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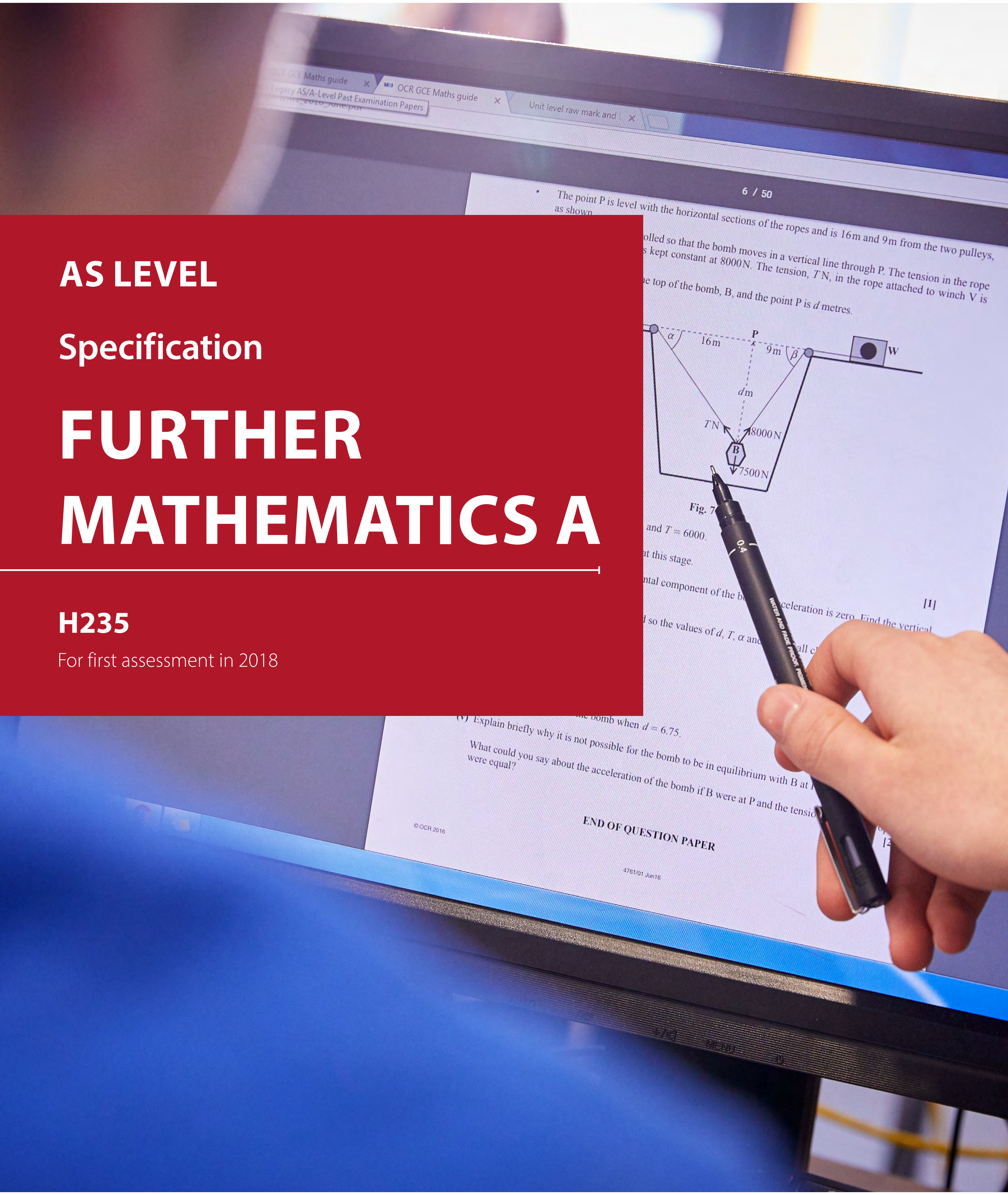

AS LEVEL

Specification

FURTHER MATHEMATICS A

H235

For first assessment in 2018

Version 1.8 (February 2024)

ocr.org.uk/aslevelfurthermaths

Registered office:
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

Contents

1 Why choose an OCR AS Level in Further Mathematics A?

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1a. Why choose an OCR qualification?

Choose OCR and you've got the reassurance that you're working with one of the UK's leading exam boards. Our new AS Level in Mathematics course has been developed in consultation with teachers, employers and Higher Education to provide learners with a qualification that's relevant to them and meets their needs.

We're part of the Cambridge Assessment Group, Europe's largest assessment agency and a department of the University of Cambridge. Cambridge Assessment plays a leading role in developing and delivering assessments throughout the world, operating in over 150 countries.

We work with a range of education providers, including schools, colleges, workplaces and other institutions in both the public and private sectors. Over 13,000 centres choose our A Levels, GCSEs and vocational qualifications including Cambridge Nationals and Cambridge Technicals.

Our Specifications

We believe in developing specifications that help you bring the subject to life and inspire your students to achieve more.

We've created teacher-friendly specifications based on extensive research and engagement with the teaching community. They're designed to be straightforward and accessible so that you can tailor the delivery of the course to suit your needs. We aim to encourage learners to become responsible for their own learning, confident in discussing ideas, innovative and engaged.

We provide a range of support services designed to help you at every stage, from preparation through to the delivery of our specifications. This includes:

- A wide range of high-quality creative resources including:
  - Delivery Guides
  - Transition Guides
  - Topic Exploration Packs
  - Lesson Elements
  - …and much more.
- Access to Subject Advisors to support you through the transition and throughout the lifetime of the specifications.
- CPD/Training for teachers including events to introduce the qualifications and prepare you for first teaching.
- Active Results – our free results analysis service to help you review the performance of individual learners or whole schools.
- ExamBuilder – our new free online past papers service that enables you to build your own test papers from past OCR exam questions can be found on the website at: www.ocr.org.uk/exambuilder

All AS level qualifications offered by OCR are accredited by Ofqual, the Regulator for qualifications offered in England. The accreditation number for OCR's AS Level in Further Mathematics A is QN603/1329/8.

1b. Why choose an OCR AS Level in Further Mathematics A?

OCR's AS Level in Further Mathematics A is a coherent course of study that supports the development of mathematically informed individuals. It encourages learners to think and act mathematically, using mathematical skills and forms of communication to analyse situations within mathematics and elsewhere.

The course provides a balance between breadth and depth of mathematical knowledge. The pure core provides the foundations for further mathematical study, onto which learners add two options taken from Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. These options provide flexibility, allowing OCR's AS Level in Further Mathematics to prepare students for further study and employment in a wide range of highly mathematical disciplines that require knowledge and understanding of sophisticated mathematical ideas and techniques.

OCR's AS Level in Further Mathematics A is designed for students who wish to study beyond an A Level in Mathematics, and provides a solid foundation for progression into further study particularly in mathematics, engineering, computer science, the sciences and economics.

OCR's AS Level in Further Mathematics A is both broader and deeper than A Level Mathematics. AS Level Further Mathematics builds from GCSE (9–1) Mathematics and AS and A Level Mathematics. As well as building on the algebra and calculus introduced in AS and A Level Mathematics, the AS Level Further Mathematics pure core content introduces complex numbers and matrices; fundamental mathematical ideas with wide applications in mathematics, engineering, physical sciences and computing. The non-core content includes different options that can enable learners to specialise in areas of mathematics that are particularly relevant to their interests and future aspirations, and gives learners the opportunity to extend their knowledge in applied mathematics and logical reasoning.

AS Level Further Mathematics A can be co-taught with the A Level Further Mathematics A as a separate qualification. It consolidates and develops GCSE (9–1) Mathematics and AS Level Mathematics and supports transition to higher education or employment in any of the many disciplines that make use of quantitative analysis, including the social sciences, business, accounting and finance, mathematics, engineering, computer science, the sciences and economics.

This qualification is part of a wide range of OCR mathematics qualifications, allowing progression from Entry Level Certificate through GCSE to Core Maths, AS and A Level.

We appreciate that one size doesn't fit all so we offer two suites of qualifications in mathematics and further mathematics.

Mathematics A builds on our existing popular course. We've based the redevelopment of our current suite around an understanding of what works well in centres and have updated areas of content and assessment where stakeholders have identified that improvements could be made. We've undertaken a significant amount of consultation through our mathematics forums (which include representatives from learned societies, HE, teaching and industry) and through focus groups with teachers.

Mathematics B (MEI) has been developed in collaboration with Mathematics in Education and Industry, and is based on the existing suite of qualifications assessed by OCR. This is a well-established partnership which provides a firm foundation for curriculum and qualification development. MEI is a long established, independent curriculum development body; in developing Mathematics B (MEI), MEI has consulted with teachers and representatives from Higher Education to decide how best to meet the long term needs of learners.

All of our specifications have been developed with subject and teaching experts. We have worked in close consultation with teachers and representatives from Higher Education (HE).

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Aims and learning outcomes

OCR's AS Level in Further Mathematics A will encourage learners to:

- understand mathematics and mathematical processes in ways that promote confidence, foster enjoyment and provide a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

1c. What are the key features of this specification?

The key features of OCR's AS Level in Further Mathematics A for you and your learners are:

- a specification developed by teachers specifically for teachers, laying out the content clearly in terms of topic area, showing clear progression through the course and supporting co-teaching with A Level Mathematics A and A Level Further Mathematics A.
- a simple assessment model featuring three papers of equal length and a free choice of two

options from four, so that learners can follow the most appropriate pathway for their interests and aspirations.

- a team of subject advisors, who can be contacted by centres for subject and assessment queries.

This specification is:

Worthwhile

- Research, international comparisons and engagement with both teachers and the wider education community have been used to enhance the reliability, validity and appeal of our assessment tasks in mathematics.
- It will encourage the teaching of interesting mathematics, aiming for mastery leading to positive exam results.

Learner-focused

- OCR's specification and assessment will consist of mathematics fit for the modern world and presented in authentic contexts.
- It will allow learners to develop mathematical independence built on a sound base of conceptual learning and understanding.
- OCR will target support and resources to develop fluency, reasoning and problem solving skills
- It will be a springboard for future progress and achievement in employment and in a variety of subjects in Higher Education.

Teacher-centred

- OCR will provide clear communication and an extensive teacher support package, including high-quality flexible resources, particularly for the new AS Level Further Mathematics subject areas and to support the use of technology, proof, modelling and problem solving.
- OCR's support and resources will focus on empowering teachers, exploring teaching methods and classroom innovation alongside more direct content-based resources.
- OCR's assessment will be solid and dependable, recognising positive achievement in candidate learning and ability.

Dependable

- OCR's high-quality assessments are backed up by sound educational principles and a belief that the utility, richness and power of mathematics should be made evident and accessible to all learners.
- An emphasis on learning and understanding mathematical concepts underpinned by a sound, reliable and valid assessment.

1d. How do I find out more information?

If you are already using OCR specifications you can contact us at: www.ocr.org.uk

If you are not already a registered OCR centre then you can find out more information on the benefits of becoming one at: www.ocr.org.uk

If you are not yet an approved centre and would like to become one go to: www.ocr.org.uk

Want to find out more?

Get in touch with one of OCR's Subject Advisors:

Email: maths@ocr.org.uk

Customer Contact Centre: 01223 553998

Visit our Online Support Centre at support.ocr.org.uk

2 The specification overview

2a. OCR's AS Level in Further Mathematics A (H235)

OCR's AS Level in Further Mathematics A is a linear qualification in which all papers must be taken in the same examination series.

All learners must take the mandatory Pure Core paper Y531 and any two* of the optional papers Y532, Y533, Y534 and Y535 to be awarded OCR's AS Level in Further Mathematics A.

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The subject content consists of a mandatory Pure Core and four optional areas: Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics.

The Overarching Themes must be applied along with associated mathematical thinking and understanding, across the whole of the subject content, see Section 2b.

*Learners may take more than two optional papers to increase the breadth of their course. For details of how their grade will be awarded, see Section 3g.

Content Overview	Assessment Overview	
Mandatory Pure Core All learners will study the content of the Pure Core. Paper Y531 assesses the content of the Pure Core and all of the Overarching Themes.	**Pure Core (Y531)** 60 marks 75 minutes written paper	**33⅓%** of total AS Level
Optional Papers Learners will study any two* areas chosen from Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. These papers assess the relevant content area and all of the Overarching Themes.	**Two of:** • Statistics (Y532) • Mechanics (Y533) • Discrete Mathematics (Y534) • Additional Pure Mathematics (Y535) Each: 60 marks 75 minutes written paper	**33⅓%** of total AS Level **33⅓%** of total AS Level

2b. Content of AS Level in Further Mathematics A (H235)

This AS level qualification builds on the skills, knowledge and understanding set out in the whole GCSE subject content for mathematics for first teaching from 2015 and in the GCE AS Level subject content for mathematics for first teaching from 2017. All of this content is assumed, but will only be explicitly assessed where it appears in this specification.

This is a linear qualification. The content is arranged by topic area and exemplifies the level of demand at AS Level. Statements have a unique reference code. For ease of comparison, planning and co-teaching the content statements in this specification have reference codes corresponding to the same statements in 'Stage 1' of OCR's A Level in Further Mathematics A (H245). Any gaps in the alphabetic referencing in this specification therefore refer to statements in similar topic areas in 'Stage 2' of OCR's A Level in Further Mathematics A (H245).
The content in these statements is identical, but the exemplification may differ as appropriate to the qualification.

This qualification is designed to be co-teachable with OCR's AS Level in Mathematics. Occasionally, knowledge and skills from the content of A Level Mathematics which is not in AS Level Mathematics are assumed for this qualification; this is indicated in the relevant content statements.

The content is separated into five areas: the Pure Core, Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. All learners must study the Pure Core and two of the remaining optional areas. Centres are free to teach the content in the order most appropriate to their learners' needs.

Sections 4, 5, 6, 7 and 8 cover the pure core, statistics, mechanics, discrete mathematics and additional pure mathematics content of AS Level Further Mathematics. In our mathematics specifications (H230 and H240) we have used the numbering 1, 2 and 3 to cover the pure mathematics, statistics and mechanics sections in order to facilitate the co-teaching of both qualifications.

