OCR
RECOGNISING ACHIEVEMENT

Wednesday 16 May 2012 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Use Lagrange's method to find the equation of the quadratic curve $y = f(x)$ that passes through the following data points.

x	-1	0	2
y	3	6	-4

Hence find the value of x for which $f(x)$ is a maximum. **[7]**

2 The number X is an approximation to an exact value x, and $X = x(1 + r)$.

(i) Show that r is the relative error in X. **[2]**

(ii) Use the binomial theorem to show that $X^n \approx x^n(1 + nr)$ provided r is small. **[2]**

(iii) The number Y is an approximation to an exact value y. The relative error in Y is 2%. State the approximate relative errors in

(A) Y^3 as an approximation to y^3,

(B) $\frac{1}{Y}$ as an approximation to $\frac{1}{y}$. **[3]**

3 **(i)** Show that the equation $x^5 = x^4 + 2$ has a root in the interval [1, 2]. **[2]**

(ii) Use the Newton-Raphson method to find this root correct to 6 decimal places. **[6]**

4 The function $g(x)$ has the values shown in the table.

x	5	5.1	5.2	5.4
$g(x)$	0.820 86	0.780 82	0.742 73	0.672 05

(i) Find three estimates of $g'(5)$ using the forward difference method with $h = 0.4, 0.2, 0.1$. **[3]**

(ii) Use these estimates to show that the forward difference method has first order convergence. **[3]**

(iii) Give the value of $g'(5)$ to the accuracy that is justified, explaining your reasoning. **[2]**

5 The cells of a spreadsheet have the formulae shown in Fig. 5a. The values displayed by the spreadsheet are shown in Fig. 5b.

	A	B	C
1	0.6		
2	=A1–0.2		
3	=A2–0.2		
4	=A3–0.2	=A4+1	=B4–1

Fig. 5a

	A	B	C
1	0.6		
2	0.4		
3	0.2		
4	–5.5E–17	1	0

Fig. 5b

(i) State what the entry in cell A4 of Fig. 5b means. Explain why it is not zero. **[3]**

(ii) What can you deduce about the way the spreadsheet stores and displays numbers from the values shown in cells B4 and C4? **[3]**

Section B (36 marks)

6 The table below gives some values of a function f(x).

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
f(x)	1.000 00	1.060 51	1.116 87	1.168 88	1.216 32	1.259 01	1.296 78	1.329 49	1.357 07

In this question, you are required to find estimates of the integral $\int_0^2 \mathrm{f}(x)\mathrm{d}x$.

(i) Find trapezium rule estimates T_1, T_2, T_4, T_8 with $h = 2, 1, 0.5, 0.25$ respectively.

Find the values of $\dfrac{T_4 - T_2}{T_2 - T_1}$ and $\dfrac{T_8 - T_4}{T_4 - T_2}$. State what these values indicate about the trapezium rule. **[7]**

(ii) Use your trapezium rule estimates from part **(i)** to find three Simpson's rule estimates of the integral.

Calculate the ratio of differences for these estimates. What does this value indicate about Simpson's rule? **[6]**

(iii) State, with reasons, the value of the integral to the accuracy that is justified if the given values of f(x) are exact.

Hence give a range within which the value of the integral lies if the given values of f(x) had been rounded to 5 decimal places. **[5]**

7 In this question you are asked to find the roots of the equation $x^2 - 1 = \sin x$, where x is in radians.

(i) Show that the equation has a root in the interval $[-1, 0]$ and another in the interval $[1, 2]$. **[3]**

(ii) Starting with the interval $[-1, 0]$, find the initial estimate of the negative root as given by the method of false position. Apply this method to find two further estimates of the negative root. Discuss briefly the accuracy to which the root has been found. **[7]**

(iii) Starting with $x_0 = 1$ and $x_1 = 2$, find the first estimate of the positive root as given by the secant method. Apply this method to find two further estimates of the positive root. Discuss briefly the accuracy with which the root has been found. **[6]**

(iv) Comment briefly on the relative merits of the method of false position and the secant rule in this case. **[2]**