OCR
RECOGNISING ACHIEVEMENT

Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate

H860/01 Paper 1

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

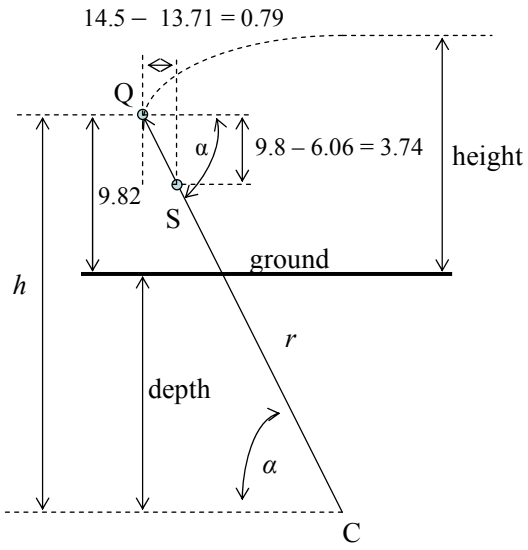
Question			Answer	Mark	Guidance
1	(a)	(i)	Length of steel for stage i is $5\left(\frac{1}{2}\right)^{i-1}$	1 [1]	Candidates should recognise this as a geometric series and use the appropriate formulae in MF1. Accept ar^{i-1} provided that correct values for a and/or r are given.
1	(a)	(ii)	Total length of steel at stage n is $\frac{5\left(1-\left(\frac{1}{2}\right)^{n}\right)}{1-\frac{1}{2}}=10\left(1-\left(\frac{1}{2}\right)^{n}\right)$ metres Total length of steel at stage 10 = $10\left(1-\left(\frac{1}{2}\right)^{10}\right)=9.9902$ metres	3 [3]	Allow 1 mark for $\frac{a(1-r^{n})}{1-r}$ if given without values for a and r and without any final answer. Allow reasonable attempts at manually summing ten values. Award 3 marks if answer is correct to 1 dp. Award 2 marks if ten individual values are correct but sum is wrong. Award 1 mark for ten reasonable (decreasing) individual values seen summed. NOTE: The question did not state that the formulae in MF1 MUST be used.
1	(a)	(iii)	Limit as $n\to\infty\left\{10\left(1-\left(\frac{1}{2}\right)^{n}\right)\right\}=10$ metres	2 [2]	Allow 1 mark for $\frac{a}{1-r}$
1	(b)	(i)	$t_i = 1 + 4(i - 1)$ or $t_i = 4i - 3$	2 [2]	Allow 1 mark for $a + (n - 1)d$ seen without values for a and d. Allow 1 mark for $t_i = t_i + 4$ OE
1	(b)	(ii)	$t_{10} = 37$	1 [1]	CAO allow answer with no working

Question			Answer	Mark	Guidance
1	(b)	(iii)	$\sum_{i=1}^{n} t_i = \frac{1}{2}n(2+4(n-1)) = \frac{1}{2}n(4n-2)$ $\sum_{i=1}^{10} t_i = \frac{1}{2}10(40-2) = 190$	1 1 **[2]**	
2			Information given A'.B'.C' = 50 A = 60 and therefore A' = 140 B = 80 and therefore B' =120 C = 105 and therefore C' = 95 A.B'.C' = 15 A'.B.C' = 20 A'.B'.C = 40 A.B.C = 20		
2	(a)	(i)	Required A'.B.C Since A'.B.C + A'.B'.C + A'.B.C' + A'.B'.C' = A' = 140 A'.B.C = 140 – A'.B'.C – A'.B.C' – A'.B'.C' = 140 – 40 – 20 – 50 = 30	 2 **[2]**	Accept answers shown in a Venn diagram OE
2	(a)	(ii)	Required A.B.C' Since A.B.C' + A.B.C + A'.B.C' + A'.B.C = B = 80 A.B.C' = 80 – A.B.C – A'.B.C' – A'.B.C = 80 – 20 – 20 – 30 = 10	 2 **[2]**	Accept answers shown in a Venn diagram OE