The italic text in the content statements provides examples and further detail of the requirements of this specification. All exemplars contained in the specification under the heading "e.g." are for illustration only and do not constitute an exhaustive list. The heading "i.e." is used to denote a complete list. For the avoidance of doubt an italic statement in square brackets indicates content which will not be tested.

The expectation is that some assessment items will require learners to use two or more content statements without further guidance. Learners are expected to have explored the connections between their optional areas and the Pure Core. Learners may be required to demonstrate their understanding of the Pure Core content, and/or the content of AS Level Mathematics (H230), within the optional papers, but that content will not be explicitly assessed.

Learners are expected to be able to use their knowledge to reason mathematically and solve problems both within mathematics and in context. Content that is covered by any statement may be required in problem solving, modelling and reasoning tasks even if that is not explicitly stated in the statement.

Problem solving, proof and mathematical modelling will be assessed in further mathematics in the context of the wider knowledge which students taking AS Level further mathematics will have studied.

In **Pure Core (section 2c)** learners will extend and deepen their knowledge of proof, algebra and vectors studied in AS Level Mathematics. They will also broaden their knowledge into other areas of pure mathematics that underpin the further study of mathematics and other numerate subjects with complex numbers and matrices.

In **Statistics (section 2d)** learners will explore the theory which underlies the statistics content in A Level Mathematics, as well as extending their tool box of statistical concepts and techniques. This area covers, probability involving combinatorics,

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probability distributions for discrete random variables, χ-squared tests, correlation and regression.

In **Mechanics (section 2e)** learners extend their knowledge of particles, kinematics and forces from A Level Mathematics, using their extended pure mathematical knowledge to explore more complex physical systems. The area covers dimensional analysis, work, energy, power, impulse, momentum and circular motion.

Discrete Mathematics (section 2f) is the part of mathematics dedicated to the study of discrete objects. Learners will study pure mathematical structures and techniques, and their application to solving real-world problems of existence, construction, enumeration and optimisation. Areas studied include counting, graphs and networks, algorithms, critical path analysis, linear programming, and game theory.

In **Additional Pure Mathematics (section 2g)** learners will broaden and deepen their knowledge of pure mathematics, studying both discrete and continuous topics which form the foundation of undergraduate study in mathematics and mathematical disciplines. This area covers recurrence relations, number theory, group theory, the vector product, surfaces and partial differentiation.

Use of technology

It is assumed that learners will have access to appropriate technology when studying this course such as mathematical and statistical graphing tools and spreadsheets. When embedded in the mathematics classroom, the use of technology can facilitate the visualisation of certain concepts and deepen learners' overall understanding. The primary use of technology at this level is to offload computation and visualisation, to enable learners to investigate and generalise from patterns. Learners are not expected to be familiar with any particular software, but they are expected to be able to use their calculator for any function it can perform, when appropriate.

Suggested applications of technology to teaching and learning may be found in the introduction to each content area.

Use of calculators

Learners are permitted to use a scientific or graphical calculator for all papers. Calculators are subject to the rules in the document Instructions for Conducting Examinations, published annually by JCQ (www.jcq.org.uk).

It is expected that calculators available in the assessment will include the following features:

- An iterative function such as an ANS key.
- The ability to perform calculations, including inversion, with matrices up to at least order 3×3.
- The ability to compute summary statistics and access probabilities from the binomial and normal distributions.

Allowable calculators can be used for any function they can perform.
When using calculators, candidates should bear in mind the following:

1. Candidates are advised to write down explicitly any expressions, including integrals, that they use the calculator to evaluate.
2. Candidates are advised to write down the values of any parameters and variables that they input into the calculator. Candidates are not expected to write down data transferred from question paper to calculator.
3. Correct mathematical notation (rather than "calculator notation") should be used; incorrect notation may result in loss of marks.

Formulae

Learners will be given a Formulae Booklet in each assessment, which includes the formulae given for OCR's AS Level in Mathematics A. See section 5e for the content of this booklet.

Pre-release

There is no pre-released large data set for this qualification.

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Simplifying expressions

It is expected that learners will simplify algebraic and numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so. For example

- $80\frac{\sqrt{3}}{2}$ should be written as $40\sqrt{3}$,
- $\frac{1}{2}(1+2x)^{-\frac{1}{2}} \times 2$ should be written as either $(1+2x)^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{1+2x}}$,
- $\ln 2 + \ln 3 - \ln 1$ should be written as $\ln 6$,
- the equation of a straight line should be given in the form $y = mx + c$ or $ax + by = c$ unless otherwise stated.

The meanings of some instructions used in examination questions

In general, learners should show sufficient detail of their working to indicate that a correct method is being used. The following command words are used to indicate when more, or less, specific detail is required.

Exact
An exact answer is one where numbers are not given in rounded form. The answer will often contain an irrational number such as $\sqrt{3}$, e or π and these numbers should be given in that form when an exact answer is required.

The use of the word 'exact' also tells learners that rigorous (exact) working is expected in the answer to the question.
e.g. Find the exact solution of $\ln x = 2$.
The correct answer is e^2 and not $7.389\,056$.
e.g. Find the exact solution of $3x = 2$.
The correct answer is $x = \frac{2}{3}$ or $x = 0.\dot{6}$, not $x = 0.67$ or similar.

Prove
Learners are given a statement and must provide a formal mathematical argument which demonstrates its validity. A formal proof requires a high level of mathematical detail, with candidates clearly defining variables, correct algebraic manipulation and a concise conclusion.

Show that
Learners are given a result and have to show that it is true. Because they are given the result, the explanation has to be sufficiently detailed to cover every step of their working.
e.g. Show that the curve $y = x\ln x$ has a stationary point $\left(\frac{1}{e}, -\frac{1}{e}\right)$.

Determine
This command word indicates that justification should be given for any results found, including working where appropriate.

Verify
A clear substitution of the given value to justify the statement is required.

Find, Solve, Calculate
These command words indicate, while working may be necessary to answer the question, no justification is required. A solution could be obtained from the efficient use of a calculator, either graphically or using a numerical method.

Give, State, Write down
These command words indicate that neither working nor justification is required.

In this question you must show detailed reasoning.
When a question includes this instruction learners must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain sufficient detail to allow the line of their argument to be followed. This is not a restriction on a learner's use of a calculator when tackling the question, e.g. for checking an answer or evaluating a function at a given point, but it is a restriction on what will be accepted as evidence of a complete method.

In these examples variations in the structure of the answers are possible, for example; giving the integral as $\ln(x + \sqrt{x^2 - 16})$ in example 2, and different intermediate steps may be given.

Example 1:
Express $-4 + 2i$ in modulus-argument form.

The answer is $\sqrt{20}(\cos 2.68 + i\sin 2.68)$,
but the learner *must* include the steps
$|-4+2i| = \sqrt{16+4} = \sqrt{20}$,
$\arg(-4+2i) = \pi - \tan^{-1}(0.5) = 2.68$. Using a

calculator in complex mode to convert to modulus-argument form would not result in a complete analytical method.

Example 2:

Evaluate $\int_4^5 \frac{1}{\sqrt{x^2-16}}\,\mathrm{d}x$.

The answer is $\ln(2)$, but the learner *must* include at least $\lim_{a\to 4}\left[\operatorname{ar\,cosh}\left(\frac{x}{4}\right)\right]_a^5$ and the substitution $\ln\left(\frac{5}{4}+\sqrt{\frac{25}{16}-1}\right)-\ln(1+\sqrt{0})$. Just writing down the answer using the definite integral function on a calculator would therefore not be awarded any marks.

Example 3:

Solve the equation $2x^3-11x^2+22x-15=0$.

The answer is $1.5,\ 2\pm\mathrm{i}$, but the learner *must* include steps to find a real root or corresponding factor, find the factor $(2x-3)$ and factorise the cubic then solve the quadratic. Just writing down the three roots by using the cubic equation solver on a calculator would not be awarded any marks.

Hence

When a question uses the word 'hence', it is an indication that the next step should be based on what has gone before. The intention is that learners should start from the indicated statement.

e.g.

You are given that $\mathrm{f}(x) = 2x^3 - x^2 - 7x + 6$.

Show that $(x-1)$ is a factor of $\mathrm{f}(x)$.

Hence find the three factors of $\mathrm{f}(x)$.

Hence or otherwise

This is used when there are multiple ways of answering a given question. Learners starting from the indicated statement may well gain some information about the solution from doing so, and may already be some way towards the answer. The command phrase is used to direct learners towards using a particular piece of information to start from or to a particular method. It also indicates to learners that valid alternate methods exist which will be given full credit, but that they may be more time-consuming or complex.

e.g. Show that $(\cos x+\sin x)^2=1+\sin 2x$ for all x.

Hence, or otherwise, find the derivative of $(\cos x+\sin x)^2$.

You may use the result

When this phrase is used it indicates a given result that learners would not normally be expected to know, but which may be useful in answering the question.

The phrase should be taken as permissive; use of the given result is not required.

Plot

Learners should mark points accurately on graph paper. They will either have been given the points or have had to calculate them. They may also need to join them with a curve or a straight line, or draw a line of best fit through them.

e.g. Plot this additional point on the scatter diagram.

Sketch

Learners should draw a diagram, not necessarily to scale, showing the main features of a curve. These are likely to include at least some of the following.

- Turning points
- Asymptotes
- Intersection with the y-axis
- Intersection with the x-axis
- Behaviour for large x (+ or −)

Any other important features should also be shown.

e.g. Sketch the curve with equation $y=\frac{1}{(x-1)}$

Draw

Learners should draw to an accuracy appropriate to the problem. They are being asked to make a sensible judgement about the level of accuracy which is appropriate.

e.g. Draw a diagram showing the forces acting on the particle.

e.g. Draw a line of best fit for the data.

Other command words

Other command words, for example "explain", will have their ordinary English meaning.

Overarching Themes

These Overarching Themes should be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content in this specification. These statements are intended to direct the teaching and learning of AS Level Further Mathematics, and they will be reflected in assessment tasks.

OT1 Mathematical argument, language and proof

	Knowledge/Skill
OT1.1	Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable
OT1.2	Understand and use mathematical language and syntax as set out in the content
OT1.3	Understand and use language and symbols associated with set theory, as set out in the content
OT1.4	Not Applicable to AS Further Mathematics
OT1.5	Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics

OT2 Mathematical problem solving

	Knowledge/Skill
OT2.1	Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved
OT2.2	Construct extended arguments to solve problems presented in an unstructured form, including problems in context
OT2.3	Interpret and communicate solutions in the context of the original problem
OT2.4	Not Applicable to AS Further Mathematics
OT2.5	Not Applicable to AS Further Mathematics
OT2.6	Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle
OT2.7	Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems

OT3 Mathematical modelling

	Knowledge/Skill
OT3.1	Translate a situation in context into a mathematical model, making simplifying assumptions
OT3.2	Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student)
OT3.3	Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student)
OT3.4	Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate
OT3.5	Understand and use modelling assumptions

2c. Content of Pure Core (Mandatory paper Y531)

Introduction to the Pure Core.

In **Pure Core** learners will extend and deepen their knowledge of proof, algebra, and vectors studied in AS Level Mathematics. They will also broaden their knowledge into other areas of pure mathematics that underpin the further study of mathematics and other numerate subjects with complex numbers and matrices.

4.01 Proof
Proof by induction is introduced, including its application in proofs on powers of matrices and divisibility.

4.02 Complex Numbers
Complex numbers and their basic arithmetic are introduced, including in modulus-argument form. They are used to solve polynomial equations with real coefficients and to define loci on the Argand diagram.

4.03 Matrices
Matrix arithmetic is introduced and applied to linear transformations in 2-D, and some in 3-D, including the concept of invariance. Determinants and inverses of 2×2 and 3×3 matrices are found and used to solve matrix equations.

4.04 Further Vectors
Vector equations of lines are studied; methods for finding angles and distances between points and lines are developed. Scalar and vector products are introduced, and used in a variety of geometrical problems.

4.05 Further Algebra
Relationships between roots of and coefficients of polynomials are explored.

Assumed knowledge

Learners are assumed to know the content of GCSE (9–1) Mathematics and AS Level Mathematics. All of this content is assumed, but will only be explicitly assessed where it appears in this specification.

Occasionally, knowledge and skills from the content of A Level Mathematics which is not in AS Level Mathematics are assumed for this qualification; this is indicated in the relevant content statements.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Graphing tools: Learners should use graphing software to investigate the relationships between graphical and algebraic representations of complex numbers and vectors.

2. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships and matrices, and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.

3. Visualisation: Learners should use appropriate software to visualise situations in 3-D relating to lines and planes, and to linear transformations.

4. Spreadsheets: Learners should use spreadsheet software for modelling and to generate tables of values for functions.

Content of the Pure Core (Mandatory paper Y531)

Any gaps in the OCR ref. in this specification refer to statements in similar topic areas in 'Stage 2' of OCR's A Level in Further Mathematics A (H245).

