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A2 GCE MATHEMATICS (MEI)

4754B Applications of Advanced Mathematics (C4) Paper B: Comprehension

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The world population

Population pressure on our planet

During the last 200 years, the human population has increased by a factor of about 7. Table 1 gives the years when it reached 1, 2, 3 and so on billions of people, where 1 billion is 10^9.

Year	1804	1927	1960	1974	1987	1999	2011
Population (billions)	1	2	3	4	5	6	7

Table 1 World population

The increase in population is placing rising demands on the resources of our planet and on the whole eco-system that supports us. This raises very important questions.

- Is the world's population going to continue to increase indefinitely or will there be a limit?
- Will the world's population reach a level that the planet is unable to support?
- Should we be taking measures to restrict the world's population, and if so what?

The first two of these questions require mathematical modelling of the situation. The third involves political and ethical decisions which should be informed by that modelling.

The modelling involved is complicated; this article introduces some of the issues involved.

The exponential model

A simple mathematical model is that the world's population is increasing at a rate which is directly proportional to its existing size,

$$\frac{dp}{dt} = kp\ ,$$

where p is the number of people,
t is time, measured in years,
k is a constant.

The solution of this differential equation is

$$p = p_0 e^{kt}$$

where p_0 is the population at the time from which t is measured.

If a model is to be valuable in this context, it must be possible to use it to predict the size of the world population in the future. So, as a test case, the first two data points in Table 1 should allow the later values to be predicted. These data points are

$$1804 \qquad t = 0,\ p = p_0 = 10^9,$$
$$1927 \qquad t = 123,\ p = 2 \times 10^9,$$

and these correspond to $k = 0.005\,63\ldots$.

With this value of k, this model would predict that the population in 2011 would be 3.2 billion but in fact it was 7 billion. This model, based on the first two data points, is clearly not suitable.

In fact no exponential model fits the data in Table 1 well. You can see this just by looking at the graph of the data in Fig. 2. The graph of an exponential function is a curve which gets steeper and steeper but for the last 50 years this graph is virtually a straight line, indicating a constant rate of growth.

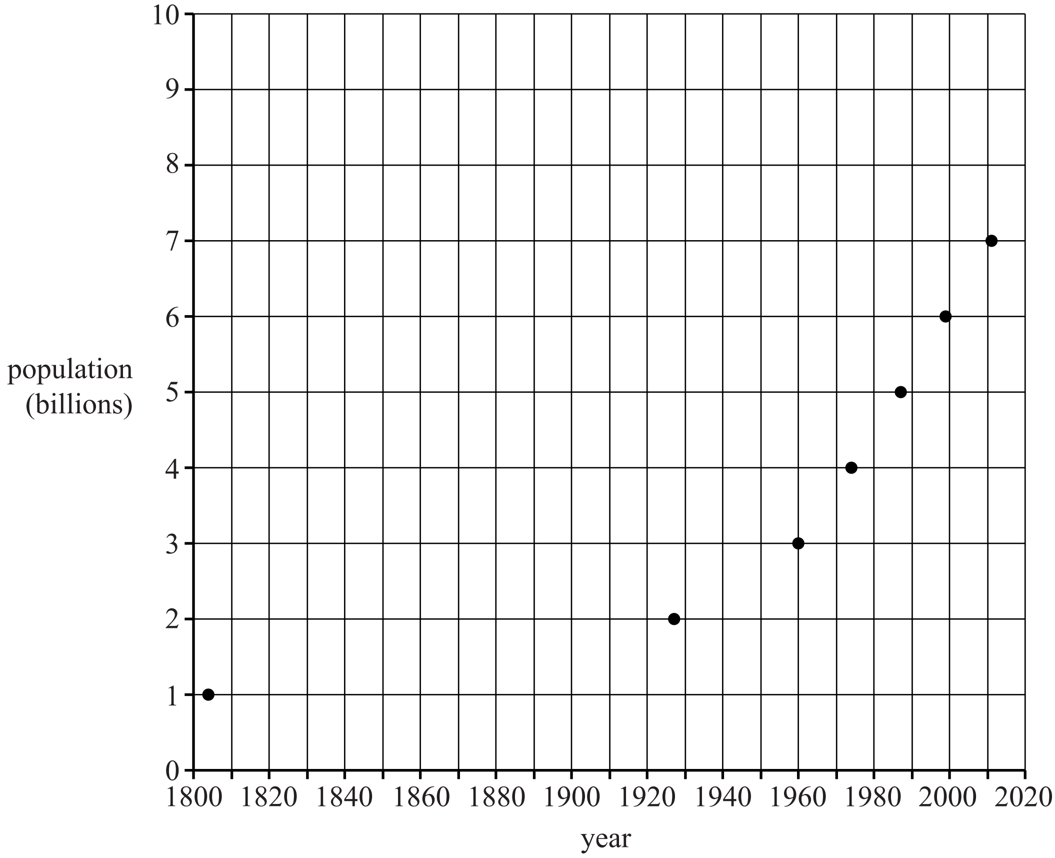

Fig. 2 World population from 1800 to the present

The logistic model

A standard mathematical model for a population which increases towards a limiting value of m is given by the differential equation

