OCR
RECOGNISING ACHIEVEMENT

Friday 18 May 2012 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/01 Paper 1

H8167601612

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 16 page Answer Booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

H8600 1

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

1 For this question you may use the formulae on page 2 of the List of Formulae (MF1) provided.

A construction engineering company specialises in the production of metal sculptures. The company has received a commission from an artist to construct a large steel equilateral triangular structure containing smaller internal equilateral triangles.

The construction starts with the large outside triangle (stage 1). Each subsequent stage of the construction involves inserting one new internal triangle as shown in Fig. 1. The structure is made from steel rod of negligible thickness.

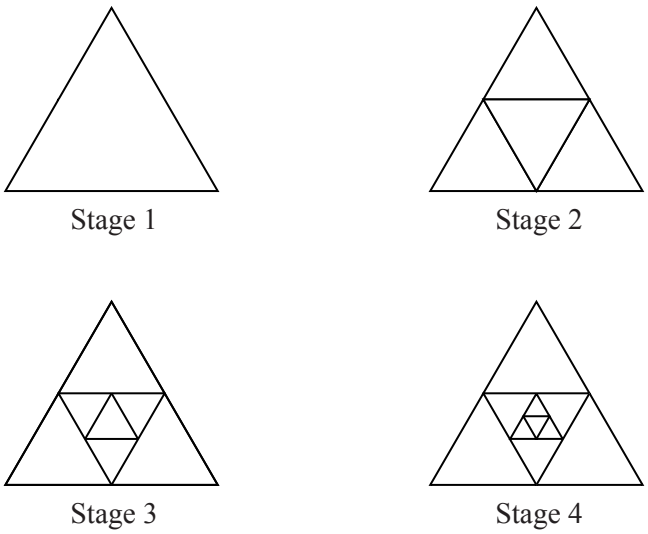

Fig. 1

The vertices of each of the newly inserted triangles are the midpoints of the sides of the triangle inserted at the previous stage. The design is to be repeated until stage 10.

(a) The length of steel used to construct all three sides of the large triangle at stage 1 is 5 m.

(i) Find a formula, in terms of i, for the length of steel required to construct the smallest triangle at stage i. **[1]**

(ii) Calculate the total length of the steel in the structure at stage 10. **[3]**

(iii) If the process of adding internal triangles could be continued indefinitely, what is the theoretical total length of the steel required? **[2]**

(b) At any stage of the construction a number of triangles of various sizes can be counted. Fig. 1 shows that at stage 2 there is a total of 5 triangles. At stage 3 there is a total of 9 triangles and at stage 4 there is a total of 13 triangles.

The total number of triangles of various sizes at stage i is represented by t_i.

(i) Find a formula for t_i in terms of i. **[2]**

(ii) Calculate the value of t_{10}. **[1]**

(iii) Find a formula, in terms of n, for

$$\sum_{i=1}^{n} t_i$$

and calculate its value when $n = 10$. **[2]**

2 A quality control department in a company conducts tests on 200 items. Of the 200 items tested, 150 had defects which were of one or more of types A, B and C. The remaining 50 items had no defects at all.

Table 2 provides a summary of the number of items with defects.

Defect	Number of items
Including type A	60
Including type B	80
Including type C	105
Only type A	15
Only type B	20
Only type C	40
Types A, B and C	20

Table 2

(a) Determine the number of items that have

(i) only defects of both types B **and** C, **[2]**

(ii) only defects of both types A **and** B. **[2]**

(b) If one of the 200 items tested is selected at random determine the following probabilities.

(i) The item will have only defects of both types A **and** C. **[2]**

(ii) The item will have exactly two types of defect. **[2]**

(c) An item is selected at random from those known to have defects of type B. What is the probability it also has a defect of type A? **[2]**

3 Fig. 3 shows a diagram of an electrical circuit containing four resistors and three DC voltage supplies.

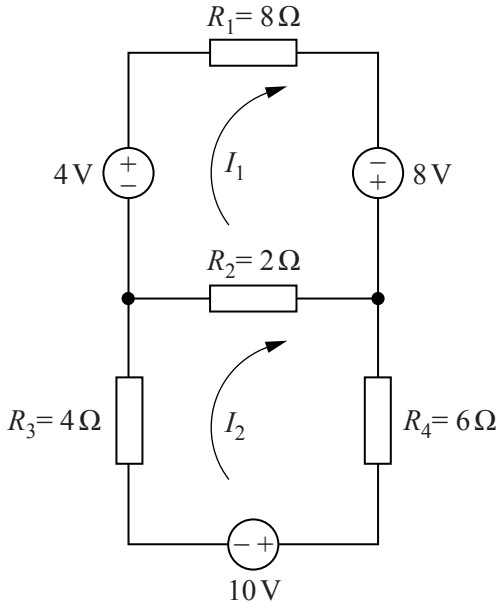

Fig. 3

From Kirchhoff's voltage law the following equations, which relate the loop currents (I_1 and I_2) to the resistor values (R_1, R_2, R_3 and R_4) and to the supply voltages, can be shown.

$$-4 + R_1 I_1 - 8 + R_2(I_1 - I_2) = 0$$

$$10 + R_3 I_2 + R_2(I_2 - I_1) + R_4 I_2 = 0$$

(a) Use the values shown in Fig. 3 and rearrange these equations into two linear simultaneous equations in the two unknowns I_1 and I_2. **[2]**

(b) Write the equations found in part **(a)** in matrix notation. **[1]**

(c) Use matrix algebra, including matrix inversion, to determine the values of I_1 and I_2. **[4]**

4 **(a)** Fig. 4a shows the arc of a circle with centre at point C, radius r and chord PR. $Q\hat{S}R$ is a right angle. S is midway between P and R.