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.01 Proof			
4.01a	Mathematical induction	a) Be able to construct proofs using mathematical induction. *This topic may be tested using any relevant content including divisibility, powers of matrices and results on powers, exponentials and factorials.* *e.g.* *Prove that* $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ *for* $n \in \mathbb{Z}^+$. *Prove that* $7^n - 3^n$ *is divisible by* 4 *for* $n \in \mathbb{Z}^+$. *Prove that* $2^n > 2n$ *for* $n \geq 3, n \in \mathbb{Z}$.	A1
4.02 Complex Numbers			
4.02a 4.02b 4.02c	The language of complex numbers	a) Understand the language of complex numbers. *Know the meaning of "real part", "imaginary part", "conjugate", "modulus" and "argument" of a complex number.* b) Be able to express a complex number z in either cartesian form $z = x + \mathrm{i}y$, where $\mathrm{i}^2 = -1$, or modulus-argument form $z = r(\cos\theta + \mathrm{i}\sin\theta) = [r, \theta] = r\mathrm{cis}\theta$, where $r \geq 0$ is the modulus of z and θ, measured in radians, is the argument of z. c) Understand and be able to use the notation: z, z^*, $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $\arg(z)$, $\lvert z \rvert$. *Includes knowing that a complex number is zero if and only if both the real and imaginary parts are zero.* *The principal argument of a complex number, for uniqueness, will be taken to lie in either of the intervals* $[0, 2\pi)$ *or* $(-\pi, \pi]$. *Learners may use either as appropriate unless the interval is specified.* *Knowledge of radians is assumed: see H240 section 1.05d.*	B2 B5 B3

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.02e 4.02f	Basic operations	e) Be able to carry out basic arithmetic operations $(+,-,\times,\div)$ on complex numbers in both cartesian and modulus-argument forms. *Knowledge of radians and compound angle formulae is assumed: see H240 sections 1.05d and 1.05l.* *Learners may use the results* $z_1z_2=[r_1r_2,\theta_1+\theta_2]$ *and* $\frac{z_1}{z_2}=\left[\frac{r_1}{r_2},\theta_1-\theta_2\right]$. f) Convert between cartesian and modulus-argument forms.	B2 B5 B6
4.02g 4.02h 4.02i 4.02j	Solution of equations	g) Know that, for a polynomial equation with real coefficients, complex roots occur in conjugate pairs. h) Be able to find algebraically the two square roots of a complex number. *e.g. By squaring and comparing real and imaginary parts.* i) Be able to solve quadratic equations with real coefficients and complex roots. j) Be able to use conjugate pairs, and the factor theorem, to solve or factorise cubic or quartic equations with real coefficients. *Where necessary, sufficient information will be given to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics.*	B1 B3
4.02k 4.02l	Argand diagrams	k) Be able to use and interpret Argand diagrams. *e.g. To represent and interpret complex numbers geometrically.* *Understand and use the terms "real axis" and "imaginary axis".* l) Understand the geometrical effects of taking the conjugate of a complex number, and adding and subtracting two complex numbers.	B4 B6

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.02o 4.02p	Loci	o) Be able to illustrate equations and inequalities involving complex numbers by means of loci in an Argand diagram. *i.e. Circle of the form* $\lvert z-a \rvert = k$, *half-lines of the form* $\arg(z-a)=b$, *lines of the form* $\mathrm{Re}(z)=k$, $\mathrm{Im}(z)=k$ *and* $\lvert z-a \rvert = \lvert z-b \rvert$, *and regions defined by inequalities in these forms.* *To include the convention of dashed and solid lines to show exclusion and inclusion respectively.* *No shading convention will be assumed. If not directed, learners should indicate clearly which regions are included.* *Knowledge of radians is assumed: see H240 section 1.05d.* p) Understand and be able to use set notation in the context of loci. *e.g. The region* $\lvert z-a \rvert > k$ *where* $z = x + \mathrm{i}y$, $a = x_a + \mathrm{i}y_a$ *and* $k > 0$ *may be represented by the set* $\{x+\mathrm{i}y : (x-x_a)^2 + (y-y_a)^2 > k^2\}$. *Knowledge of radians is assumed: see H240 section 1.05d.*	B7

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.03 Matrices			
4.03a	The language of matrices	a) Understand the language of matrices. *Understand the meaning of "conformable", "equal", "square", "rectangular", "m by n", "determinant", "zero" and "null", "transpose" and "identity" when applied to matrices.* *Learners should be familiar with real matrices and complex matrices.*	C2
4.03b	Matrix addition and multiplication	b) Be able to add, subtract and multiply conformable matrices; multiply a matrix by a scalar. *Learners may perform any operations involving entirely numerical matrices by calculator.* *Includes raising square matrices to positive integer powers.* *Learners should understand the effects on a matrix of adding the zero matrix to it, multiplying it by the zero matrix and multiplying it by the identity matrix.*	C1
4.03c		c) Understand that matrix multiplication is associative but not commutative. *Understand the terms "associative" and "commutative".*	

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.03d	Linear transformations	d) Be able to find and use matrices to represent linear transformations in 2-D. *Includes:* • *reflection in either coordinate axis and in the lines* $y = \pm x$ • *rotation about the origin* *(defined by the angle of rotation* θ*, where the direction of positive rotation is taken to be anticlockwise)* • *enlargement centre the origin* *(defined by the the scale factor)* • *stretch parallel to either coordinate axis* *(defined by the invariant axis and scale factor)* • *shear parallel to either coordinate axis* *(defined by the invariant axis and the image of a transformed point).* *Includes the terms “object” and “image”.*	C3
4.03e		e) Be able to find and use matrices to represent successive transformations. *Includes understanding and being able to use the result that the matrix product* $\mathbf{AB}$ *represents the transformation that results from the transformation represented by* $\mathbf{B}$ *followed by the transformation represented by* $\mathbf{A}$*.*	
4.03f		f) Be able to use matrices to represent single linear transformations in 3-D. *3-D transformations will be confined to reflection in one of the planes* $x = 0,\ y = 0,\ z = 0$ *or rotation about one of the coordinate axes. The direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation.* *Includes the terms “plane of reflection” and “axis of rotation”.* *Knowledge of 3-D vectors is assumed: see H240 section 1.10b.*	

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.03g	Invariance	g) Be able to find invariant points and lines for a linear transformation. *Includes the distinction between invariant lines and lines of invariant points.* [*The 3-D transformations in section 4.03f are excluded.*]	C4
4.03h	Determinants	h) Be able to find the determinant of a 2×2 matrix with and without a calculator. *Use and understand the notation* $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ *or* $\lvert \mathbf{M} \rvert$ *or* $\det \mathbf{M}$.	C5
4.03i		i) Know that the determinant of a 2×2 matrix is the area scale factor of the transformation defined by that matrix, including the effect on the orientation of the image. *Learners should know that a transformation preserves the orientation of the object if the determinant of the matrix which represents it is positive and that the transformation reverses orientation if the determinant is negative, and be able to interpret this geometrically.*	
4.03j		j) Be able to find the determinant of a 3×3 matrix with and without a calculator.	
4.03k		k) Know that the determinant of a 3×3 matrix is the volume scale factor of the transformation defined by that matrix, including the effect on the preservation of the orientation of the image. *Learners should know that the sign of the determinant determines whether or not the corresponding transformation preserves orientation, but do not need to understand the geometric interpretation of this in 3-D.*	
4.03l		l) Understand and be able to use singular and non-singular matrices. *Includes understanding the significance of a zero determinant.*	
4.03m		m) Know and be able to use the result that $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$.	

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.03n 4.03o 4.03p 4.03q	Inverses	n) Be able to find and use the inverse of a non-singular 2×2 matrix with and without a calculator. o) Be able to find and use the inverse of a non-singular 3×3 matrix with and without a calculator. p) Understand and be able to use simple properties of inverse matrices. *e.g. The result that* $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. q) Understand and be able to use the connection between inverse matrices and inverse transformations.	C6
4.03r	Solution of simultaneous equations	r) Be able to solve two or three linear simultaneous equations in two or three variables by the use of an inverse matrix, where a unique solution exists.	C7
4.04 Further Vectors			
4.04a	Equation of a straight line	a) Understand and be able to use the equation of a straight line, in 2-D and 3-D, in cartesian and vector form. *Learners should know and be able to use the forms:* $y = mx + c$, $ax + by = c$ *and* $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ *in 2-D and* $\frac{x - a_1}{u_1} = \frac{y - a_2}{u_2} = \frac{z - a_3}{u_3} (= \lambda)$ *and* $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ *in 3-D.* *Includes being able to convert from one form to another.*	F1
4.04c	Scalar product	c) Be able to calculate the scalar product and use it both to calculate the angles between vectors and/or lines, and also as a test for perpendicularity. *Includes the notation* $\mathbf{a.b}$	F3 F4
4.04e	Intersections	e) Be able to find, where it exists, the point of intersection between two lines. *Includes determining whether or not lines intersect, are parallel or are skew.*	F5

OCR Ref.	Subject Content	AS Level learners should…	DfE Ref.
4.04g	Vector product	g) Be able to use the vector product to find a vector perpendicular to two given vectors. *Includes the notation* $\mathbf{a} \times \mathbf{b}$. *When the vector product is required, either a calculator or a formula may be used. The formula below will be given:* $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}.$ [*The magnitude of the vector product is excluded.*]	Essential content for F2, F3, F5, C6
4.05 Further Algebra			
4.05a	Roots of equations	a) Understand and be able to use the relationships between the symmetric functions of the roots of polynomial equations and the coefficients. *Up to, and including, quartic equations.* *e.g. For the quadratic equation* $ax^2 + bx + c = 0$ *with roots* α *and* β, $\alpha + \beta = -\frac{b}{a}$ *and* $\alpha\beta = \frac{c}{a}$.	D1
4.05b	Transformation of equations	b) Be able to use a substitution to obtain an equation whose roots are related to those of the original equation. *Equations will be of at least cubic degree.*	D2

2d. Content of Statistics (Optional paper Y532)

Introduction to Statistics.

In **Statistics** learners will explore the theory which underlies the statistics content in AS Level Mathematics, as well as extending their tool box of statistical concepts and techniques. This area covers probability involving combinatorics, probability distributions for discrete random variables, chi-squared tests, correlation and regression.

5.01 Probability
The work on probability in A Level Mathematics is extended to include problems involving arrangements and selections.

5.02 Discrete Random Variables
The general concept of a discrete random variable introduced in AS Level Mathematics is further developed, along with the calculation of expectation and variance. The discrete uniform, binomial, geometric and Poisson distributions are studied.

5.06 Chi-squared Tests
The use of a chi-squared test to test for independence and goodness of fit is explored, including the interpretation of the results.

5.08 Correlation
The concept of correlation introduced in AS Level Mathematics is formalised and explored further, including the study of rank correlation.

5.09 Linear Regression
Regressions lines are calculated and used in context for estimation.

Hypothesis Tests

Hypotheses should be stated in terms of parameter values (where relevant) and the meanings of symbols should be stated. For example, “ $\mathrm{H}_0 : p = 0.7, \mathrm{H}_1 : p < 0.7$, where p is the population proportion in favour of the resolution”.

Conclusions should be stated in such a way as to reflect the fact that they are not certain. For example, “There is evidence at the 5% level to reject H_0. It is likely that the mean mass is less than 500 g.”
“There is no evidence at the 2% level to reject H_0. There is no reason to suppose that the mean journey time has changed.”

Some examples of incorrect conclusion are as follows:
“H_0 is rejected. Waiting times have increased.”
“Accept H_0. Plants in this area have the same height as plants in other areas.”

Assumed knowledge

Learners are assumed to know the content of GCSE (9–1) Mathematics and AS Level Mathematics. They are also assumed to know the content of the Pure Core (Y531). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

Occasionally, knowledge and skills from the content of A Level Mathematics which is not in AS Level Mathematics are assumed for this qualification; this is indicated in the relevant content statements.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Learners should use spreadsheets or statistical software to generate tables and diagrams, and to perform standard statistical calculations.
2. Hypothesis tests: Learners should use spreadsheets or statistical software to carry out hypothesis tests using the techniques in this paper.
3. Probability: Learners should use random number generators, including spreadsheets, to simulate tossing coins, rolling dice etc, and to investigate probability distributions.

Content of Statistics (Optional paper Y532)

Any gaps in the OCR ref. in this specification refer to statements in similar topic areas in 'Stage 2' of OCR's A Level in Further Mathematics A (H245).

OCR Ref.	Subject Content	AS Level learners should…
5.01 Probability		
5.01a	Probability	a) Be able to evaluate probabilities by calculation using permutations and combinations. *Includes the terms "permutation" and "combination".* *Includes the notation* ${}_n\mathrm{P}_r = {}^n\mathrm{P}_r$ *and* ${}_n\mathrm{C}_r = {}^n\mathrm{C}_r$. *For underlying content on probability see H230 section 2.03.*
5.01b		b) Be able to evaluate probabilities by calculation in contexts involving selections and arrangements. *Selection problems include, for example, finding the probability that 3 vowels and 2 consonants are chosen when 5 letters are chosen at random from the word 'CALCULATOR'.* *Arrangement problems only involve arrangement of objects in a line and include:* *1. repetition, e.g. the probability that the word 'ARTIST' is formed when the letters of the word 'STRAIT' are chosen at random.* *2. restriction, e.g. the probability that two consonants are (or are not) next to each other when the letters of the word 'TRAITS' are placed in a random order.*

OCR Ref.	Subject Content	AS Level learners should…
5.02 Discrete Random Variables		
5.02a	Probability distributions for general discrete random variables	a) Understand and be able to use discrete probability distributions. *Includes using and constructing probability distribution tables and functions relating to a given situation involving a discrete random variable.* *Any defined non-standard distribution will be finite.*
5.02b		b) Understand and be able to calculate the expectation and variance of a discrete random variable. *Includes knowing and being able to use the formulae* $\mu = \mathrm{E}(X) = \sum x_i p_i$ $\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$. [*Proof of these results is excluded.*]
5.02c		c) Know and be able to use the effects of linear coding on the mean and variance of a random variable.
5.02d	The binomial distribution	d) Know and be able to use the formulae $\mu = np$ and $\sigma^2 = np(1-p)$ for a binomial distribution. *[Proof of these results is excluded.]* *For the underlying content on binomial distributions, see H230 sections 2.04b and 2.04c.*
5.02e	The discrete uniform distribution	e) Know and be able to use the conditions under which a random variable will have a discrete uniform distribution, and be able to calculate probabilities and the mean and variance for a given discrete uniform distribution. *Includes use of the notation $X \sim \mathrm{U(n)}$ for the uniform distribution over the interval $[1, n]$.*

OCR Ref.	Subject Content	AS Level learners should…
5.02f	The geometric distribution	f) Know and be able to use the conditions under which a random variable will have a geometric distribution. *Includes use of the notation* $X \sim \mathrm{Geo}(p)$*, where* X *is the number of trials up to and including the first success.*
5.02g		g) Be able to calculate probabilities using the geometric distribution. *Learners may use the formulae* $\mathrm{P}(X = x) = (1-p)^{x-1}p$ *and* $\mathrm{P}(X > x) = (1-p)^x$.
5.02h		h) Know and be able to use the formulae $\mu = \frac{1}{p}$ and $\sigma^2 = \frac{1-p}{p^2}$ for a geometric distribution. [*Proof of these results is excluded.*]
5.02i	The Poisson distribution	i) Understand informally the relevance of the Poisson distribution to the distribution of random events, and be able to use the Poisson distribution as a model. *Includes use of the notation* $X \sim \mathrm{Po}(\lambda)$*, where* X *is the number of events in a given interval.*
5.02j		j) Understand and be able to use the formula $\mathrm{P}(X = x) = \mathrm{e}^{-\lambda}\frac{\lambda^x}{x!}$.
5.02k		k) Be able to calculate probabilities using the Poisson distribution, using appropriate calculator functions. *Learners are expected to have a calculator with the ability to access probabilities from the Poisson distribution.* [*Use of the Poisson distribution to calculate numerical approximations for a binomial distribution is excluded.*]
5.02l		l) Know and be able use the conditions under which a random variable will have a Poisson distribution. *Learners will be expected to identify which of the modelling conditions [assumptions] is/are relevant to a given scenario and to explain them in context.*
5.02m		m) Be able to use the result that if $X \sim \mathrm{Po}(\lambda)$ then the mean and variance of X are each equal to λ.
5.02n		n) Know and be able to use the result that the sum of independent Poisson variables has a Poisson distribution.

