

Mathematics for Engineering

OCR Level 3 Certificate

H860/02 Paper 2

Mark Scheme for June 2012

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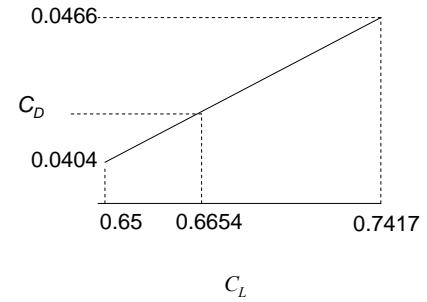
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Question			Answer	Marks	Guidance
1	(a)	(i)	$L = 40000g = 392 \text{ kN}$	1 [1]	For level flight $L = W = mg$
		(ii)	$D = \frac{1}{2} C_D \rho v^2 S$ $D = \frac{1}{2} 0.035 \times 0.5 \times 176^2 \times 120 \approx 33 \text{ kN}$	1 [1]	
	(b)	(i)	For constant speed $D = F$ From $D = \frac{1}{2} C_D \rho v^2 S$ $v^2 = \frac{2D}{C_D \rho S} = \frac{2 \times 25000}{0.03 \times 0.5 \times 120} = 27777.7$ $v = 166.6 \text{ m s}^{-1}$	1 1 [2]	
		(ii)	For level flight $L = W$ From $L = \frac{1}{2} C_L \rho v^2 S$ $C_L = \frac{2W}{\rho v^2 S} = \frac{2 \times 40000 \times 9.8}{0.5 \times 27777.7 \times 120} = 0.4704$	1 1 [2]	Alternatively using $D/L = C_D/C_L$ $C_L = C_D L/D$ $= \frac{0.03 \times 40000 \times 9.8}{25000} = 0.4704$
		(iii)	The increase in thrust will mean initially that $D < F$. This will cause a steady <u>increase in velocity</u> . The increased velocity will cause an increased lift force which will cause the aircraft to <u>gain height</u> .	1 1 [2]	

Question			Answer	Marks	Guidance
2	(a)	(i)	From $L = \frac{1}{2} C_L \rho v^2 S$ $L = \frac{1}{2} 0.665 \times 0.5 \times 180^2 \times 100 = 538650 \approx 539 \text{ kN}$	1	
			$W = mg = 55000 \times 9.8 = 539 \text{ kN}$	1 [2]	
		(ii)	$D = \frac{1}{2} C_D \rho v^2 S$ $D = \frac{1}{2} 0.042 \times 0.5 \times 180^2 \times 100 = 34020 \approx 34 \text{ kN}$	1 [1]	
		(iii)	$\beta = \frac{D}{L} = \frac{34}{539} \approx 0.0631$ $f = \frac{F}{W} = \frac{60}{539} \approx 0.1113$	1 1 [2]	Also accept $\beta = \frac{C_D}{C_L} = \frac{0.042}{0.665} \approx 0.0632$
		(iv)	$f - \beta = \sin \gamma \Rightarrow \gamma = \sin^{-1}(f - \beta)$ $= \sin^{-1}(0.1113 - 0.0631) \approx 2.76^\circ$	1 1 [2]	Also allow $F = D + W \sin(\gamma) \Rightarrow \lambda = \sin^{-1}\left(\frac{F - D}{W}\right)$ $= \sin^{-1}\left(\frac{60000 - 34000}{539000}\right) \approx 2.76^\circ$ Allow 1 mark only for $\cos \gamma = \frac{L}{W} \Rightarrow \lambda = \cos^{-1}\left(\frac{L}{W}\right)$ $= \cos^{-1}\left(\frac{538650}{539000}\right) \approx 2.07^\circ$

Question		Answer	Marks	Guidance
(b)	(i)	$C_L = \frac{2W}{\rho v^2 S} = \frac{2 \times 55000 \times 9.8}{0.5 \times 180^2 \times 100} \approx 0.6654$	2 [2]	Allow rounding to 0.67
	(ii)	Using the table and linear interpolation $\frac{0.6654 - 0.65}{C_D - 0.0404} = \frac{0.7417 - 0.65}{0.0466 - 0.0404}$ $C_D = \left(\frac{0.6654 - 0.65}{0.7417 - 0.65} \right) \times (0.0466 - 0.0404) + 0.0404 \approx 0.0414$	1 1 [2]	 <p>The answer must refer to the values in the table and show that a linear relationship has been used. Award no marks for a value with no working. Award 1 mark for answer between 0.0404 and 0.0466 with some evidence of reasoning.</p>
	(iii)	$\beta = \frac{C_D}{C_L} = \frac{0.0414}{0.6654} \approx 0.0622$ using $\frac{F}{W} - \beta = \sin \gamma$ $F = W \times (\sin \gamma + \beta)$ $= 539000 \times (\sin(4) + 0.0622) \approx 71 \text{ kN}$	1 1 1 [3]	Allow ECF from (b)(i) and (b)(ii). Allow alternative solution $F = D + W \sin \gamma$ $= 34 + 550000g \sin 4$ $\approx 71 \text{ kN}$

