

Wednesday 30 May 2012 – Afternoon

FSMQ ADVANCED LEVEL

6993 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 6993

Other materials required:

- Scientific or graphical calculator

Duration: 2 hours



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

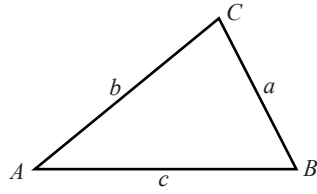
INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Formulae Sheet: 6993 Additional Mathematics

In any triangle ABC

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



Binomial expansion

When n is a positive integer

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

- 1 (i) Find the range of values of x satisfying $x^2 - 4x + 3 \leq 0$. [3]
- (ii) Show this range on the number line provided. [1]
- 2 A die has 6 faces numbered one to six. The die is biased so that when it is thrown the probability of obtaining a six is $\frac{1}{5}$.
- The die is thrown 5 times.
- Find the probability of obtaining
- (i) at least 1 six, [2]
- (ii) exactly 3 sixes. [4]
- 3 The function $f(x) = x^3 + ax + 6$ is such that when $f(x)$ is divided by $(x - 3)$ the remainder is 12.
- (i) Show that the value of a is -7 . [2]
- (ii) Factorise $f(x)$. [3]
- 4 A car moves from rest with constant acceleration on a straight road. When the car passes a point A it is travelling at 10 m s^{-1} and when it passes a point B further along the road it is travelling at 16 m s^{-1} .
- The car takes 10 seconds to travel from A to B.
- Find
- the distance AB,
 - the constant acceleration. [4]

5 (i) Show that the equation $3\cos^2\theta = \sin\theta + 1$ can be written as $3\sin^2\theta + \sin\theta - 2 = 0$. [2]

(ii) Solve this equation to find values of θ in the range $0^\circ < \theta < 360^\circ$ that satisfy

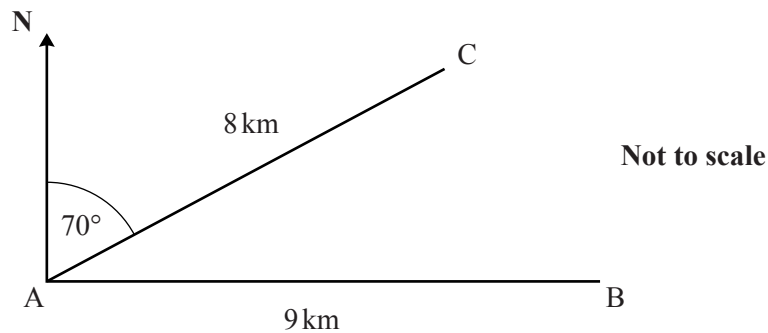
$$3\cos^2\theta = \sin\theta + 1. \quad [4]$$

6 The equation of a curve is $y = 2x^3 - 9x^2 + 12x$.

(i) Show that the curve has a stationary point where $x = 2$. [4]

(ii) Determine whether the stationary value where $x = 2$ is a maximum or minimum. [2]

7 A yachtsman wishes to sail from a port, A, to another port, B, which is 9 km due East of A. Because of the wind he is unable to sail directly East and sails 8 km on a bearing of 070° to point C.



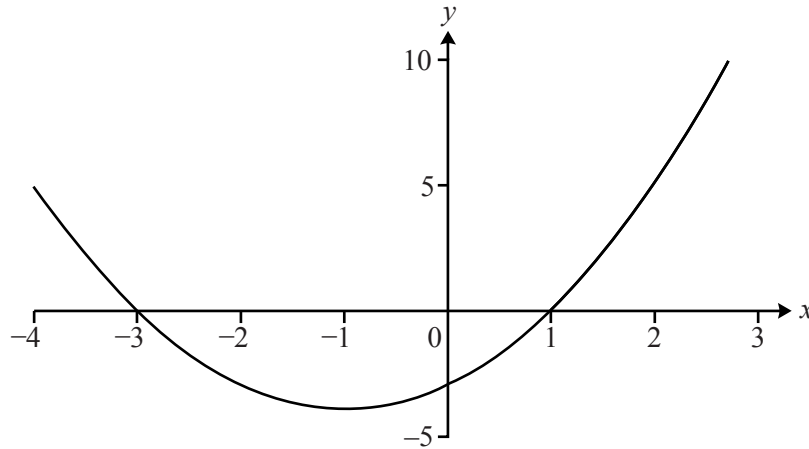
Calculate

(i) the distance he is now from port B, [3]

(ii) the angle ABC and hence the bearing on which he must sail to reach port B from point C, correct to the nearest degree. [4]

- 8 (i) Show that $\int_0^2 (x^2 + 2x - 3) dx = \frac{2}{3}$. [3]

The diagram shows part of the curve $y = x^2 + 2x - 3$.