OCR Ref.	Subject Content	AS Level learners should…
5.06 Chi-squared Tests		
5.06a	Contingency tables	a) Be able to use a chi-squared (χ^2) test with the appropriate number of degrees of freedom to test for independence in a contingency table and interpret the results of such a test. *Rows or columns, as appropriate, should be combined so that each expected frequency is at least 5, and Yates' correction should be used in the special case of a 2×2 table.* *A table of critical values of the χ^2 distribution will be provided.* *Includes calculation of expected frequencies and contributions to the test statistic.* *Questions may require candidates to calculate some expected frequencies and contributions to the test statistic, but will not involve lengthy calculations.*
5.06b	Fitting a theoretical distribution	b) Be able to fit a theoretical distribution, as prescribed by a given hypothesis involving a given ratio, proportion or discrete uniform distribution, to given data. *Questions may require candidates to calculate some expected frequencies, but will not involve lengthy calculations.*
5.06d	Goodness of fit test	d) Be able to use a χ^2 test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit test. *Where necessary, adjacent classes should be combined so that each expected frequency is at least 5.* *A table of critical values of the χ^2 distribution will be provided.*

OCR Ref.	Subject Content	AS Level learners should…
5.08 Correlation		
5.08a	Pearson's product-moment correlation coefficient	a) Be able to calculate the product-moment correlation coefficient (pmcc) for a set of bivariate data; raw data or summarised data may be given. *Use of appropriate calculator functions is expected.* *Learners will not be required to enter large amounts of data into a calculator during the examination.*
5.08b		b) Understand that the value of a correlation coefficient is unaffected by linear coding of the variables.
5.08c		c) Understand Pearson's product-moment correlation coefficient as a measure of how close data points lie to a straight line.
5.08d	Hypothesis tests using Pearson's product-moment correlation coefficient	d) Use and be able to interpret Pearson's product-moment correlation coefficient in hypothesis tests, using either a given critical value or a p-value and a table of critical values. *When using Pearson's coefficient in a hypothesis test, the data may be assumed to come from a bivariate normal distribution.* *A table of critical values of Pearson's coefficient will be provided*.
5.08e	Spearman's rank correlation coefficient	e) Be able to calculate Spearman's rank correlation coefficient for a maximum of 10 pairs of data values or ranks. *Includes being able to draw basic conclusions about the meaning of a value of the coefficient in relation to the ranks before, or without, carrying out a hypothesis test.* *Includes understanding the conditions under which the use of rank correlation may be appropriate.* [*Tied ranks are excluded.*]
5.08f	Hypothesis tests using Spearman's coefficient	f) Be able to carry out a hypothesis test for association in a population. *Includes understanding that this is a non-parametric test, as it makes no assumptions about the population.* *Tables of critical values of Spearman's coefficient will be provided.*
5.08g	Comparison of coefficients	g) Be able to choose between Pearson's product-moment correlation coefficient and Spearman's rank correlation coefficient for a given context. *Includes interpreting a scatter diagram and distinguishing between linear correlation and association.*

OCR Ref.	Subject Content	AS Level learners should…
5.09 Linear Regression		
5.09a	Dependent and independent variables	a) Understand the difference between an independent (or controlled) variable and a dependent (or response) variable. *Includes appreciating that, in a given situation, neither parameter may be independent.*
5.09b	Calculation of the equation of the regression line	b) Understand the concepts of least squares and regression lines in the context of a scatter diagram.
5.09c		c) Be able to calculate, both from raw data and from summarised data, the equation of the regression line of y on x, where the independent variable (if any) is x. [*The regression line of x on y is excluded in the case when x is independent.*]
5.09d		d) Understand the effect on a regression line of linear coding on one or both variables.
5.09e	Use of the regression line	e) Be able to use, in the context of a problem, the regression line of y on x to estimate a value of y, and be able to interpret in context the uncertainties of such an estimate.

2

2e. Content of Mechanics (Optional paper Y533)

Introduction to Mechanics.

In **Mechanics** learners extend their knowledge of particles, kinematics and forces from A Level Mathematics, using their extended pure mathematical knowledge to explore more complex physical systems. The area covers dimensional analysis, work, energy, power, impulse, momentum and circular motion.

6.01 Dimensional Analysis
The relationships between physical quantities are analysed by considering their dimensions (length, mass and time), in order to construct or check models.

6.02 Work, Energy and Power
The fundamental concepts of work, energy and power are introduced, including kinetic energy and gravitational potential energy. The principle of conservation of mechanical energy is used to solve problems.

6.03 Impulse and Momentum
Problems involving collisions in a straight line are studied, using the principal of conservation of linear momentum and Newton's experimental law.

6.05 Motion in a circle
The motion of a particle in a horizontal or vertical circle is explored, including using energy considerations to study motion with variable speed.

Assumed knowledge

Learners are assumed to know the content of GCSE (9–1) Mathematics and AS Level Mathematics. They are also assumed to know the content of the Pure Core (Y531). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

The technique of **resolving forces** is found in 'Stage 2' of the A Level mathematics content, and therefore not in the AS Level Mathematics content assumed, but it is a vital underlying skill in the more advanced mechanics topics met in this paper. It is therefore taken as assumed knowledge, though it will not be assessed in isolation. This includes both being able to express a force as two mutually perpendicular components, and being able to find the resultant of two or more forces acting at a point. See sections 6.02b, 6.02l and 6.05c.

Use of technology
To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Spreadsheets: Learners should use spreadsheets to generate tables of values for functions and to investigate functions numerically.

2. Graphing Tools: Learners should use graphing software for modelling, including kinematics and projectiles, and in visualising physical systems.

3. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships, including derivatives and integrals, and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.

4. Complex problem solving: Learners could use CAS to perform computation when solving complex problems in mechanics, including those which lead to equations or systems that they cannot solve analytically.

5. Practical mechanics: Learners could use computers and/or mobile phones to enrich practical mechanics tasks, using them for data logging, to create videos of moving objects, or to share and analyse data.

Content of Mechanics (Optional paper Y533)

Any gaps in the OCR ref. in this specification refer to statements in similar topic areas in 'Stage 2' of OCR's A Level in Further Mathematics A (H245).

OCR Ref.	Subject Content	AS Level learners should…
6.01 Dimensional Analysis		
6.01a	Dimensional analysis	a) Be able to find the dimensions of a quantity in terms of M, L and T, and understand that some quantities are dimensionless. *Includes understanding and using the notation $[d]$ for the dimension of the quantity d.* *Learners are expected to know or be able to derive the dimensions of any quantity for which they know the units. Dimensions of other quantities will be given, or their derivation will be the focus of assessment.*
6.01b		b) Understand and be able to use the relationship between the units of a quantity and its dimensions.
6.01c		c) Be able to use dimensional analysis as an error check. *e.g. Verify the relationship that power is proportional to the product of the driving force and the velocity.*
6.01d		d) Be able to use dimensional analysis to determine unknown indices in a proposed formulation. *e.g. Determine the period of oscillation of a simple pendulum in terms of its length, mass and the acceleration due to gravity, g.*
6.01e		e) Be able to formulate models and derive equations of motion using a dimensional argument.

OCR Ref.	Subject Content	AS Level learners should…
6.02 Work, Energy and Power		
6.02a 6.02b	Work	a) Understand the concept of work done by a force. b) Be able to calculate the work done by a constant force. *The force may not act in the direction of motion of the body and so learners will be expected to resolve forces in two dimensions.*
6.02d 6.02e	Energy	d) Understand the concept of the mechanical energy of a body. *i.e. The kinetic and potential energy.* e) Be able to calculate the gravitational potential energy (mgh) and kinetic energy $(\frac{1}{2}mv^2)$ of a body.
6.02i	Conservation of energy	i) Understand and be able to use the principle of the conservation of mechanical energy and the work-energy principle for dynamic systems, including consideration of energy loss. *Learners will only need to consider kinetic and gravitational potential energy.*
6.02k 6.02l	Power	k) Understand and be able to use the definition of power (the rate at which a force does work). *Includes* $\text{average power} = \dfrac{\text{work done}}{\text{time elapsed}}$. l) Be able to use the relationship between power, the tractive force and velocity $(P = Fv)$ to solve problems. *e.g. Motion on an inclined plane.* *Includes maximum velocity and speed.* *Learners will be required to resolve forces in two dimensions.*

OCR Ref.	Subject Content	AS Level learners should…
6.03 Impulse and Momentum		
6.03a 6.03b	Linear momentum	a) Recall and be able to use the definition of linear momentum in one dimension. b) Understand and be able to apply the principle of conservation of linear momentum in one dimension applied to two particles. *Includes using the formula* $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.
6.03e 6.03f	Impulse	e) Understand and be able to use the concept of the impulse imparted by a force. f) Be able to use the relationship between the instantaneous impulse of a force and the change in momentum $(I = mv - mu)$. *The instantaneous impulse is the impulse associated with an instantaneous change in velocity.* *Learners will only be required to apply this to instantaneous events in one dimension.* *e.g.* *The direct impact of two smooth spheres.* *An impulsive force acting in the direction of an inelastic string.* *Questions involving collision(s) between particles may include multiple collisions and the conditions under which further collisions occur.*

OCR Ref.	Subject Content	AS Level learners should…
6.03i	Restitution	i) Recall and be able to use the definition of the coefficient of restitution, including $0 \leq e \leq 1$. [*Superelastic collisions are excluded.*]
6.03j		j) Understand and be able to use the terms “perfectly elastic” $(e = 1)$ and “inelastic” $(e = 0)$ for describing collisions. *Learners should know that for perfectly elastic collisions there will be no loss of kinetic energy and for inelastic collisions the bodies coalesce and there is maximum loss of kinetic energy.*
6.03k		k) Recall and be able to use Newton’s experimental law in one dimension for problems of direct impact. e.g. *Between two smooth spheres* $(v_1 - v_2 = -e(u_1 - u_2))$ *and a smooth sphere with a fixed plane surface* $(v = -eu)$*, where* u *and* v *are the velocities before and after impact.*
6.05 Motion in a Circle		
6.05a	Uniform motion in a circle	a) Understand and be able to use the definitions of angular velocity, velocity, speed and acceleration in relation to a particle moving in a circular path, or a point rotating in a circle, with constant speed. *Includes the use of both* ω *and* $\dot{\theta}$.
6.05b		b) Be able to use and apply the relationships $v = r\dot{\theta}$ and $a = \frac{v^2}{r} = r\dot{\theta}^2 = v\dot{\theta}$ for motion in a circle with constant speed.
6.05c		c) Be able to solve problems regarding motion in a horizontal circle. *e.g.* *Motion of a conical pendulum.* *Motion on a banked track.* *Problems will be restricted to those involving constant forces but learners will be required to resolve forces in two dimensions.*
6.05d	Motion in a vertical circle	d) Understand the motion of a particle in a circle with variable speed. *Learners will be expected to use energy considerations to calculate the speed of a particle at a given point on a circular path but knowledge of the radial and tangential components of the acceleration will not be required.*

2f. Content of Discrete Mathematics (Optional paper Y534)

Introduction to Discrete Mathematics.

Discrete Mathematics is the part of mathematics dedicated to the study of discrete objects. Learners will study pure mathematical structures and techniques, and their application to solving real-world problems of existence, construction, enumeration and optimisation. Areas studied include counting, graphs and networks, algorithms, critical path analysis, linear programming and game theory.

7.01 Mathematical Preliminaries
Learners are introduced to the fundamental categorisation of problems as existence, construction, enumeration and optimisation. They are also introduced to counting techniques that have a wide application across Discrete Mathematics.

7.02 Graphs and Networks
Graphs and networks are introduced as mathematical objects that can be used to model real world systems involving connections and relationships. The pure mathematics of graph theory is studied including isomorphism and Eulerian graphs.

7.03 Algorithms
The algorithmic approach to problem solving in introduced via sorting and packing problems. The run-time and order of an algorithm are studied.

7.04 Network Algorithms
Problems involving networks are introduced: shortest path and minimum connector.
Standard network algorithms are studied and used to solve these problems.

7.05 Decision Making in Project Management
Networks are applied to decision making, in particular to activity networks and critical path analysis, including scheduling.

7.06 Graphical Linear Programming
The concept of linear programming is explored as a tool for optimisation. Linear programmes are solved graphically.

7.08 Game Theory
Problems of conflict and cooperation are explored using game theory, including both pure and mixed strategies.

Assumed knowledge
Learners are assumed to know the content of GCSE (9–1) Mathematics and AS Level Mathematics. They are also assumed to know the content of the Pure Core (Y531). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

The use of algorithms
Learners will only be expected to use specific algorithms if instructed to do so in the question. For example a list may be sorted by inspection unless a question specifically asks for the use of a sorting algorithm, and lengths of shortest paths may be found by inspection unless a question specifically asks for the use of Dijkstra's algorithm.

The Formulae Booklet contains sketches of some of the algorithms found in this area. The focus of the study of algorithms in this area should be on understanding the theory and processes, not on memorisation or practising the rote application of algorithms by hand. In the classroom, technology should be used to demonstrate the power of these algorithms in large scale problems; in the assessment, learners will be asked to demonstrate the application of algorithms to small scale problems only.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Graphing tools: Learners could use graphing software to perform graphical linear programming, and to investigate the effects on the solution of changing coefficients and parameters.