Question	Answer	Marks	Guidance
3	$\beta = \frac{4\alpha^2 + 10\alpha + 200}{10000} \times (0.09\alpha + 0.1)^{-1}$ $\frac{d\beta}{d\alpha} = -0.09(0.09\alpha + 0.1)^{-2} \times \left(\frac{4\alpha^2 + 10\alpha + 200}{10000} \right) + \left(\frac{8\alpha + 10}{10000} \right) \times (0.09\alpha + 0.1)^{-1}$ <p>For minimum α, $\frac{d\beta}{d\alpha} = 0$</p>	1 3	Award 1 mark for $8\alpha + 10$ seen Also accept for 3 marks $\frac{(900\alpha + 1000)(8\alpha + 10) - (4\alpha^2 + 10\alpha + 10)900}{(900\alpha + 1000)^2}$ OE
	$-0.09 \times (4\alpha^2 + 10\alpha + 200) + (8\alpha + 10) \times (0.09\alpha + 0.1) = 0$ $(-0.36 + 0.72)\alpha^2 + (-0.9 + 0.8 + 0.9)\alpha + (-18 + 1) = 0$ $0.36\alpha^2 + 0.8\alpha - 17 = 0$ $\alpha = \frac{-0.8 \pm \sqrt{0.8^2 + 4 \times 0.36 \times 17}}{2 \times 0.36}$ $= \frac{-0.8 \pm 5.012}{0.72}$ <p>α must be +ve so $\alpha = 5.85$</p>	1 1 1 [7]	

Question			Answer	Marks	Guidance
4	(a)	(i)	$v = v_S$ when $L = W = 45000 \times 9.8 = 441000$ $v_S^2 = \frac{2W}{\rho S C_L} = \frac{2 \times 441000}{1.22 \times 120 \times 1.2} = 5020.4918$ $v_S = \sqrt{5020.4918} = 70.8554 \text{ m s}^{-1}$	1 1 [2]	
		(ii)	$a = \frac{150000}{45000}$ $t = \frac{v_S}{a} = \frac{70.8554 \times 45000}{150000} = 21.2566 \text{ s}$	1 1 [2]	
4	(b)	(i)	$m \frac{dv}{dt} = F - D - D_R$ $m \frac{dv}{dt} = F - \frac{1}{2} \rho v^2 C_D S - \mu_R (W - \frac{1}{2} C_L \rho v^2 S)$ $m \frac{dv}{dt} = F - (\frac{1}{2} \rho v^2 S (C_D - \mu_R C_L) + \mu_R W)$	2 [2]	

Question	Answer	Marks	Guidance
(ii)	$\frac{1}{v_s} \int_0^{v_s} F - \left(\frac{1}{2} \rho v^2 S(C_D - \mu_R C_L) + \mu_R W \right) dv$ $\frac{1}{v_s} \left[Fv - \left(\frac{1}{2} \rho \frac{v^3}{3} S(C_D - \mu_R C_L) + \mu_R Wv \right) \right]_0^{v_s}$ $= F - \left(\frac{1}{2} \rho \frac{v_s^2}{3} S(C_D - \mu_R C_L) + \mu_R W \right)$ $= 150000 - \left(\frac{1}{2} \times 1.22 \times 120 \times \frac{5020.4918}{3} \times (0.07 - 0.025 \times 1.2) + 0.025 \times 441000 \right)$ $= 134\,075 \text{ N}$	<p>1</p> <p>1</p> <p>1</p> <p>[3]</p>	<p>Accept numerical equivalent</p> $\frac{1}{70.8554} \int_0^{70.8554} 138975 - 2.928v^2 dv$ $\frac{1}{70.8554} \left[138975v - \frac{2.928v^3}{3} \right]_0^{70.8554}$ $= 138975 - \frac{2.928(70.8554)^2}{3}$ $= 134\,075 \text{ N}$ <p>Accept answer rounded to the nearest kN ($\approx 134 \text{ kN}$)</p>
(iii)	<p>From $F = ma$</p> $134075 = m \frac{dv}{dt}$ $134075t = mv$ $t = \frac{mv}{134075} = \frac{45000 \times 70.8554}{134075} \approx 23.78 \text{ s}$	<p>1</p> <p>1</p> <p>[2]</p>	<p>Accept rounded answer to 24 s</p>

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