- (ii) Marc claims that the total area between the curve, the x -axis and the lines $x = 0$ and $x = 2$ is $\frac{2}{3}$.
Explain why he is wrong. [1]
- (iii) Calculate the total area between the curve, the x -axis and the lines $x = 0$ and $x = 2$. [3]
- 9 The height above the ground of a seat on a fairground big wheel is h metres. At time t **minutes** after the wheel starts, h is given by
- $$h = 7 - 5\cos(480t)^\circ.$$
- (i) Write down the initial height above the ground of the seat (when $t = 0$). [1]
- (ii) Find the greatest height reached by the seat. [2]
- (iii) Calculate the time of the first occasion when the seat is 9 metres above the ground.
Give your answer correct to the nearest second. [4]

Section B

10 A (1, 10), B (8, 9) and C (7, 2) are three points.

(i) Find the coordinates of the midpoint, M, of AC. [1]

(ii) Find the equation of the circle with AC as diameter. [4]

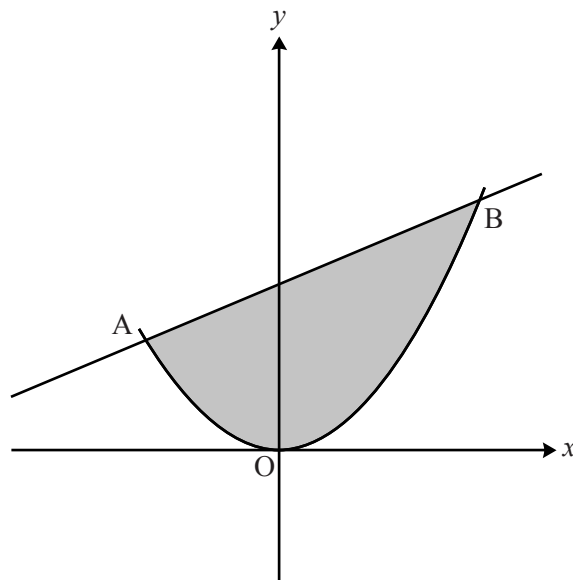
(iii) Show that B lies on this circle. [1]

(iv) Prove that AM and BM are perpendicular. [3]

(v) BD is a diameter of this circle. Find the coordinates of D. [3]

11 The shaded region in the diagram shows a wooden shape.

The curve has equation $y = \frac{1}{2}x^2$ and the coordinates of A are $(-2, 2)$.



The line AB is the normal to the curve at the point A.

(i) Find the equation of the line AB. [5]

(ii) Find the coordinates of the point B where the line AB meets the curve again. [3]

(iii) Find the shaded area. [4]

- 12** The Highway Code gives a table of shortest stopping distances (d feet) for a vehicle travelling at v miles per hour.

The formula used for this table is given by

$$d = av^2 + bv.$$

Two entries in the table are given below.

v mph	d feet
30	75
60	240

- (i)** By forming and solving a pair of simultaneous equations in a and b , show that the formula is

$$d = \frac{v^2}{20} + v. \quad [5]$$

- (ii)** Find the difference between the stopping distances for a car travelling at 65 mph and a car travelling at 70 mph. [3]

- (iii)** Many drivers maintain a distance of 50 feet or less when driving on a motorway.

Use the formula in part **(i)** to find the speed at which the shortest stopping distance is 50 feet. [4]

Question 13 is printed overleaf

- 13 (i) Find the coefficients a , b and c in the expansion

$$(2 + h)^3 = 8 + ah + bh^2 + ch^3. \quad [3]$$

- (ii) The graph of the equation $y = x^3$ passes through the points P and Q which have x -coordinates 2 and $2 + h$ respectively.

Show that the gradient of the chord PQ is $\frac{(2 + h)^3 - 8}{h}$. [3]

- (iii) Express $\frac{(2 + h)^3 - 8}{h}$ as a quadratic function of h . [2]

- (iv) As the value of h decreases, the point Q gets closer and closer to the point P on the curve. As h gets closer to 0 the chord PQ gets closer to being the tangent to the curve at P.

Deduce the value of the gradient of the tangent at P. [1]

- (v) Karen uses the same method to deduce the value of the gradient of the tangent at the point (2, 16) on the curve $y = x^4$.

The first three lines of her working are given below and in the answer booklet.

Take P to be the point (2, 16)

Take Q to be the point (2 + h, (2 + h)⁴)

The gradient of the chord PQ is given by $\frac{(2+h)^4 - 16}{h} =$

Complete Karen's working. [3]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.