2. Networks and network algorithms: Learners could use software to investigate networks and to implement network algorithms, in particular for networks which are too large to work with by hand.

3. Algorithms: Learners could use spreadsheets or a suitable programming language to implement simple algorithms and to understand how to create algorithms to perform simple tasks.

4. Computer Algebra System (CAS): Learners could use CAS software to draw and manipulate graphs, to explore algebraic relationships. This is best done in conjunction with other software such as graphing tools and spreadsheets.

5. Simulation: Learners could use spreadsheets to simulate contexts in game theory, including investigating the long term effects of particular strategies.

Content of Discrete Mathematics (Optional paper Y534)

Any gaps in the OCR ref. in this specification refer to statements in similar topic areas in 'Stage 2' of OCR's A Level in Further Mathematics A (H245).

OCR Ref.	Subject Content	AS Level learners should…
7.01 Mathematical Preliminaries		
7.01a	Types of problem	a) Understand and be able to use the terms "existence", "construction", "enumeration" and "optimisation" in the context of problem solving. *Includes classifying a given problem into one or more of these categories.*
7.01b	Set notation	b) Understand and be able to use the basic language and notation of sets. *Includes the term "partition" and counting the number of partitions of a set including with constraints.*
7.01c	The pigeonhole principle	c) Be able to use the pigeonhole principle in solving problems.
7.01d	Arrangement and selection problems	d) Understand and use the multiplicative principle. *Includes knowing that the number of arrangements of* n *distinct objects is* $\prod_{r=1}^{n} r = n!$.
7.01e		e) Be able to enumerate the number of ways of obtaining an ordered subset (permutation) of r elements from a set of n distinct elements. *Includes using the notation* ${}_{n}\mathrm{P}_{r} = {}^{n}\mathrm{P}_{r}$.
7.01f		f) Be able to enumerate the number of ways of obtaining an unordered subset (combination) of r elements from a set of n distinct elements. *Includes using the notation* ${}_{n}\mathrm{C}_{r} = {}^{n}\mathrm{C}_{r}$.

OCR Ref.	Subject Content	AS Level learners should…
7.01g 7.01i		g) Be able to solve problems about enumerating the number of arrangements of objects in a line, including those involving: 1. repetition, *e.g. how many different eight digit numbers can be made from the digit of 12333210?* 2. restriction, *e.g. how many different eight digit numbers can be made from the digits of 12333210 if the two 2s cannot be next to each other?* i) Be able to solve problems about selections, including with constraints. *e.g. Find the number of ways in which a team of 3 men and 2 women can be selected from a group of 6 men and 5 women*
7.01k	The inclusion-exclusion principle	k) Be able to use the inclusion-exclusion principle for two sets in solving problems. *e.g.* $n(\mathrm{A} \cup \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$. *Venn diagrams may be used.* *e.g. How many integers in $\{1, 2, \ldots, 100\}$ are not divisible by 2 or 3?*
7.02 Graphs and Networks		
7.02a 7.02b 7.02c	Terminology and notation	a) Understand the meaning of the terms “vertex” (or “node”) and “arc” (or “edge”). *Includes the concept of the “degree” of a vertex as the number of arcs “incident” to the vertex.* *Includes the term “adjacent” for pairs of vertices or edges.* b) Understand the meaning of the terms “tree”, “simple”, “connected” and “simply connected” as they refer to graphs. *Includes understanding and using the restrictions on the vertex degrees implied by these conditions.* c) Understand the meaning of the terms “walk”, “trail”, “path”, “cycle” and “route”. *A “walk” is a set of arcs where the end vertex of one is the start vertex of the next.* *A “trail” is a walk in which no arcs are repeated.* *A “path” is a trail in which no nodes are repeated.* *A “cycle” is a closed path.* *A “route” can be a walk, a trail or a path, or may be a closed walk, trail or path.*

OCR Ref.	Subject Content	AS Level learners should…
7.02d	Complete graphs	d) Understand and be able to use the term "complete" and the notation K_n for a complete graph on *n* vertices. *Includes knowing that K_n has $\frac{1}{2}n(n-1)$ arcs.*
7.02e	Bipartite graphs	e) Understand and be able to use bipartite graphs and the notation $K_{m,n}$ for a complete bipartite graph connecting m vertices to *n* vertices. *Includes knowing that $K_{m,n}$ has mn arcs.*
7.02g	Eulerian graphs	g) Use the degrees of vertices to determine whether a given graph is Eulerian, semi-Eulerian or neither. Understand what these terms mean in terms of traversing the graph.
7.02j	Isomorphism	j) Understand what it means to say that two graphs are isomorphic. Construct an isomorphism either by a reasoned argument or by explicit labelling of vertices. *Includes understanding that having the same degree sequence (ordered list of vertex degrees) is necessary but not sufficient to show isomorphism.* *Includes the term "non-isomorphic".*
7.02k	Digraphs	k) Understand and be able to use digraphs. *Includes the terms "indegree" and "outdegree".*
7.02p	Using graphs and networks	p) Understand that a network is a weighted graph. Use graphs and networks to model the connections between objects. *Graphs and networks may be directed or undirected.*
7.02q		q) Use an adjacency matrix representation of a graph and a weighted matrix representation of a network.
7.02r		r) Be able to model problems using graphs or networks, and solve them.

OCR Ref.	Subject Content	AS Level learners should…
7.03 Algorithms		
7.03a	Definition of an algorithm	a) Understand that an algorithm has an input and an output, is deterministic and finite. *Includes the use of a counter and the use of a stopping condition in an algorithm.* *Be familiar with the terms "greedy", "heuristic" and "recursive" in the context of algorithms.*
7.03b	Awareness of the uses and practical limitations of algorithms	b) Appreciate why an algorithmic approach to problem-solving is generally preferable to ad hoc methods, and understand the limitations of algorithmic methods. *Includes understanding that algorithmic methods are used by computers for solving large scale problems and that small scale problems are only being used to demonstrate how a given algorithm works.*
7.03c	Working with algorithms	c) Trace through an algorithm and interpret what the algorithm has achieved. Algorithms may be presented as flow diagrams, listed in words, or written in simple pseudo-code. *Includes understanding and being able to use the functions* $\text{INT}(x)$ *and* $\text{ABS}(x)$. *Learners may find it useful to have a calculator with these functions, but large numbers of repeated applications will not be required in the assessment.* *Includes adapting or altering an algorithm to achieve a given purpose, and adjusting a short set of instructions to create an algorithm.* [*Programming skills will not be required*.]
7.03d	The order of an algorithm	d) Use the order of an algorithm to calculate an approximate run-time for a large problem by scaling up a given run-time. *Includes understanding that when the "maximum run-time" of an algorithm is represented as a function of the "size" of the problem, the order of the algorithm, for very large sized problems, is given by the dominant term.* *Learners should know that the sum of the first* n *positive integers is* $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$. *Learners should be familiar with the notation* $\text{O}(n^4)$ *and the concept of dominance in an informal sense only.*

OCR Ref.	Subject Content	AS Level learners should…
7.03e	Efficiency and complexity	e) Compare the efficiency of two algorithms that achieve the same end result by considering a given aspect of the run-time in a specific case. *e.g. The number of swaps or comparisons to sort a given list.*
7.03f		f) Calculate worst case time complexity, the “maximum run-time” $\mathrm{T}(n)$, as a function of the size of a problem by considering the worst case for a specific problem. *Includes cases of the algorithms for sorting and standard network problems studied in this specification.* *Includes an informal understanding that, for example* $\mathrm{T}(n) = n^2 + n^4$ *is order* n^4*, or equivalently* $\mathrm{O}(n^4)$*.*
7.03g		g) Be familiar with $\mathrm{O}(n^k)$, where n is a measure of the size of the problems and $k = 0, 1, 2, 3$ or 4.
7.03j	Strategies for sorting	j) Be able to sort a list using bubble sort and using shuttle sort. *Bubble sort and shuttle sort will start at the left-hand end of the list, unless specified otherwise in the question.* *Includes knowing that, in general, sorting algorithms have quadratic order as a function of the length of the list.*
7.03l	Strategies for packing	l) Be familiar with the next-fit, first-fit, first-fit decreasing and full bin methods for one-dimensional packing problems. *Includes knowing that these are heuristic algorithms.* *Includes the terms “online” and “offline”.*

OCR Ref.	Subject Content	AS Level learners should…
7.04 Network Algorithms		
7.04a	Least weight path between two vertices	a) Be able to use examples to demonstrate understanding and use of Dijkstra's algorithm to find the length and route of a least weight (shortest) path. *Solve problems that require a least weight (shortest) path as part of their solution.* *Know that Dijkstra's algorithm has quadratic order (as a function of the number of vertices).*
7.04b	Least weight set of arcs connecting all vertices	b) Be able to use examples to demonstrate understanding and use of Prim's algorithm (both in graphical and tabular/matrix form) and Kruskal's algorithm to find a minimum connector (minimum spanning tree) for a network. *Solve problems that require a minimum spanning tree as part of their solution.* *Includes adapting a solution to deal with practical issues.* *Know that Prim's algorithm and Kruskal's algorithm have cubic order (as a function of the number of vertices).*
7.04f	Network problems	f) Be able to choose an appropriate algorithm to solve a practical problem. *Includes adapting an algorithm or a solution to deal with practical issues.*
7.05 Decision Making in Project Management		
7.05a	Critical path analysis	a) Be able to construct and interpret activity networks using activity on arc. *Appreciate that a path of critical activities (a critical path) is a longest path in a directed network.*
7.05b		b) Be able to carry out a forward pass to determine earliest start times and find the minimum project completion time, and to carry out a backward pass to determine latest finish times and find the critical activities. *Includes understanding and using the terms "burst" and "merge".*
7.05c		c) Understand, and be able to calculate, (total) float.

OCR Ref.	Subject Content	AS Level learners should…
7.06 Graphical Linear Programming		
7.06a	Formulating LP problems	a) be able to set up a linear programming formulation in the form "maximise (or minimise) objective subject to inequality constraints, and trivial constraints of the form variable ≥ 0". *Includes:* *1. identifying relevant variables, including units when appropriate,* *2. formulating constraints in these variables, including when the information is given in ratio form,* *3. writing down an objective function and stating whether it is to be maximised or minimised.*
7.06c	Working with constraints	c) Be able to investigate constraints and objectives in numerical cases using algebra and ad hoc methods.
7.06d	Graphical solutions	d) Be able to carry out and interpret a graphical solution for problems where the objective is a function of two variables, including cases where integer solutions are required. *The region where each inequality is not satisfied will be shaded, leaving the feasible region as the unshaded part of the graph.*
7.08 Game Theory		
7.08a 7.08b	Pay-off matrix	a) Understand the idea of a zero-sum game and its representation by means of a pay-off matrix. *Includes converting a game to a zero-sum form, where appropriate.* b) Be able to reduce a matrix using a dominance argument.
7.08c	Pure strategies	c) Be able to identify play-safe strategies and stable solutions and understand what they represent.
7.08e	Mixed strategies	e) Be able to determine an optimal mixed strategy for a game with no stable solution by reducing to two variables and using simultaneous equations or a graphical method, where possible. *Includes knowing that the optimum may occur at an extreme value ($p = 0$ or $p = 1$).*

2g. Content of Additional Pure Mathematics (Optional paper Y535)

Introduction to Additional Pure Mathematics.

In **Additional Pure Mathematics** learners will broaden and deepen their knowledge of pure mathematics, studying both discrete and continuous topics which form the foundation of undergraduate study in mathematics and mathematical disciplines. This area covers recurrence relations, number theory, group theory, the vector product, surfaces and partial differentiation.

8.01 Sequences and Series
Recurrence relations are explored, including their long term behaviour and solution of first order recurrence relations.

8.02 Number Theory
Number theory is introduced through number bases, modular arithmetic, divisibility algorithms and solving linear congruences.

8.03 Groups
Group axioms and examples of finite groups of small order are studied, including cyclic groups.

8.04 Further Vectors
The concept of vector product introduced in the Pure Core is developed and extended to finding areas and solving problems using vector methods.

8.05 Surfaces and Partial Differentiation
Surfaces in 3-D are explored, including contours and sections, and applying partial differentiation to find stationary points.

Assumed knowledge
Learners are assumed to know the content of GCSE (9–1) Mathematics and AS Level Mathematics. They are also assumed to know the content of the Pure Core (Y531).

All of this content is assumed, but will only be explicitly assessed where it appears in this section. Occasionally, knowledge and skills from the content of A Level Mathematics which is not in AS Level Mathematics are assumed for this qualification; this is indicated in the relevant content statements.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Graphing tools: Learners could use graphing software to investigate the relationships between graphical and algebraic representations, including 3-D plots of surfaces and solids of revolution.

2. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships, including manipulation of matrices, evaluating integrals, solving recurrence systems and solving equations and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.

3. Visualisation: Learners could use appropriate software to visualise situations in 3-D relating to surfaces, and to linear transformations and invariance.

4. Spreadsheets: Learners could use spreadsheet software to investigate sequences and series, for modelling and to generate tables of values for functions.

Content of Additional Pure Mathematics (Optional paper Y535)

Any gaps in the OCR ref. in this specification refer to statements in similar topic areas in 'Stage 2' of OCR's A Level in Further Mathematics A (H245).

