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Thursday 24 May 2012 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/02 Paper 2

INSERT

Duration: 1 hour 30 minutes

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Aircraft Flight Dynamics

Introduction

The analysis of aircraft flight dynamics involves many interrelated variables, some of which are difficult to quantify accurately. These variables depend on the aircraft's design, flight speed, altitude, fuel consumption, atmospheric conditions and many other factors. Simplifications can often be made to construct mathematical models that provide approximate results. These simplifications often include neglecting factors that are assumed to have little effect, or replacing variables by constants for particular situations. This document describes some of the fundamental equations that can be used to approximate important flight characteristics. Only straight flight paths are considered and no attempt is made to analyse turning manoeuvres, aircraft pitching or varying weather conditions.

Throughout this document a number of symbols are used to represent physical quantities. The appendix provides a list of these symbols, together with their meaning and associated units.

Acceleration due to gravity is taken as $9.8\,\mathrm{m\,s^{-2}}$ in all numerical calculations.

Forces acting on an aircraft

The forces acting on an aircraft in steady, horizontal flight and in ideal 'still air' conditions are shown in Fig. 1.

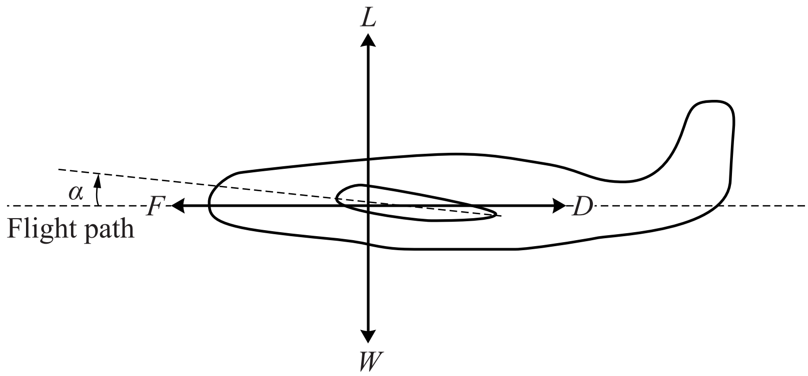


Fig. 1

The orientation of the wing with respect to the direction of travel (ie flight path) is at a small angle α, whose value is chosen to influence the values of critical forces. The angle α is referred to as the *angle of attack*.

The only forces acting on the aircraft are assumed to be as follows.

W The total weight of the aircraft is a constant acting vertically downwards.

F The force generated by the engines, referred to as the thrust, acting in the same direction as the flight path.

D The drag force created as the aircraft moves through the air, acting in the direction opposite to that of the flight path.

L The lift force created by pressure differences around the wing. The shape of the wing and its orientation with respect to the direction of flight cause the pressure on the lower surface to be greater than that on the upper surface. This causes an upward lift force and the direction of the lift force will always be perpendicular to the flight path.

The drag force, D, is described by the formula

$$D = \tfrac{1}{2} C_D \rho v^2 S$$

where
- C_D is the drag coefficient,
- ρ is the air density, which varies with height and temperature,
- v is the speed of the aircraft in the direction of the flight path,
- S is the effective wing area.

(*See the appendix for the units of the quantities in this formula.*)

The lift force, L, is described by the formula

$$L = \tfrac{1}{2} C_L \rho v^2 S$$

where
- C_L is the lift coefficient.

For a given aircraft design both C_D and C_L are dependent upon the angle of attack.

Example 1

Consider a medium-size, passenger aircraft with a mass of 50 000 kg and an effective wing area of 100 m^2, travelling at a constant speed at an altitude where the air density is 0.4 kg m^{-3}. Assume that $C_L = 0.6$ and $C_D = 0.035$.

