OCR
RECOGNISING ACHIEVEMENT

Thursday 24 May 2012 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/02 Paper 2

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 8 page Answer Booklet (sent with general stationery)
- Insert (inserted)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **40**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

Please refer to the appendix of the pre-release document for the symbols used in these questions and for their associated units.

For all of the questions in this paper only straight line aircraft motion is considered. In each question the mass of the aircraft may be taken as constant.

Use Table 1 in the pre-release document for question **2(b)(ii)** only.

1 An aircraft has a mass of 40 000 kg and an effective wing area of 120 m^2. The aircraft is travelling at a constant speed and at a constant altitude where the air density is 0.5 kg m^{-3}.

(a) The aircraft is travelling at 176 m s^{-1} and the drag coefficient, C_D, is 0.035.

Calculate the following forces rounding your answers to the nearest kN.

(i) Lift force. **[1]**

(ii) Drag force. **[1]**

(b) In another part of the flight the engines provide a constant thrust of 25 kN and the drag coefficient, C_D, is 0.03. The mass of the aircraft, effective wing area and air density still have the values given above.

(i) Determine the constant speed of the aircraft. **[2]**

(ii) The aircraft maintains this constant speed and constant altitude. Calculate the value of the lift coefficient, C_L. **[2]**

(iii) If the thrust is now increased to 35 kN, state with reasons what effect this will have on the motion of the aircraft. **[2]**

2 An aircraft with mass 55 000 kg and an effective wing area of 100 m^2 is climbing at a **small** constant angle of γ with a constant speed of 180 m s^{-1}. Assume that during the climbing period the density of air remains constant at 0.5 kg m^{-3}.

(a) For one particular phase of the flight, the lift coefficient, C_L, is 0.665 and the drag coefficient, C_D, is 0.042. The engines provide a constant thrust of 60 kN.

(i) Calculate the lift force and confirm that it is approximately equal to the aircraft's weight. **[2]**

(ii) Calculate the drag force. **[1]**

(iii) Calculate the drag-to-lift ratio, β, and the thrust-to-weight ratio, f. **[2]**

(iv) Calculate the angle of climb, γ. **[2]**

(b) In another phase of the flight the aircraft climbs at a constant angle of 4° with a speed of 180 m s^{-1}.

(i) Calculate the lift coefficient, C_L, required for this phase of the flight. **[2]**

(ii) Using Table 1 in the pre-release document and assuming a linear relationship between the lift coefficient and the drag coefficient in the region in which your value of C_L lies, estimate the corresponding drag coefficient C_D. **[2]**

(iii) Calculate the thrust required to maintain the aircraft in this phase. **[3]**

3 An aircraft has lift coefficient, C_L, and drag coefficient, C_D, which are both related to the angle α as follows.

$$C_L = 0.09\alpha + 0.1$$

$$C_D = \frac{4\alpha^2 + 10\alpha + 200}{10\,000}$$

where $\alpha < 14$.

Given that $\beta = \frac{C_D}{C_L}$, use calculus to determine the value of α for which β has a minimum value. **[7]**

4 An aircraft with a mass of 45 000 kg and an effective wing area of 120 m^2 is taking off on a level runway. During the initial acceleration phase, which starts from rest and increases to stall speed, v_S, flaps and slats on the wings are extended so that $C_L = 1.2$ and $C_D = 0.07$. The runway is at an altitude where the density of air is 1.22 kg m^{-3}. During this phase the engines provide a total constant thrust, F, of 150 kN.

(a) Ignoring all forces apart from thrust, weight and lift, and assuming constant acceleration calculate

(i) the stall speed, **[2]**

(ii) the time taken to reach stall speed. **[2]**

(b) The motion of the aircraft during the acceleration phase may instead be modelled by the following equation.

$$m\frac{dv}{dt} = F - D - D_R$$

where $D_R = \mu_R(W - L)$ and μ_R is the rolling resistance coefficient.

(i) By making appropriate substitutions show that

$$m\frac{dv}{dt} = F - \left(\tfrac{1}{2}\rho v^2 S(C_D - \mu_R C_L) + \mu_R W\right).$$ **[2]**

(ii) The average total force for the period in which the aircraft accelerates from rest to stall speed is approximated by

$$\frac{1}{v_S}\int_0^{v_S}\left(\mathrm{F} - \left(\tfrac{1}{2}\rho v^2 S(C_D - \mu_R C_L) + \mu_R W\right)\right)dv.$$

With $\mu_R = 0.025$, the value of v_S as calculated in part **(a)(i)** and the other quantities with the values given, calculate the average total force. **[3]**

(iii) Using this average total force, the value for v_S found in part **(a)(i)** and assuming constant acceleration, calculate another estimate of the time it will take the aircraft to reach stall speed. **[2]**