$$\frac{dp}{dt} = kp(m-p) .$$

This is known as the *logistic equation*. A typical solution curve is shown in Fig. 3.

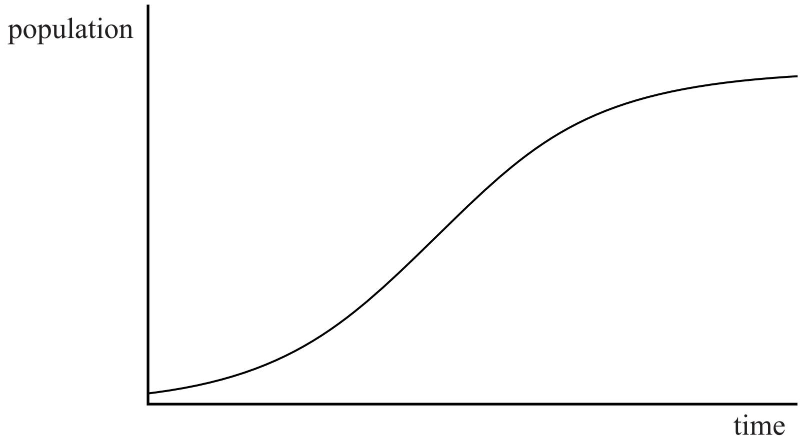

Fig. 3 The logistic model

While this looks as though it starts with the same sort of shape as a curve through the data points in Fig. 2, the resemblance is only superficial; it is not actually possible to find values of k and m that produce anything like a good fit. So this model is also unsatisfactory.

Like the previous model, this is an attempt to find a simple, neat solution to a very complicated problem. A different approach is needed and a starting point is provided by population profiles.