The downward force is the weight, W, where

$$W = 50\,000 \times 9.8 = 490\,\text{kN}.$$

In order to maintain the aircraft in horizontal flight (with no vertical acceleration), the magnitude of the lift force, L, must equal the magnitude of the weight, W. Then

$$L = \tfrac{1}{2} C_L \rho v^2 S = W.$$

By re-arranging this formula, the speed of the aircraft is

$$v = \sqrt{\frac{2W}{C_L \rho S}} = \sqrt{\frac{2 \times 490\,000}{0.6 \times 0.4 \times 100}} \approx 202\,\text{m s}^{-1}. \text{ (Correct to 3 significant figures.)}$$

In order to maintain the aircraft at a constant speed the thrust and the drag forces must have the same magnitude.

The drag force is given by

$$D = \tfrac{1}{2} C_D \rho v^2 S.$$

This gives

$$D = 0.5 \times 0.035 \times 0.4 \times 202^2 \times 100 \approx 29\,\text{kN}. \text{ (Correct to the nearest kN.)}$$

The thrust force required to maintain the aircraft in this situation is therefore about 29 kN.

By manipulation of the formulae for drag and lift forces, it can be shown that

$$\frac{D}{L} = \frac{C_D}{C_L}$$

In this case the magnitude of D could have been calculated as follows.

$$D = \frac{C_D}{C_L} L = \frac{C_D}{C_L} W \approx 29\,\text{kN}. \text{ (Correct to the nearest kN.)}$$

Relationship between C_D and C_L

Both C_D and C_L are related to the angle of attack. For a particular aircraft design, typical values are given in Table 1. The drag-to-lift ratio β is $\frac{D}{L}$ which equals $\frac{C_D}{C_L}$.

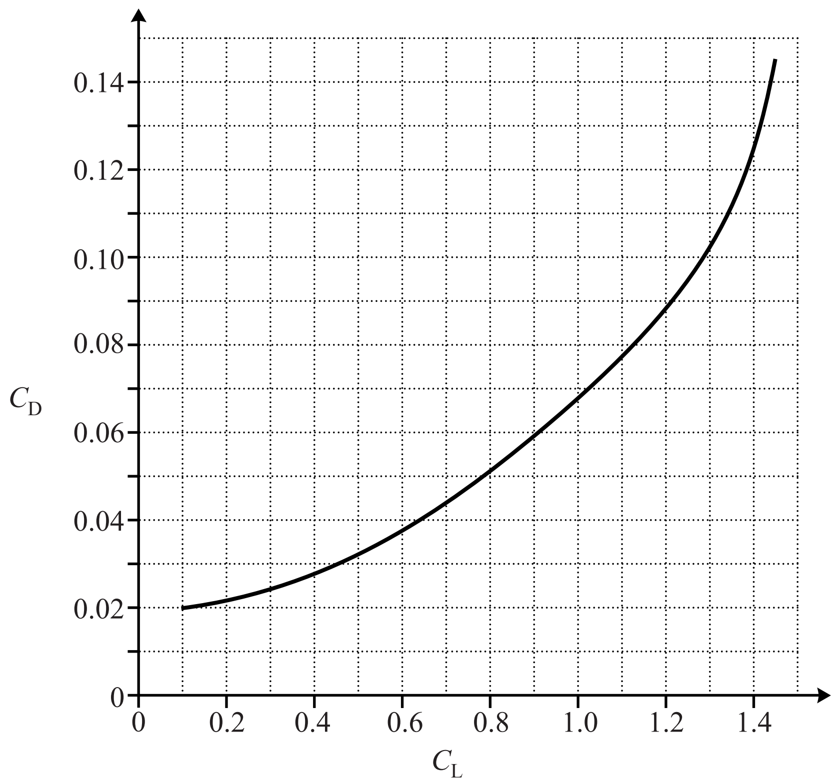
$\alpha°$	C_L	C_D	β	$\alpha°$	C_L	C_D	β
0	0.1000	0.0200	0.2000	8	0.8333	0.0536	0.0643
1	0.1917	0.0214	0.1116	9	0.9250	0.0614	0.0664
2	0.2833	0.0236	0.0833	10	1.0167	0.0700	0.0689
3	0.3750	0.0266	0.0709	11	1.1083	0.0794	0.0716
4	0.4667	0.0304	0.0651	12	1.2000	0.0896	0.0747
5	0.5583	0.0350	0.0627	13	1.2917	0.1006	0.0779
6	0.6500	0.0404	0.0622	14	1.3833	0.1200	0.0867
7	0.7417	0.0466	0.0628	15	1.4500	0.1450	0.1000