OCR Ref.	Subject Content	AS Level learners should…
8.01 Sequences and Series		
8.01a	Recurrence relations	a) Be able to work with general sequences given as recurrence relations or by position-to-term (closed form) formulae $u_n = \mathrm{f}(n)$. *The notation $\{u_n\}$ for sequences, which may or may not include a zeroth term, should be recognised.*
8.01b		b) Use induction to prove results relating to both sequences and series.
8.01c	Properties of sequences	c) Understand and be able to describe various possibilities for the behaviour of sequences. *Learners are expected to be able to use the terms "periodic", "convergence", "divergence" "oscillating", "monotonic".* *Note that a periodic sequence with period two may be referred to as "oscillating", but that both convergent and divergent sequences can oscillate. "Divergence" can refer to sequences that are bounded or unbounded.*
8.01d		d) Identify and be able to use the limit of the nth term of a sequence as $n \to \infty$, including steady-states. *Includes forming sequences from other sequences, for example, finding differences or ratios of successive terms of a sequence.* [*Rates of convergence are excluded.*]
8.01e	Fibonacci and related numbers	e) Be able to work with the Fibonacci numbers (and other Fibonacci-like sequences, such as the Lucas numbers), and understand their properties. *Includes recognising and using the properties of ϕ, both numerical and algebraic, and its role in the Fibonacci sequence.*

OCR Ref.	Subject Content	AS Level learners should…
8.01f	Solving recurrence systems	f) Be able to solve a first-order linear recurrence relation with constant coefficients, using the associated auxiliary equation and complementary function. *Includes finding both general and particular solutions.* *Includes homogeneous and non-homogeneous recurrence relations of the form* $u_{n+1} = au_n + \mathrm{f}(n)$*, where* $\mathrm{f}(n)$ *may be a polynomial function or of the form* dk^n*.* *Includes knowing the terms, "closed form" and "position-to-term".* *Includes understanding that a "recurrence system" consists of a "recurrence relation", an "initial condition" and the range of the variable* n*.*
8.01h	Modelling	h) Be able to apply their knowledge of recurrence relations to modelling. *Includes birth- and/or death-rates and the use of the* $\mathrm{INT}(x)$ *function for discrete models. Learners may find it useful to have a calculator with this function, but large numbers of repeated applications will not be required in the assessment.*
8.02 Number Theory		
8.02a	Number bases	a) Understand and be able to work with numbers written in base n, where n is a positive integer. *The standard notation for number bases will be used.* *i.e.* 2013_n *will denote the number* $2n^3 + n + 3$ *(with* $n > 3$ *in this example) and the letters* A*–*F *will be used to represent the integers 10–15 respectively when* $11 \le n \le 16$*.*
8.02b 8.02c	Divisibility tests	b) Be able to use (without proof) standard tests for divisibility by $2, 3, 4, 5, 8, 9$ and 11. *Includes knowing that repeated tests can be used to establish divisibility by composite numbers.* c) Be able to establish suitable (algorithmic) tests for divisibility by other primes less than 50. *For integers* a *and* b*, the notation* $a \mid b$ will be used for *"*a *divides exactly into* b*" ("*a *is a factor of* b*", "*b *is a multiple of* a*", etc.).*

OCR Ref.	Subject Content	AS Level learners should…
8.02d	The division algorithm	d) Appreciate that, for any pair of positive integers a, b with $0 < b \leq a$, we can uniquely express a as $a = bq + r$ where q (the quotient) and r (the residue, or remainder, when a is divided by b) are both positive integers and $r < b$.
8.02e 8.02f	Finite (modular) arithmetics	e) Understand and be able to use finite arithmetics (the arithmetic of integers modulo n for $n \geq 2$). f) Be able to solve single linear congruences of the form $a\,x \equiv b \pmod{n}$.
8.02i 8.02j	Prime numbers	i) Understand the concepts of prime numbers, composite numbers, highest common factors (hcf), and coprimality (relative primeness). *Knowledge of the fundamental theorem of arithmetic will be expected, but proof of the result will not be required.* j) Know and be able to apply the result that $a \mid b$ and $a \mid c \Rightarrow a \mid (bx + cy)$ for any integers x and y. *Includes using this result, for example to test for common factors or coprimality.*
8.02k	Euclid's lemma	k) Know and be able to use Euclid's lemma: if $a \mid rs$ and $\mathrm{hcf}(a, r) = 1$ then $a \mid s$.
8.03 Groups		
8.03a 8.03b	Binary operations	a) Be able to work with binary operations and their properties when defined on given sets. *Includes knowing and understanding the terms "associativity" and "commutativity".* b) Be able to construct Cayley tables for given finite sets under the action of a given binary operation. *Multiplicative notation and/or terminology will generally be used, when appropriate.*

OCR Ref.	Subject Content	AS Level learners should…
8.03c 8.03d	Definition of a group	c) Recall and be able to use the definition of a group, for example to show that a given structure is, or is not, a group. *e.g. Questions may be set on groups of integers modulo n (for $n \geq 2$), functions, matrices, transformations, the symmetries of given geometrical shapes and complex numbers.* *Groups may be referred to in either of the forms:* *1. by the given set and associated binary operation $(G, \circ)$,* *2. as "G", where the operation is understood, or* *3. as "the set G with the operation $\circ$".* *To include knowing the meaning of the terms "identity" and "closed", and that in an abelian group the operation is commutative.* d) Recognise and be able to use the Latin square property for group tables.
8.03e	Orders of elements and groups	e) Recall the meaning of the term "order", as applied both to groups and to elements of a group, and be able to determine the orders of elements in a given group. *Includes knowing and being able to use the fact that the order of an element is a factor of the order of the group.*
8.03f	Subgroups	f) Understand and be able to use the definition of a subgroup, find subgroups and show that given subsets are, or are not, proper subgroups.
8.03g	Cyclic groups	g) Recall the meaning of the term "cyclic" as applied to groups.
8.03h	Generators	h) Understand that a cyclic group is generated by "powers" of a single element (generator), that there may be more than one such element within a group, and that other (non-cyclic) groups may be generated by two or more elements along with their "powers" and "products".
8.03i	Properties of groups	i) Be familiar with the structure of finite groups up to, and including, order seven, and be able to apply this knowledge in solving problems.

OCR Ref.	Subject Content	AS Level learners should…
8.04 Further Vectors		
8.04a	Vector product	a) Understand and be able to use the definition, in geometrical terms, of the vector product and be able to form the vector product in magnitude and direction, and in component form. *Includes use of the formula* $\mathbf{a} \times \mathbf{b} = \lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \sin\theta\, \hat{\mathbf{n}}$, *where* $\mathbf{a}, \mathbf{b}, \hat{\mathbf{n}}$, *in that order (and the vectors* $\mathbf{i}, \mathbf{j}, \mathbf{k}$, *in that order) form a right-handed triple.*
8.04b		b) Understand the anti-commutative and distributive properties of the vector product.
8.04c		c) Be able to use the vector product to calculate areas of triangles and parallelograms.
8.04d		d) Understand the significance of $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. *e.g. The equation of a line in the form* $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$.

OCR Ref.	Subject Content	AS Level learners should…
8.05 Surfaces and Partial Differentiation		
8.05a	3-D surfaces	a) Be able to work with functions of two variables, given either explicitly in the form $z = \mathrm{f}(x, y)$ or implicitly in the form $\mathrm{g}(x, y, z) = c$, and understand and use the fact that this equation, and its partial derivatives, relate to a 3-D surface. *An informal understanding only of how the partial derivatives relate to the surface is required.* *Functions* $\mathrm{f}(x, y)$ *will involve sums and products of powers of* x *and* y *only. Issues relating to domains and ranges will not be considered beyond the appreciation that, for example the surface* $z = \frac{x}{y}$ *has no point at which* $y = 0$.
8.05c	Sections and contours	c) Be able to sketch sections and contours, and know how these are related to the surface. *i.e. Sections of the form* $z = \mathrm{f}(a, y)$ *or* $z = \mathrm{f}(x, b)$ *and contours of the form* $c = \mathrm{f}(x, y)$.
8.05d	Partial differentiation	d) Be able to find first and second derivatives, including mixed derivatives. *Learners will be expected to recognise and use both notations for first- and second-order partial derivatives, including mixed ones.* *e g.* $\frac{\partial \mathrm{f}}{\partial x}, \frac{\partial^2 \mathrm{f}}{\partial x \partial y}$ and $\mathrm{f}_x, \mathrm{f}_{xy}$. *Includes the Mixed derivative theorem; namely, that* $\frac{\partial^2 \mathrm{f}}{\partial x \partial y} = \frac{\partial^2 \mathrm{f}}{\partial y \partial x}$ *or* $\mathrm{f}_{xy} = \mathrm{f}_{yx}$ *for suitably well-defined, continuous functions* f.
8.05e	Stationary points	e) Understand and be able to apply the concept that stationary points of z arise when $\frac{\partial \mathrm{f}}{\partial x} = \frac{\partial \mathrm{f}}{\partial y} = 0$ (or $\mathrm{f}_x = \mathrm{f}_y = 0$) and that these can be maxima, minima or saddle-points. *Learners should know and understand the basic properties of these stationary points.* *Learners will only be required to find stationary points, but will not be required to determine their natures.*

2h. Prior knowledge, learning and progression

- It is assumed that learners are familiar with the content of GCSE (9–1) Mathematics for first teaching from 2015 and the content of GCE AS Level Mathematics for first teaching from September 2017. Occasionally knowledge and skills from the content of A Level Mathematics which is not in AS Level Mathematics are assumed; this is indicated in the relevant content statements.

- OCR's AS Level in Further Mathematics A is designed for learners who wish to study more mathematics alongside an AS Level in Mathematics, and provides a solid foundation for progression into further study particularly in mathematics, engineering, computer science, the sciences and economics.

- OCR's AS Level in Further Mathematics A is both broader and deeper than AS Level Mathematics. AS Level Further Mathematics builds from GCSE (9–1) Mathematics and AS Level Mathematics. As well as building on the algebra and calculus introduced in AS Level Mathematics, the AS Level Further Mathematics pure core content introduces complex numbers and matrices, and gives learners the opportunity to extend their knowledge in applied mathematics and logical reasoning. The non-core content includes different options that can enable learners to specialise in areas of mathematics that are particularly relevant to their interests and future aspirations. AS Level Further Mathematics provides a valuable breadth and depth of study for supporting the transition to degree level work and employment in disciplines that make use of quantitative analysis, including the social sciences, business, accounting and finance, mathematics, engineering, computer science, the sciences and economics.

- Some learners may wish to follow a further mathematics course only up to AS Level, in order to broaden their curriculum, and to develop their interest and understanding of different areas of the subject.

- A Level Further Mathematics A can be co-taught with AS Level Further Mathematics A as a separate qualification. It consolidates and develops GCSE (9–1) Mathematics, A Level Mathematics and AS Level Further Mathematics, and prepares students for further study and employment in highly mathematical disciplines that require knowledge and understanding of sophisticated mathematical ideas and techniques.

There are a number of Mathematics specifications at OCR. Find out more at www.ocr.org.uk

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3 Assessment of AS Level in Further Mathematics A

3a. Forms of assessment

OCR's AS Level in Further Mathematics A consists of one mandatory paper and two optional papers all of which are externally assessed.

OCR's AS Level in Further Mathematics A is a linear qualification in which all papers must be taken in the same examination series.

3

Mandatory Pure Core

All learners will study the content of the Pure Core. This Pure Core is assessed through one paper and synoptically within the optional papers as appropriate.

Optional papers

Learners will study at least two areas chosen from Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. Each area is assessed in a single paper.

All five papers (Y531 – Y535) contain assessment of the Overarching Themes and some extended response questions.

Any valid combination of three papers will include at least one unstructured problem solving question, which addresses multiple areas of the problem solving cycle as set out in the Overarching Themes in section 2b.

Any valid combination of three papers will include at least two problem solving questions, which address the first two bullet points of Assessment Objective 3 in combination and at least two modelling questions, which address the last three bullet points of Assessment Objective 3 in combination. See Section 3b.

All examinations have a duration of 75 minutes.

Allowable calculators can be used for any function they can perform.

In each question paper, learners are expected to support their answers with appropriate working.

See Section 2b for use of calculators.

Pure Core (Y531)
This paper is worth 33⅓% of the total AS Level. All questions are compulsory and there are 60 marks in total. The paper assesses content from the Pure Core section of the specification, in the context of all of the Overarching Themes. The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Statistics (Y532)
This paper is worth 33⅓% of the total AS Level. All questions are compulsory and there are 60 marks in total. The paper assesses content from the Statistics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes. The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Mechanics (Y533)
This paper is worth 33⅓% of the total AS Level. All questions are compulsory and there are 60 marks in total.

The paper assesses content from the Mechanics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Discrete Mathematics (Y534)
This paper is worth 33⅓% of the total AS Level. All questions are compulsory and there are 60 marks in total.

The paper assesses content from the Discrete Mathematics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Additional Pure Mathematics (Y535)
This paper is worth 33⅓% of the total AS Level. All questions are compulsory and there are 60 marks in total.

The paper assesses content from the Additional Pure Mathematics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

3b. Assessment Objectives (AO)

There are 3 Assessment Objectives in OCR AS Level in Further Mathematics A. These are detailed in the table below.

	Assessment Objectives	Weightings AS Level
AO1	**Use and apply standard techniques** Learners should be able to: • select and correctly carry out routine procedures; and • accurately recall facts, terminology and definitions.	60% (±2%)
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • construct rigorous mathematical arguments (including proofs); • make deductions and inferences; • assess the validity of mathematical arguments; • explain their reasoning; and • use mathematical language and notation correctly. *Where questions/tasks targeting this assessment objective will also credit learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and other contexts' (AO3) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s).*	At least 10% for any valid combination of papers
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical and non-mathematical contexts into mathematical processes; • interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations; • translate situations in context into mathematical models; • use mathematical models; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. *Where questions/tasks targeting this assessment objective will also credit learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s).*	At least 10% for any valid combination of papers

AO weightings in AS Level in Further Mathematics

The target number of marks allocated to each AO for each paper, out of a total of 60 for each paper, is given in the table below.

The number of marks for AO2 and AO3 reflects the appropriate balance of assessment for the content in each paper.

Paper	Number of marks		
	AO1	AO2	AO3
Pure Core (Y531)	35–37 Marks	14–16 Marks	8–10 Marks
Statistics (Y532)	35–37 Marks	11–13 Marks	11–13 Marks
Mechanics (Y533)	35–37 Marks	5–7 Marks	17–19 Marks
Discrete Mathematics (Y534)	35–37 Marks	11–13 Marks	11–13 Marks
Additional Pure Mathematics (Y535)	35–37 Marks	14–16 Marks	8–10 Marks

Each set of assessments which constitutes a valid combination will achieve the weighting shown in the following table.

Each of these sets of assessment gives an overall percentage of 60% (±2%) for AO1 and at least 10% for each of AO2 and AO3 as required.