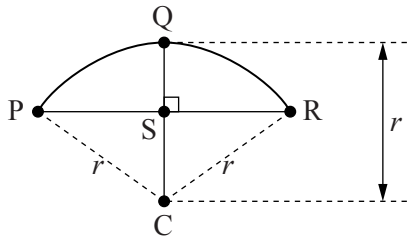

Fig. 4a

Show that

$$r = \frac{SQ^2 + SR^2}{2SQ}.$$

[3]

(b) Fig. 4b shows the profile of a large building with a roof in the shape of the circular arc, PR, of radius r and centre C.

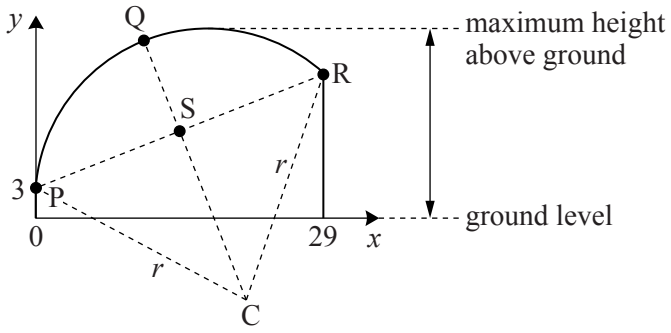

Fig. 4b

The coordinates of the points P, R and Q are referred to the x-axis and y-axis shown in Fig. 4b. The units of the axes are metres.

Point P (0, 3) Point R (29, 9.12) Point Q (13.71, 9.8)

Point Q is the midpoint of the arc PR and $Q\hat{S}R$ is a right angle.

(i) Calculate the length PR. **[1]**

(ii) Calculate the length SQ. **[2]**

(iii) Calculate the radius r. **[1]**

(iv) Calculate the vertical height of point Q above point C. **[2]**

(v) Calculate the maximum height of the roof above ground level. **[2]**

5 For this question you may use the following identity.

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

(a) Two functions S_1 and S_2 are given by

$$S_1 = \sin(2\pi ft) \quad \text{and} \quad S_2 = \sin(2\pi ft + \phi)$$

Using the identity given above, rewrite $S_1 + S_2$ in the form

$$2\sin p\cos q$$

where p and q are to be determined.

Simplify your expression when

(i) $\phi = \frac{2\pi}{3}$,

(ii) $\phi = \pi$. **[5]**

(b) Show that

$$\sin(2\pi f_1 t) + \sin(2\pi f_2 t) = 2\cos(\pi(f_1 - f_2)t)\sin(\pi(f_1 + f_2)t).$$

[2]

(c) Given

$$\sin(2\pi f_1 t) + \sin(2\pi f_2 t) = 2\cos(2\pi \times 5t)\sin(2\pi \times 50t),$$

calculate the two frequencies f_1 and f_2. **[2]**

6 An important technique in telecommunications engineering is the representation of periodic signals using series involving sine and cosine terms.

Fig. 6 shows a graph of a periodic signal with amplitude $g(t)$.

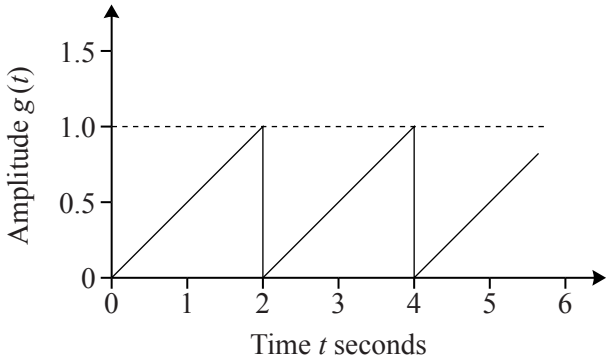

Fig. 6

(a) State the values of the period, T, and the frequency, f, of the signal. **[1]**

(b) State the formula for the function, $g(t)$, for the value of the amplitude in the time interval $0 < t < 2$. **[1]**

(c) Prove that

$$\int t \sin at \, \mathrm{d}t = \frac{\sin at}{a^2} - \frac{t \cos at}{a} + C .$$

[4]

(d) The amplitude of the signal at time t can be expressed by a series involving a constant c and further constants $a_1, a_2, a_3, a_4, \ldots$

where

$$c = \frac{2}{T}\int_0^T g(t)\,\mathrm{d}t \qquad \text{and} \qquad a_n = \frac{2}{T}\int_0^T g(t)\sin 2\pi fnt \,\mathrm{d}t$$

(i) Calculate the value of c. **[2]**

(ii) Show that $a_n = -\frac{1}{n\pi}$. **[4]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.