Table 1

For an angle of attack up to about 12°, C_L is approximately a linear function of α. The relationship between C_D and α is not linear, but up to about 12° could be approximated by a quadratic in α.

The most efficient aircraft operation occurs when β is a minimum. Note that the table shows an approximate minimum value of β of 0.0622.

Graph 1 shows the relationship between C_L and C_D using the values from Table 1.



Graph 1

Climbing

The forces acting on an aircraft maintaining a steady climb in ideal 'still air' conditions are shown in Fig. 2.

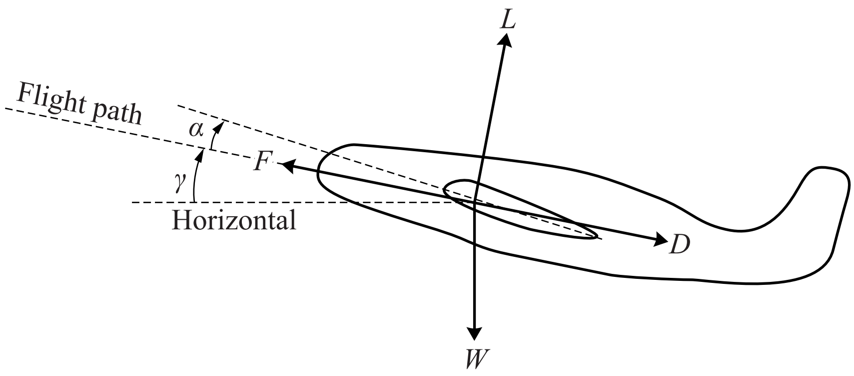


Fig. 2

In order to maintain the aircraft in a steady climb with an angle of γ and with zero acceleration, the forces must be in equilibrium.

Equating forces acting perpendicular to the flight path

$$L = W\cos\gamma.$$

Equating forces acting along the flight path

$$F = D + W\sin\gamma.$$

In addition to the drag-to-lift ratio it is also useful to define the thrust-to-weight ratio, f, where

$$f = \frac{F}{W}.$$

By manipulating these formulae it can be shown that

$$f = \beta\cos\gamma + \sin\gamma.$$

For small angles of climb, the approximation $\cos\gamma = 1$ is often used to simplify calculations. This results in the equation

$$f - \beta = \sin\gamma.$$

This formula, together with the data given in Table 1, can then be used to calculate the forces required for the aircraft to maintain constant speed along a uniformly inclined flight path.

Example 2

Consider an aircraft of mass 50 000 kg and an effective wing area of 100 m^2 travelling with a constant climb angle of 5° at a constant speed of 160 $m\,s^{-1}$. The air density remains constant at 1 $kg\,m^{-3}$ for the duration of the flight. The requirement is to find the drag coefficient and the constant thrust (F) in this situation.

The weight, W, is 490 kN.

Using the simplification $\cos\gamma = 1$, the lift force L is 490 kN.

Substituting L, ρ, v and S into $C_L = \dfrac{2L}{\rho v^2 S}$ gives

$$C_L = \frac{2 \times 490\,000}{1 \times 160^2 \times 100} = 0.3828. \text{ (Correct to 4 significant figures.)}$$

The value of 0.3828 lies between 0.3750 and 0.4667 given in the column headed C_L of Table 1. Between these values of C_L there is an approximate linear relationship between C_L and C_D. Inspection of Graph 1 confirms this approximate linearity. A calculation based on this approximation shows that

$$C_D \approx 0.026\,92.$$

You are encouraged to confirm this result.