Pure Core + choice of two options	% of overall AS Level in Further Mathematics A (H235)		
	AO1	AO2	AO3
Statistics and Mechanics	105–111 marks	30–36 marks	36–42 marks
Statistics and Discrete Mathematics	105–111 marks	36–42 marks	30–36 marks
Statistics and Additional Pure Mathematics	105–111 marks	39–45 marks	27–33 marks
Mechanics and Discrete Mathematics	105–111 marks	30–36 marks	36–42 marks
Mechanics and Additional Pure Mathematics	105–111 marks	33–39 marks	33–39 marks
Discrete Mathematics and Additional Pure Mathematics	105–111 marks	39–45 marks	27–33 marks
Total	60% (±2%)	At least 10%	At least 10%

3c. Total qualification time

Total qualification time (TQT) is the total amount of time, in hours, expected to be spent by a learner to achieve a qualification. It includes both guided learning hours and hours spent in preparation, study, and assessment. The total qualification time for AS Level in Further Mathematics A is 180 hours. The total guided learning time is 180 hours.

3d. Qualification availability outside of England

This qualification is available in England. For Wales and Northern Ireland please check the Qualifications in Wales Portal (QIW) or the Northern Ireland Department of Education Performance Measures / Northern Ireland Entitlement Framework Qualifications Accreditation Number (NIEFQAN) list to see current availability.

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3e. Language

This qualification is available in English only. All assessment materials are available in English only and all candidate work must be in English.

3f. Assessment availability

There will be one examination series available each year in May/June to **all** learners.

All examined papers must be taken in the same examination series at the end of the course.
This specification will be certificated from the June 2018 examination series onwards.

3g. Retaking the qualification

Learners can retake the qualification as many times as they wish. They must retake a complete valid combination of papers for the qualification as detailed in section 2a.

3h. Assessment of extended response

The assessment materials for this qualification provide learners with the opportunity to demonstrate their ability to construct and develop a sustained and coherent line of reasoning and marks for extended responses are integrated into the marking criteria. Tasks which offer this opportunity will be found across all five papers.

3i. Synoptic assessment

Mathematics is by nature, a synoptic subject. The assessment in this specification allows learners to demonstrate the understanding they have acquired from the course as a whole and their ability to integrate and apply that understanding. This level of understanding is needed for successful use of the knowledge and skills from this course in future life, work and study.

In all the examination papers, learners will be required to integrate and apply their understanding in order to address problems which require both breadth and depth of understanding in order to reach a satisfactory solution.

Learners will be expected to reflect on and interpret solutions, drawing on their understanding of different aspects of the course.

Tasks which offer this opportunity will be found in all papers.

3j. Calculating qualification results

A learner's overall qualification grade for AS Level in Further Mathematics A will be calculated by adding together their marks from the three papers taken to give their total raw mark. This mark will then be compared to the qualification level grade boundaries that apply for the combination of papers taken by the learner and for the relevant exam series to determine the learner's overall qualification grade.

Where learners take more than the required number of optional papers, the combination of papers that result in the best grade will be used.

Note: this may NOT be the combination with the highest number of raw marks.

The total raw mark will be the total from the combination that leads to the best grade.

4 Admin: what you need to know

The information in this section is designed to give an overview of the processes involved in administering this qualification so that you can speak to your exams officer. All of the following processes require you to submit something to OCR by a specific deadline.

More information about the processes and deadlines involved at each stage of the assessment cycle can be found in the Administration area of the OCR website

OCR's *Admin overview* is available on the OCR website at http://www.ocr.org.uk/administration.

4a. Pre-assessment

Estimated entries

Estimated entries are your best projection of the number of learners who will be entered for a qualification in a particular series. Estimated entries should be submitted to OCR by the specified deadline. They are free and do not commit your centre in any way.

Final entries

Final entries provide OCR with detailed data for each learner, showing each assessment to be taken. It is essential that you use the correct entry code, considering the relevant entry rules.

Final entries must be submitted to OCR by the published deadlines or late entry fees will apply.

All learners taking an AS Level in Further Mathematics must be entered for H235.

All learners must also be entered for each of the papers they are taking using the relevant entry codes.

Paper entry codes are given in the table below.

All learners take Y531 and **at least** two of the optional papers Y532, Y533, Y534 and Y535 to be awarded OCR's AS Level in Further Mathematics A.

Where learners take more than the required number of optional papers, the combination of papers that result in the best grade will be used.

Note: this may NOT be the combination with the highest number of raw marks.

The total raw mark will be the total from the combination that leads to the best grade.

Entry code	Title	Paper code	Paper title	Assessment type
H235	Further Mathematics A	Y531	Pure Core	External Assessment (Mandatory)
		Y532	Statistics	External Assessment (Optional)
		Y533	Mechanics	External Assessment (Optional)
		Y534	Discrete Mathematics	External Assessment (Optional)
		Y535	Additional Pure Mathematics	External Assessment (Optional)

Collecting evidence of student performance to ensure resilience in the qualifications system

Regulators have published guidance on collecting evidence of student performance as part of long-term contingency arrangements to improve the resilience of the qualifications system. You should review and consider this guidance when delivering this qualification to students at your centre.

For more detailed information on collecting of evidence of student performance please visit our website at: https://www.ocr.org.uk/administration/general-qualifications/assessment/

4b. Special consideration

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Special consideration is a post–assessment adjustment to marks or grades to reflect temporary injury, illness or other indisposition at the time the assessment was taken.

Detailed information about eligibility for special consideration can be found in the JCQ publication *A guide to the special consideration process.*

4c. External assessment arrangements

Regulations governing examination arrangements are contained in the JCQ *Instructions for conducting examinations*.

Head of centre annual declaration

The Head of Centre is required to provide a declaration to the JCQ as part of the annual NCN update, conducted in the autumn term, to confirm that the centre is meeting all of the requirements detailed in the specification. Any failure by a centre to provide the Head of Centre Annual Declaration will result in your centre status being suspended and could lead to the withdrawal of our approval for you to operate as a centre.

Private candidates

Private candidates may enter for OCR assessments.

A private candidate is someone who pursues a course of study independently but takes an examination or assessment at an approved examination centre. A private candidate may be a part-time student, someone taking a distance learning course, or someone being tutored privately. They must be based in the UK.

Private candidates need to contact OCR approved centres to establish whether they are prepared to host them as a private candidate. The centre may charge for this facility and OCR recommends that the arrangement is made early in the course.

Further guidance for private candidates may be found on the OCR website: https://www.ocr.org.uk

4d. Results and certificates

Grade Scale

AS Level qualifications are graded on the scale: A, B, C, D, E, where A is the highest. Learners who fail to reach the minimum standard for E will be Unclassified (U). Only subjects in which grades A to E are attained will be recorded on certificates.

Papers are graded on the scale a, b, c, d, e, where a is the highest. Learners who fail to reach the minimum standard for e with be unclassified (u). Individual paper results will not be recorded on certificates.

Results

Results are released to centres and learners for information and to allow any queries to be resolved before certificates are issued.

Centres will have access to the following results information for each learner:

- the grade for the qualification
- the raw mark for each paper
- the total raw mark for the qualification.

The following supporting information will be available:

- raw mark grade boundaries for each paper
- raw mark grade boundaries for all combinations of paper.

Until certificates are issued, results are deemed to be provisional and may be subject to amendment.

A learner's final results will be recorded on an OCR certificate.

The qualification title will be shown on the certificate as 'OCR Level 3 Advanced Subsidiary GCE in Further Mathematics A'.

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4e. Post-results services

A number of post-results services are available:

- **Review of marking** – If you are not happy with the outcome of a learner's results, centres may request a review of marking. Full details of the post-results services are provided on the OCR website.
- **Missing and incomplete results** – This service should be used if an individual subject result for a learner is missing, or the learner has been omitted entirely from the results supplied.
- **Access to scripts** – Centres can request access to marked scripts.

4f. Malpractice

Any breach of the regulations for the conduct of examinations and non-exam assessment work may constitute malpractice (which includes maladministration) and must be reported to OCR as soon as it is detected.

Detailed information on malpractice can be found in the JCQ publication *Suspected Malpractice in Examinations and Assessments: Policies and Procedures*.

5 Appendices

5a. Overlap with other qualifications

This qualification overlaps with A Level Further Mathematics A and with other specifications in A Level Further Mathematics and AS Further Mathematics.

5b. Accessibility

Reasonable adjustments and access arrangements allow learners with special educational needs, disabilities or temporary injuries to access the assessment and show what they know and can do, without changing the demands of the assessment. Applications for these should be made before the examination series. Detailed information about eligibility for access arrangements can be found in the JCQ *Access Arrangements and Reasonable Adjustments.*

The A Level qualification and subject criteria have been reviewed in order to identify any feature which could disadvantage learners who share a protected Characteristic as defined by the Equality Act 2010. All reasonable steps have been taken to minimise any such disadvantage.

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5c. Mathematical notation

The tables below set out the notation that must be used by AS Level Mathematics and Further Mathematics specifications. Learners will be expected to understand this notation without need for further explanation. Any additional notation required is listed in the relevant content statement in section 2 of the specification.

1	**Set Notation**	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1,\ x_2,\ \ldots$
1.6	$\{x : \ldots\}$	the set of all x such that $\ldots$
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$

1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \le x \le b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R} : a \le x < b\}$
1.22	$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \le b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
1.24	$\mathbb{C}$	the set of complex numbers
2	**Miscellaneous Symbols**	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \le$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \ge$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
3	**Operations**	
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b
3.3	$a \times b, ab, a.b$	a multiplied by b

3.4	$a \div b, \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1,\ n \in \mathbb{N};\ 0! = 1$
3.10	$\binom{n}{r}, {}^nC_r, {}_nC_r$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leqslant n$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$
4	**Functions**	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
4.5	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x, \delta x$	an increment of x
4.7	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x),\ \mathrm{f}''(x),\ \ldots,\ \mathrm{f}^{(n)}(x)$	the first, second, ..., nth derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x},\ \ddot{x},\ \ldots$	the first, second, ... derivatives of x with respect to t
4.11	$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
5	**Exponential and Logarithmic Functions**	
5.1	e	base of natural logarithms
5.2	$\mathrm{e}^x,\ \exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x,\ \log_{\mathrm{e}} x$	natural logarithm of x

6	**Trigonometric Functions**	
6.1	sin, cos, tan, cosec, sec, cot	the trigonometric functions
6.2	$\sin^{-1}, \cos^{-1}, \tan^{-1}$ arcsin, arccos, arctan	the inverse trigonometric functions
6.3	$\circ$	degrees
6.4	rad	radians
7	**Complex Numbers**	
7.1	i, j	square root of –1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
7.5	$\mathrm{Re}(z)$	the real part of z, $\mathrm{Re}(z) = x$
7.6	$\mathrm{Im}(z)$	the imaginary part of z, $\mathrm{Im}(z) = y$
7.7	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
7.8	$\arg(z)$	the argument of z, $\arg(z) = \theta$, $-\pi < \theta \le \pi$
7.9	z^*	the complex conjugate of z, $x - \mathrm{i}y$
8	**Matrices**	
8.1	$\mathbf{M}$	a matrix $\mathbf{M}$
8.2	$\mathbf{0}$	zero matrix
8.3	$\mathbf{I}$	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
8.6	Δ, det $\mathbf{M}$ or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix $\mathbf{M}$
8.7	$\mathbf{Mr}$	image of column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$
9	**Vectors**	
9.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector $\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment AB

9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of **a**
9.4	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert \mathbf{a} \rvert, \ a$	the magnitude of **a**
9.6	$\lvert \overrightarrow{AB} \rvert, AB$	the magnitude of $\overrightarrow{AB}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}, \ a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.12	$\mathbf{a.b}$	the scalar product of **a** and **b**
11	**Probability and Statistics**	
11.1	$A, B, C,$ etc.	events
11.4	$\mathrm{P}(A)$	probability of the event A
11.5	A'	complement of the event A
11.7	$X, Y, R,$ etc.	random variables
11.8	$x, y, r,$ etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \ldots$	observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x), \mathrm{P}(X = x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n, \ p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean

11.26	s^2	sample variance
11.27	s	sample standard deviation
11.28	H_0	Null hypothesis
11.29	H_1	Alternative hypothesis
11.30	r	product-moment correlation coefficient for a sample
11.31	ρ	product-moment correlation coefficient for a population
12	**Mechanics**	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres
12.4	m/s, $\mathrm{m\,s^{-1}}$	metres per second (velocity)
12.5	$\mathrm{m/s^2}$, $\mathrm{m\,s^{-2}}$	metres per second per second (acceleration)
12.6	F	force or resultant force
12.7	N	Newton
12.8	N m	Newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction

5d. Mathematical formulae and identities

Learners must be able to use the following formulae and identities for AS Level Further Mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms.

These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

<u>Pure Mathematics</u>

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$$a^x a^y \equiv a^{x+y}$$
$$a^x \div a^y \equiv a^{x-y}$$
$$(a^x)^y \equiv a^{xy}$$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x + \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$

$$k \log_a x \equiv \log_a (x^k)$$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$$y - y_1 = m(x - x_1)$$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Trigonometry

In the triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area $= \dfrac{1}{2} ab \sin C$

$$\cos^2 A + \sin^2 A \equiv 1$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Mensuration

Circumference and area of circle, radius r and diameter d

$C = 2\pi r = \pi d \qquad A = \pi r^2$

Pythagoras' theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse

$c^2 = a^2 + b^2$

Area of a trapezium $= \frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length l and encloses an associated sector of area a

$l = r\theta \qquad a = \frac{1}{2}r^2\theta$

Complex Numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

Loci in the Argand diagram

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix $\mathbf{AB}$ is the transformation represented by matrix $\mathbf{B}$ followed by the transformation represented by matrix $\mathbf{A}$.