Question			Answer	Mark	Guidance
2	(b)	(i)	Required P (A.B'.C) Since A.B.C + A.B'.C + A.B.C' + A.B'.C' = A = 60 A.B'.C = 60 – A.B.C – A.B.C'– A.B'.C' = 60 – 20 – 10 – 15 = 15 15/200 = 3/40	2 **[2]**	Allow 1 mark here OR in part (b)(ii) if 200 seen as the denominator.
2	(b)	(ii)	Required P ((A.B.C') + (A.B'.C) + (A'.B.C)) (10 + 15 + 30)/200 = 55/200 = 11/40	2 **[2]**	See note above.
2	(c)		Required P{(A.B)/B} = (A.B.C' + A.B.C)/B = (10 + 20)/80 = 3/8	2 **[2]**	Allow 1 mark if reasonable attempt seen for conditional probability.
3	(a)		$-4 + R_1 I_1 - 8 + R_2 (I_1 - I_2) = 0$ $\quad (R_1 + R_2)I_1 - R_2 I_2 = 12 \quad 10I_1 - 2I_2 = 12$ OR $5I_1 - I_2 = 6$ $10 + R_3 I_2 + R_2 (I_2 - I_1) + R_4 I_2 = 0 \quad -R_2 I_1 + (R_2 + R_3 + R_4)I_2 = -10 \quad -2I_1 + 12I_2 = -10$ OR $-I_1 + 6I_2 = -5$	1 1 **[2]**	Also allow $10I_1 - 2I_2 - 12 = 0$ OE Also allow $-2I_1 + 12I_2 + 10 = 0$ OE
3	(b)		$\begin{bmatrix} 10 & -2 \\ -2 & 12 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \end{bmatrix}$ $\quad \mathbf{A}\,\underline{I} = \underline{b}$	1 **[1]**	Correct matrix notation MUST be used

Question			Answer	Mark	Guidance
3	(c)		$\det \mathbf{A} = \begin{vmatrix} 10 & -2 \\ -2 & 12 \end{vmatrix} = 120 - 4 = 116$	1	Accept any correct alternative method of solution.
			or $\det \mathrm{A} = \begin{vmatrix} 5 & -1 \\ -1 & 6 \end{vmatrix} = 30 - 1 = 29$		
			cofactors		
			$\mathbf{C} = \begin{pmatrix} 12 & 2 \\ 2 & 10 \end{pmatrix}$	1	
			or $\begin{pmatrix} 6 & 1 \\ 1 & 5 \end{pmatrix}$		
			$\mathbf{A}^{-1} = \frac{1}{\det A}\mathbf{C}^{\mathrm{T}} = \frac{1}{116}\begin{bmatrix} 12 & 2 \\ 2 & 10 \end{bmatrix}$	1	
			or $\frac{1}{29}\begin{bmatrix} 6 & 1 \\ 1 & 5 \end{bmatrix}$		
			$\mathbf{I} = \mathbf{A}^{-1}\underline{b}$		
			$\frac{1}{116}\begin{bmatrix} 12 & 2 \\ 2 & 10 \end{bmatrix}\begin{bmatrix} 12 \\ -10 \end{bmatrix} = \begin{bmatrix} (12\times 12 - 2\times 10)/116 \\ (2\times 12 - 10\times 10)/116 \end{bmatrix} = \begin{bmatrix} 31/29 \\ -19/29 \end{bmatrix}$		
			or $\frac{1}{29}\begin{bmatrix} 6 & 1 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 \\ -5 \end{bmatrix} = \begin{bmatrix} (6\times 6 - 1\times 5)/29 \\ (1\times 6 - 5\times 5)/29 \end{bmatrix} = \begin{bmatrix} 31/29 \\ -19/29 \end{bmatrix}$		
			$I_1 = 31/29$ A		
			$I_2 = -19/29$ A	1	
				[4]	Allow 2 marks for any correct method Allow 2 marks for accuracy

Question			Answer	Mark	Guidance
4	(a)		$(r - SQ)^2 + SR^2 = r^2$ $r^2 - 2SQr + SQ^2 + SR^2 = r^2$ $2SQr = SQ^2 + SR^2$ $r = \frac{SQ^2 + SR^2}{2SQ}$	1 1 1 **[3]**	Answer must clearly show correct reasoning
4	(b)	(i)	PR = $\sqrt{29^2 + (9.12 - 3)^2} = 29.6387$ m	1 **[1]**	
4	(b)	(ii)	S at (29/2, 3 + (9.12 – 3)/2) = (14.5, 6.06) SQ = $\sqrt{(14.5 - 13.71)^2 + (9.8 - 6.06)^2} \approx 3.82$ m	2 **[2]**	Allow alternative solution with ECF: SR = PR/2 = 14.8194 $QR^2 = (29 - 13.17)^2 - (9.8 - 9.12)^2$ $= 233.3217$ or QR = 15.2749 Then SQ = $\sqrt{QR^2 - SR^2} \approx 3.83$
4	(b)	(iii)	$r = \frac{3.8225^2 + \left(\frac{PR}{2}\right)^2}{2 \times 3.8225} = 30.6376$ m	1 **[1]**	Allow ECF