Population profiles

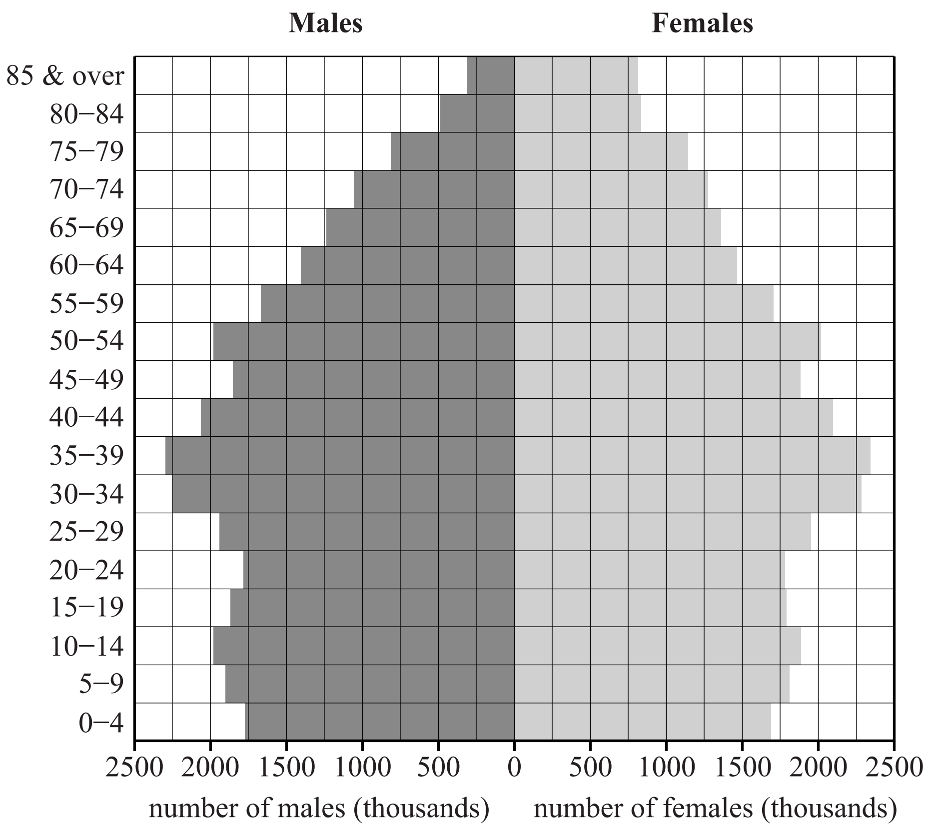

Fig. 4 Population profile of the UK in 2001

Population profiles are often illustrated by population pyramids, like that in Fig. 4. The lengths of the horizontal bars indicate the numbers of males and females in the UK population in 2001, in 5-year age intervals. In this case, the numbers on the horizontal scale are in thousands. Those on the vertical scale refer to age in completed years so that, for example, 10–14 means from 10 years 0 days to 14 years 364 days.

The UK population profile shows that in 2001 the number of children in the 0–4 age range was among the lowest for 50 years. Because there are fewer people in that age group, they in turn can be expected to have fewer children.

The shapes of the population profiles vary considerably between countries. In some countries the profiles have very wide bases, indicating large numbers of children.

It is worth noting that population figures for the UK are often affected by emigration and immigration.

Modelling using population profiles

If the individual population profiles of the large number of countries in the world are combined, a profile for the whole world emerges. It is possible to predict the changes in any country's profile in the years ahead, and hence the changes in the world's population. Each country is different and so needs to be looked at separately before combining the profiles.

The following model, for an imaginary country, is designed to highlight the key factors. Table 5 illustrates its profile in 2010 and part of that for 2030.

Age group	2010	2030
80+	1	?
60–79	10	?
40–59	20	20
20–39	20	20
0–19	20	?
Total	**71**	**?**

Table 5 Population profile of an imaginary country (in millions)

In 2010, this country has a stable population with the same numbers in the youngest three age groups, up to the age of 60; however, life expectancy is quite low with very few people reaching the age of 80.

Two of the figures for 2030 have also been filled in. The 20 million people in the 0–19 age group in 2010 will move into the 20–39 group. Similarly those in the 20–39 group will move into the 40–59 group. (It is assumed, for simplicity, that no one in these age groups dies.) What will the other figures for 2030 be?

Two different factors are involved: the birth rate and the life expectancy.

The 2010 profile in Table 5 was constructed using a number of assumptions:

- that those in the 0–19 age group are all children of females in the 20–39 group;
- that 50% of those in the 20–39 age group are female;
- that on average each female has 2 children;
- that there is no immigration or emigration.

While these assumptions are obviously somewhat artificial, particularly with regard to the age at which women have children, they are good enough to demonstrate the key features of a country's population.

Throughout the world, life expectancy is rising. The proportion of the population in Table 5 who reach the age of 80 could be expected to increase.

In Table 6, the population profile of the country in Table 5 is predicted for the next 100 years, on the basis of the following new assumptions about the birth rate and life expectancy.

- Every 20 years, each group of people moves up a level.
- The average figure of 2 children per female is assumed to fall to 1.8, from 2010 onwards.
- The proportion of those in the 40–59 age group surviving into the 60–79 group increases from the 2010 figure of 50%; similarly there is an increase in survival from the 60–79 group into the 80+ group.

The figures used in these assumptions have been chosen to illustrate the modelling process. Their use does not mean that they will actually apply to the population of any real country.

Age group	**2010**	**2030**	**2050**	**2070**	**2090**	**2110**
80+	1	4	6.4	6.4	6.4	5.76
60–79	10	16	16	16	14.4	12.96
40–59	20	20	20	18	16.2	14.58
20–39	20	20	18	16.2	14.58	13.12
0–19	20	18	16.2	14.58	13.12	11.81
Total	**71**	**78**	**76.6**	**71.18**	**64.7**	**58.23**

Table 6 Population profile of an imaginary country (in millions, to 4 significant figures)

The figures in Table 6 show the total population rising quite sharply to a maximum and then reducing, initially rather slowly but then more quickly. Patterns like this are observed in many countries. In some, like Japan, the population has passed its maximum and is declining. In most, however, it is still increasing and consequently the population of the world as a whole is still increasing.

An important feature of Table 6 is that it is based on a low birth rate of 1.8 children per female. In many countries the birth rate is much higher than the stable level of 2 children per female.

Birth rate and life expectancy

Two key factors that determine the change in a country's population have been identified as its birth rate and its life expectancy. Data show that these are closely associated. Countries with high birth rates tend to have low life expectancy and those with low birth rates have high life expectancy. This is illustrated in Fig. 7 for all 221 countries; the data were drawn from the CIA World Factbook for 2009.