For matrices $\mathbf{A}$, $\mathbf{B}$:

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Algebra

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

For $ax^2 + bx + c = 0$ with roots α and β:

$\alpha + \beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$

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For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ:

$$\sum \alpha = \frac{-b}{a} \qquad \sum \alpha\beta = \frac{c}{a} \quad \alpha\beta\gamma = \frac{-d}{a}$$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
e^{kx}	$k\mathrm{e}^{kx}$
$\mathrm{f}(x) + \mathrm{g}(x)$	$\mathrm{f}'(x) + \mathrm{g}'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1} + c, \; n \neq -1$
$\mathrm{f}'(x) + \mathrm{g}'(x)$	$\mathrm{f}(x) + \mathrm{g}(x) + c$

Area under a curve $= \int_a^b y \, \mathrm{d}x \; (y \geq 0)$

Vectors

$|x\mathbf{i} + y\mathbf{j}| = \sqrt{x^2 + y^2}$

$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$

Scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$$

where θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$

The equation of the line through the point with position vector $\mathbf{a}$ parallel to vector $\mathbf{b}$ is

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

Mechanics

Forces and Equilibrium

Weight $=$ mass $\times\, g$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration

$v = \dfrac{\mathrm{d}r}{\mathrm{d}t}$ $\quad a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$

$r = \int v \, \mathrm{d}t$ $\quad v = \int a \, \mathrm{d}t$

$v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$ $\quad a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2}$

$s = \int v \, \mathrm{d}t$ $\quad v = \int a \, \mathrm{d}t$

Statistics

The mean of a set of data: $\bar{x} = \dfrac{\sum x}{n} = \dfrac{\sum fx}{\sum f}$

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Learners will be given the following formulae in the Formulae Booklet in each assessment.

Pure Mathematics

Binomial series

$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \ldots + {}^nC_r\, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \begin{pmatrix} n \\ r \end{pmatrix} = \dfrac{n!}{r!\,(n-r)!}$

Matrix transformations

Reflection in the line $y = \pm x$: $\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Rotations through θ about the coordinate axes. The direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation.

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Complex numbers

Circles: $|z - a| = k$

Half lines: $\arg(z - a) = \alpha$

Lines: $|z - a| = |z - b|$

Vectors and 3-D coordinate geometry

Cartesian equation of the line through the point A with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ in direction $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ is $\dfrac{x - a_1}{u_1} = \dfrac{y - a_2}{u_2} = \dfrac{z - a_3}{u_3} \;(= \lambda)$

Vector product: $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

Statistics

Standard deviation

$$\sqrt{\frac{\sum (x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \textbf{ or } \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $P(X = x_i) = p_i$

Expectation: $\mu = E(X) = \sum x_i p_i$

Variance: $\sigma^2 = Var(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	$P(X = x)$	$E(X)$	$Var(X)$
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Uniform distribution over 1, 2, …, n U(n)	$\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2-1)$
Geometric distribution Geo(p)	$(1-p)^{x-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson Po(λ)	$e^{-\lambda}\frac{\lambda^x}{x!}$	λ	λ

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_v$

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Correlation and regression

For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}, S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n},$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

Product-moment correlation coefficient: $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

Spearman's rank correlation coefficient: $r_s = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}$

Critical values for the product moment correlation coefficient, r

1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					n				
1	–	–	–	–	31	0.3009	0.3550	0.4158	0.4556
2	–	–	–	–	32	0.2960	0.3494	0.4093	0.4487
3	0.9877	0.9969	0.9995	0.9999	33	0.2913	0.3440	0.4032	0.4421
4	0.9000	0.9500	0.9800	0.9900	34	0.2869	0.3388	0.3972	0.4357
5	0.8054	0.8783	0.9343	0.9587	35	0.2826	0.3338	0.3916	0.4296
6	0.7293	0.8114	0.8822	0.9172	36	0.2785	0.3291	0.3862	0.4238
7	0.6694	0.7545	0.8329	0.8745	37	0.2746	0.3246	0.3810	0.4182
8	0.6215	0.7067	0.7887	0.8343	38	0.2709	0.3202	0.3760	0.4128
9	0.5822	0.6664	0.7498	0.7977	39	0.2673	0.3160	0.3712	0.4076
10	0.5494	0.6319	0.7155	0.7646	40	0.2638	0.3120	0.3665	0.4026
11	0.5214	0.6021	0.6851	0.7348	41	0.2605	0.3081	0.3621	0.3978
12	0.4973	0.5760	0.6581	0.7079	42	0.2573	0.3044	0.3578	0.3932
13	0.4762	0.5529	0.6339	0.6835	43	0.2542	0.3008	0.3536	0.3887
14	0.4575	0.5324	0.6120	0.6614	44	0.2512	0.2973	0.3496	0.3843
15	0.4409	0.5140	0.5923	0.6411	45	0.2483	0.2940	0.3457	0.3801
16	0.4259	0.4973	0.5742	0.6226	46	0.2455	0.2907	0.3420	0.3761
17	0.4124	0.4821	0.5577	0.6055	47	0.2429	0.2876	0.3384	0.3721
18	0.4000	0.4683	0.5425	0.5897	48	0.2403	0.2845	0.3348	0.3683
19	0.3887	0.4555	0.5285	0.5751	49	0.2377	0.2816	0.3314	0.3646
20	0.3783	0.4438	0.5155	0.5614	50	0.2353	0.2787	0.3281	0.3610
21	0.3687	0.4329	0.5034	0.5487	51	0.2329	0.2759	0.3249	0.3575
22	0.3598	0.4227	0.4921	0.5368	52	0.2306	0.2732	0.3218	0.3542
23	0.3515	0.4132	0.4815	0.5256	53	0.2284	0.2706	0.3188	0.3509
24	0.3438	0.4044	0.4716	0.5151	54	0.2262	0.2681	0.3158	0.3477
25	0.3365	0.3961	0.4622	0.5052	55	0.2241	0.2656	0.3129	0.3445
26	0.3297	0.3882	0.4534	0.4958	56	0.2221	0.2632	0.3102	0.3415
27	0.3233	0.3809	0.4451	0.4869	57	0.2201	0.2609	0.3074	0.3385
28	0.3172	0.3739	0.4372	0.4785	58	0.2181	0.2586	0.3048	0.3357
29	0.3115	0.3673	0.4297	0.4705	59	0.2162	0.2564	0.3022	0.3328
30	0.3061	0.3610	0.4226	0.4629	60	0.2144	0.2542	0.2997	0.3301

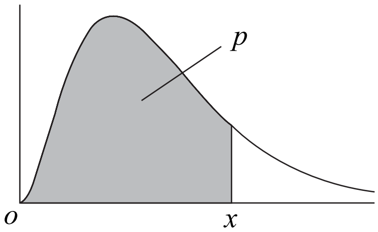
Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					n				
1	–	–	–	–	31	0.3012	0.3560	0.4185	0.4593
2	–	–	–	–	32	0.2962	0.3504	0.4117	0.4523
3	–	–	–	–	33	0.2914	0.3449	0.4054	0.4455
4	1.0000	–	–	–	34	0.2871	0.3396	0.3995	0.4390
5	0.9000	1.0000	1.0000	–	35	0.2829	0.3347	0.3936	0.4328
6	0.8286	0.8857	0.9429	1.0000	36	0.2788	0.3300	0.3882	0.4268
7	0.7143	0.7857	0.8929	0.9286	37	0.2748	0.3253	0.3829	0.4211
8	0.6429	0.7381	0.8333	0.8810	38	0.2710	0.3209	0.3778	0.4155
9	0.6000	0.7000	0.7833	0.8333	39	0.2674	0.3168	0.3729	0.4103
10	0.5636	0.6485	0.7455	0.7939	40	0.2640	0.3128	0.3681	0.4051
11	0.5364	0.6182	0.7091	0.7545	41	0.2606	0.3087	0.3636	0.4002
12	0.5035	0.5874	0.6783	0.7273	42	0.2574	0.3051	0.3594	0.3955
13	0.4835	0.5604	0.6484	0.7033	43	0.2543	0.3014	0.3550	0.3908
14	0.4637	0.5385	0.6264	0.6791	44	0.2513	0.2978	0.3511	0.3865
15	0.4464	0.5214	0.6036	0.6536	45	0.2484	0.2945	0.3470	0.3822
16	0.4294	0.5029	0.5824	0.6353	46	0.2456	0.2913	0.3433	0.3781
17	0.4142	0.4877	0.5662	0.6176	47	0.2429	0.2880	0.3396	0.3741
18	0.4014	0.4716	0.5501	0.5996	48	0.2403	0.2850	0.3361	0.3702
19	0.3912	0.4596	0.5351	0.5842	49	0.2378	0.2820	0.3326	0.3664
20	0.3805	0.4466	0.5218	0.5699	50	0.2353	0.2791	0.3293	0.3628
21	0.3701	0.4364	0.5091	0.5558	51	0.2329	0.2764	0.3260	0.3592
22	0.3608	0.4252	0.4975	0.5438	52	0.2307	0.2736	0.3228	0.3558
23	0.3528	0.4160	0.4862	0.5316	53	0.2284	0.2710	0.3198	0.3524
24	0.3443	0.4070	0.4757	0.5209	54	0.2262	0.2685	0.3168	0.3492
25	0.3369	0.3977	0.4662	0.5108	55	0.2242	0.2659	0.3139	0.3460
26	0.3306	0.3901	0.4571	0.5009	56	0.2221	0.2636	0.3111	0.3429
27	0.3242	0.3828	0.4487	0.4915	57	0.2201	0.2612	0.3083	0.3400
28	0.3180	0.3755	0.4401	0.4828	58	0.2181	0.2589	0.3057	0.3370
29	0.3118	0.3685	0.4325	0.4749	59	0.2162	0.2567	0.3030	0.3342
30	0.3063	0.3624	0.4251	0.4670	60	0.2144	0.2545	0.3005	0.3314

Critical values for the χ^2 distribution

If X has a χ^2 distribution with v degrees of freedom then, for each pair of values of p and v, the table gives the value of x such that

$P(X \le x) = p$.



p	0.01	0.025	0.05	0.90	0.95	0.975	0.99	0.995	0.999
$v = 1$	$0.0^3 1571$	$0.0^3 9821$	$0.0^2 3932$	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

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Mechanics

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Newton's experimental law

Between two smooth spheres $v_1 - v_2 = -e(u_1 - u_2)$

Between a smooth sphere with a fixed plane surface $v = -eu$

Motion in a circle

Tangential velocity is $v = r\dot{\theta}$

Radial acceleration is $\frac{v^2}{r}$ or $r\dot{\theta}^2$ towards the centre

Tangential acceleration is $\dot{v} = r\ddot{\theta}$

Discrete Mathematics

Sorting algorithms

Bubble sort:

> Start at the left hand end of the list unless specified otherwise.
>
> Compare the first and second values and swap if necessary. Then compare the (new) second value with the third value and swap if necessary. Continue in this way until all values have been considered.
>
> Fix the last value then repeat with the reduced list until either there is a pass in which no swaps occur or the list is reduced to length 1, then stop.

Shuttle sort:

> Start at the left hand end of the list unless specified otherwise.
>
> Compare the second value with the first and swap if necessary, this completes the first pass. Next compare the third value with the second and swap if necessary, if a swap happened shuttle back to compare the (new) second with the first as in the first pass, this completes the second pass.
>
> Next compare the fourth value with the third and swap if necessary, if a swap happened shuttle back to compare the (new) third value with the second as in the second pass (so if a swap happened shuttle back again). Continue in this way for $n - 1$ passes, where n is the length of the list.

Network algorithms

Dijkstra's algorithm

START with a graph G. At each vertex draw a box, the lower area for temporary labels, the upper left hand area for the order of becoming permanent and the upper right hand area for the permanent label.

STEP 1 Make the given start vertex permanent by giving it permanent label 0 and order label 1.

STEP 2 For each vertex that is not permanent and is connected by an arc to the vertex that has just been made permanent (with permanent label = P), add the arc weight to P. If this is smaller than the best temporary label at the vertex, write this value as the new best temporary label.

STEP 3 Choose the vertex that is not yet permanent which has the smallest best temporary label. If there is more than one such vertex, choose any one of them. Make this vertex permanent and assign it the next order label.

STEP 4 If every vertex is now permanent, or if the target vertex is permanent, use 'trace back' to find the routes or route, then STOP; otherwise return to STEP 2.

Prim's algorithm (graphical version)

START with an arbitrary vertex of G.

STEP 1 Add an edge of minimum weight joining a vertex already included to a vertex not already included.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Prim's algorithm (tabular version)

START with a table (or matrix) of weights for a connected weighted graph.

STEP 1 Cross through the entries in an arbitrary row, and mark the corresponding column.

STEP 2 Choose a minimum entry from the uncircled entries in the marked column(s).

STEP 3 If no such entry exists STOP; otherwise go to STEP 4.

STEP 4 Circle the weight w_{ij} found in STEP 2; mark column i; cross through row i.

STEP 5 Return to STEP 2.

Kruskal's algorithm

START with all the vertices of G, but no edges; list the edges in increasing order of weight.

STEP 1 Add an edge of G of minimum weight in such a way that no cycles are created.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Additional Pure Mathematics

Vector product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$, where $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$, in that order, form a right-handed triple.

Summary of updates

Date	Version	Section	Title of section	Change
June 2018	1.1	Front cover	Disclaimer	Addition of Disclaimer
September 2019	1.2	Multiple 2b	 Content of AS Level in Further Mathematics A (H235)	Correction of minor typographical errors Insertion of additional command words
April 2020	1.3	1d	How do I find out more information?	Insertion of the Online Support Centre link
February 2021	1.4			Update to specification covers to meet digital accessibility standards
April 2022	1.5	5d	Mathematical formulae and identities	Correction of minor typographical error
March 2023	1.6	3c	Total qualification time	Update to include total qualification time and guided learning hours (TQT/GLH) to comply with Qualifications in Wales regulations
July 2023	1.7	4c	External Assessment Arrangements	Fixed link.
February 2024	1.8	3d, 3e 4a Checklist	Qualification availability, Language Pre-assessment	Inclusion of disclaimer regarding availability and language Update to include resilience guidance Inclusion of Teach Cambridge

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YOUR CHECKLIST

Our aim is to provide you with all the information and support you need to deliver our specifications.

- ☐ Bookmark OCR website for all the latest information and news on AS Level Further Mathematics A
- ☐ Sign up for Teach Cambridge: our personalised and secure website that provides teachers with access to all planning, teaching and assessment support materials
- ☐ Be among the first to hear about support materials and resources as they become available – register for AS Level Further Mathematics A
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01223 553998

maths@ocr.org.uk

@OCR_Maths

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