Question			Answer	Mark	Guidance
4	(b)	(iv)	 $\alpha = \tan^{-1}\frac{3.74}{0.79} = 1.363 \text{ rads} = 78.07°$ $h = r \sin \alpha = 29.9759$	1 1 **[2]**	Diagram **not** required for full marks
4	(b)	(v)	depth = h – 9.8 = 20.1759 height = r – depth = 30.6376 – 20.1759 = 10.4617 m	1 1 **[2]**	

Question			Answer	Mark	Guidance
5	(a)		$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$		
			$S_1 = \sin(2\pi ft) \quad \text{and} \quad S_2 = \sin(2\pi ft + \phi)$		
			$S_1 + S_2 = 2\sin\left(\frac{2\pi ft + 2\pi ft + \phi}{2}\right)\cos\left(\frac{2\pi ft - 2\pi ft - \phi}{2}\right) =$		Award 2 marks for a response that uses $\cos\left(-\frac{\phi}{2}\right)$ or $\cos\left(\frac{\phi}{2}\right)$
			$2\sin\left(2\pi ft + \frac{\phi}{2}\right)\cos\left(-\frac{\phi}{2}\right) = 2\sin\left(2\pi ft + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)$	2	
		i	When $\phi = \frac{2\pi}{3}$ $S_1 + S_2 = 2\sin\left(2\pi ft + \frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) = \sin\left(2\pi ft + \frac{\pi}{3}\right)$	2	Answer must cancel 2 and $\cos\left(\frac{\pi}{3}\right)$
		ii	When $\phi = \pi$ $S_1 + S_2 = 2\sin\left(2\pi ft + \frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) = 0$	1	CAO
				[5]	
5	(b)		$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) =$		
			$\sin 2\pi f_1 t + \sin 2\pi f_2 t = 2\sin\left(\frac{2\pi f_1 t + 2\pi f_2 t}{2}\right)\cos\left(\frac{2\pi f_1 t - 2\pi f_2 t}{2}\right) =$		
			$2\sin\left(\left(\frac{f_1 + f_2}{2}\right)2\pi t\right)\cos\left(\left(\frac{f_1 - f_2}{2}\right)2\pi t\right) =$		
			$2\cos\left(\pi(f_1 - f_2)t\right)\sin\left(\pi(f_1 + f_2)t\right)$	2	
				[2]	

Question			Answer	Mark	Guidance
5	(c)		$\frac{f_1 + f_2}{2} = 50$ $\frac{f_1 - f_2}{2} = 5$ $f_1 + f_2 = 100$ $f_1 - f_2 = 10$ $2f_1 = 110$ $f_1 = 55$ Hz and $f_2 = 45$ Hz	2 **[2]**	Award 1 each for 55 and 45 seen
6	(a)		Period = 2 s Frequency = 0.5 Hz	1 **[1]**	
6	(b)		$g(t) = t/2$	1 **[1]**	
6	(c)		$\int uv' = uv - \int vu'$ $\int t \sin at \mathrm{d}t = -\frac{t\cos at}{a} - \int -\frac{\cos at}{a} dt =$ $-\frac{t\cos at}{a} + \frac{\sin at}{a^2} + c =$ $\frac{\sin at}{a^2} - \frac{t\cos at}{a} + c$	1 2 1 **[4]**	1 mark for use of correct formula 1 mark for $-t\cos at - \int -\cos at \mathrm{d}t$

Question			Answer	Mark	Guidance
6	(d)	(i)	$c = \frac{2}{T}\int_0^T g(t)\mathrm{d}t \Rightarrow\ c = \frac{2}{2}\int_0^2 \frac{t}{2}\mathrm{d}t = \left[\frac{t^2}{4}\right]_0^2 = 1$	2 **[2]**	Accept ECF
6	(d)	(ii)	$a_n = \frac{2}{T}\int_0^T g(t)\sin 2\pi fnt\ \ dt = \frac{2}{2}\int_0^2 \frac{t}{2}\sin 2\pi\frac{1}{2}nt\ \ \ dt = \frac{1}{2}\int_0^2 t\sin \pi nt\ dt =$ $\frac{1}{2}\left[\frac{\sin \pi nt}{(\pi n)^2} - \frac{t\cos \pi nt}{\pi n}\right]_0^2 =$ $\frac{1}{2}\left[\left(0 - \frac{2}{\pi n}\right) - (0-0)\right] = -\frac{1}{\pi n}$	1 1 2 **[4]**	Accept ECF From given result in part c

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553