Using $C_D = 0.026\,92$ and $C_L = 0.3828$

$$\beta = \frac{C_D}{C_L} \approx 0.070\,32.$$

Using the approximation $f - \beta \approx \sin\gamma$ with $\gamma = 5$ and $f = \dfrac{F}{W}$

$$\frac{F}{W} - 0.070\,32 \approx 0.087\,16.$$

This gives the required thrust

$$F = (0.087\,16 + 0.070\,32) \times 490\,000 \approx 77\text{ kN. (Correct to the nearest kN.)}$$

Taking off

During the take-off procedure, when the aircraft is travelling along the runway, flaps and slats on the wings are extended to provide the high lift coefficient necessary for the lift-off speed to be reached within a reasonable distance. As the aircraft accelerates along the runway it is acted upon by the thrust of the engines, the increasing drag force, a force due to the rolling resistance, an increasing lift force and any other forces due to the inclination of the runway and weather conditions.

The force opposing motion due to the rolling resistance, D_R, is given by

$$D_R = \mu_R(W - L) \qquad \text{while on the runway,}$$
$$D_R = 0 \qquad \text{otherwise.}$$

where μ_R is the rolling resistance coefficient; a typical value is about 0.025.

The motion of the aircraft as it starts from rest and travels along a level runway to the lift-off speed is modelled by the differential equation

$$m\frac{dv}{dt} = F - D - D_R$$

where

m is the mass of the aircraft.

A considerable simplification of the situation can be made by neglecting drag and rolling resistance forces and assuming a constant thrust force, and therefore a constant acceleration.

In this case

$$m\frac{dv}{dt} = F$$

and so the constant acceleration, a, is given by

$$a = \frac{F}{m}.$$

As the velocity of the aircraft increases, the lift force also increases and this will eventually become equal to the aircraft's weight. At this time

$$L = W = \tfrac{1}{2}C_L \rho v^2 S$$

and therefore

$$v^2 = \frac{2W}{\rho S C_L}.$$

The speed at which $L = W$ is referred to as the stall speed, v_S. The lift-off speed is slightly greater than this and typically is $1.1v_S$.

By calculating v_S and by assuming a constant acceleration, the time taken to reach the speed v_S and the distance travelled during this time can readily be approximated.

For a more rigorous analysis, the differential equation of motion

$$m\frac{dv}{dt} = F - D - D_R$$

must be solved.

In this equation F, D and D_R are all dependent on the speed v, which results in a non-trivial differential equation. Simplification can be made for the total effect of these forces. The subsequent solution of the simplified equation will provide fairly good approximations to the required results.

The next time you visit an airport try to observe the time it takes for a medium-size, passenger aircraft to reach lift-off speed from when the brakes are released and the initial engine thrust is applied. Also observe how the slats and flaps are extended during the take-off and landing periods and how these are retracted during normal flight.

Appendix

List of symbols and units of measurement.

Symbol	Units	Meaning
L	N	lift force
D	N	drag force
F	N	thrust
W	N	weight of aircraft
D_R	N	rolling resistance force
M	kg	mass of aircraft
C_L	none	lift coefficient
C_D	none	drag coefficient
β	none	drag-to-lift ratio
f	none	thrust-to-weight ratio
μ_R	none	rolling resistance coefficient
S	m^2	effective wing area
α	degrees	angle of attack
γ	degrees	angle of climb
ρ	$kg\,m^{-3}$	air density
v	$m\,s^{-1}$	speed
v_S	$m\,s^{-1}$	stall speed
a	$m\,s^{-2}$	acceleration
g	$m\,s^{-2}$	acceleration due to gravity