circ) + 3\sin 6\theta + \sin(6\theta + 15^\circ)}{\cos(6\theta - 15^\circ) + 3\cos 6\theta + \cos(6\theta + 15^\circ)} = k.$$ **[4]**