Notice that in Fig. 7 the birth rate is the number of births per 1000 of the population per year. It is thus a different measure from that used so far in this article which is mean births per female over her lifetime. So, for example, a country with a population of 80 million people and 1.2 million births per year has a birth rate of 15 births per 1000 per year.

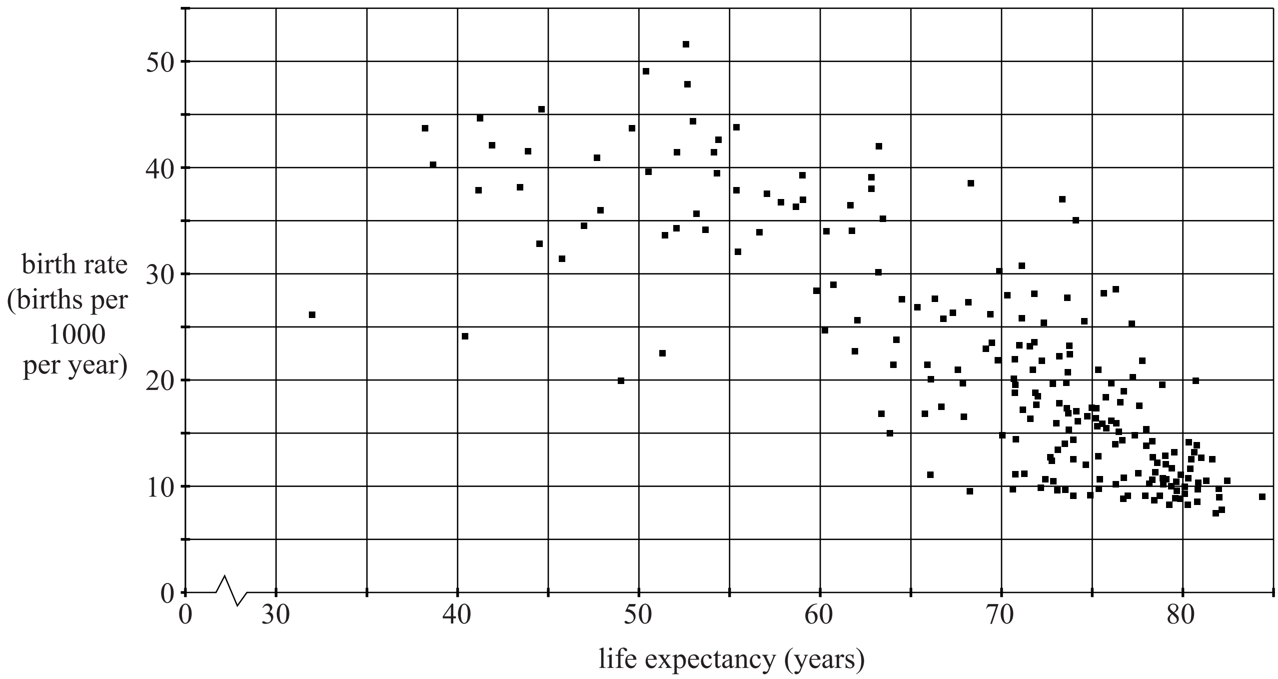

Fig. 7 Scatter diagram showing birth rate against life expectancy for the countries of the world

Conclusion

Table 8 gives the data for some selected countries in 2009.

Country	Life expectancy	Birth rate
Japan	82.12	7.64
Sweden	80.86	10.13
Italy	80.20	8.18
UK	79.01	10.65
USA	78.11	13.82
Tunisia	75.78	15.42
Jamaica	73.53	19.68
China	73.47	14.00
Brazil	71.99	18.43
India	69.89	21.76
Bangladesh	60.25	24.68
Ghana	59.85	28.58
Uganda	52.72	47.84
Afghanistan	44.64	45.46

Table 8 Life expectancy and birth rates of selected countries (2009)

The data in Table 8 illustrate the observation that countries with low birth rate and high life expectancy tend to be those with developed economies. Studies over time indicate that as they develop, countries follow a path from high birth rate and low life expectancy to low birth rate and high life expectancy. So it is reasonable to expect that at some time in the future, the world's population will attain a maximum value and then start to decline.

When that maximum occurs, and how great the population then is, will depend on how quickly countries progress along that path. Consequently modelling the world's population requires an understanding of the factors involved. Then it will be possible to determine what can be done to match the population to the planet's resources.