

Monday 25 June 2012 – Afternoon

A2 GCE MATHEMATICS

4731 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4731
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 A uniform square lamina, of mass 4.5 kg and side 0.6 m, is rotating about a fixed vertical axis which is perpendicular to the lamina and passes through its centre. A stationary particle becomes attached to the lamina at one of its corners, and this causes the angular speed of the lamina to change instantaneously from 2.2 rad s^{-1} to 1.5 rad s^{-1} .

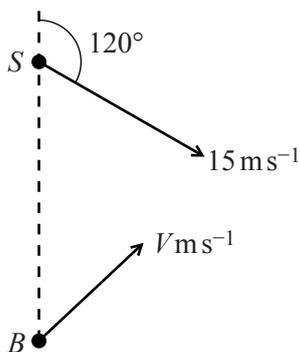
(i) Find the mass of the particle. [4]

The lamina then slows down with constant angular deceleration. It turns through 36 radians as its angular speed reduces from 1.5 rad s^{-1} to zero.

(ii) Find the time taken for the lamina to come to rest. [2]

- 2 A uniform solid of revolution is formed by rotating the region bounded by the x -axis and the curve $y = x \left(1 - \frac{x^2}{a^2} \right)$ for $0 \leq x \leq a$, where a is a constant, about the x -axis. Find the x -coordinate of the centre of mass of this solid. [7]

3



A ship S is travelling with constant velocity 15 m s^{-1} on a course with bearing 120° . A patrol boat B observes the ship when S is due north of B . The patrol boat B then moves with constant speed $V \text{ m s}^{-1}$ in a straight line (see diagram).

(i) Given that $V = 18$, find the bearing of the course of B such that B intercepts S . [3]

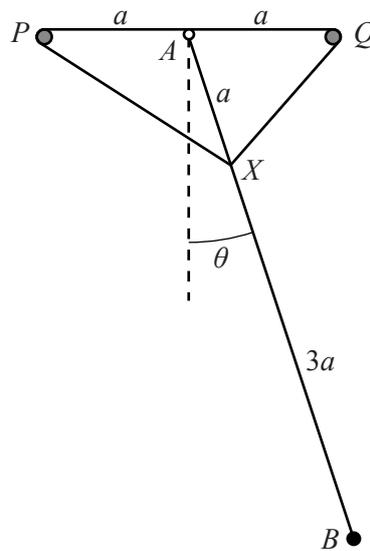
(ii) Given instead that $V = 9$, find the bearing of the course of B such that B passes as close as possible to S . [5]

(iii) Find the smallest value of V for which it is possible for B to intercept S . [2]

- 4 A uniform lamina of mass 18 kg occupies the region bounded by the x -axis, the y -axis, the line $x = \ln 9$ and the curve $y = e^{\frac{1}{2}x}$ for $0 \leq x \leq \ln 9$. The unit of length is the metre. Find the moment of inertia of this lamina about the x -axis. [7]

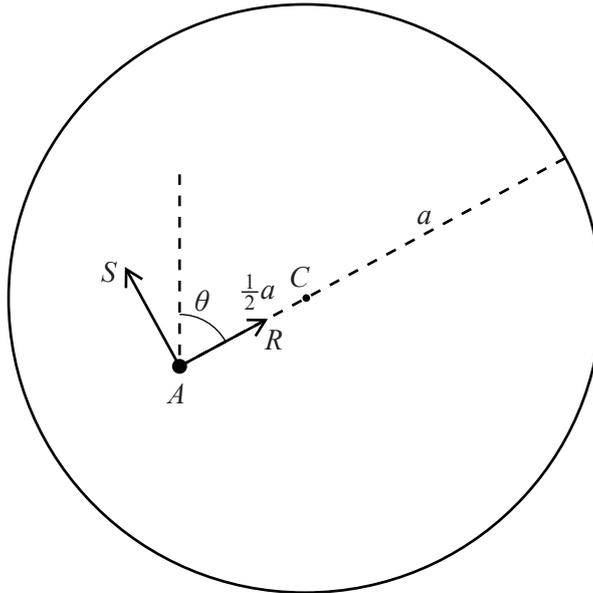
- 5 A uniform rod of mass 4 kg and length 2.4 m can rotate in a vertical plane about a fixed horizontal axis through one end of the rod. The rod is released from rest in a horizontal position and a frictional couple of constant moment 20 N m opposes the motion.
- (i) Find the angular acceleration of the rod immediately after it is released. [4]
- (ii) Find the angle that the rod makes with the horizontal when its angular acceleration is zero. [3]
- (iii) Find the maximum angular speed of the rod. [5]
- (iv) The rod first comes to instantaneous rest after rotating through an angle θ radians from its initial position. Find an equation for θ , and verify that $2.0 < \theta < 2.1$. [3]

6



Two small smooth pegs P and Q are fixed at a distance $2a$ apart on the same horizontal level, and A is the mid-point of PQ . A light rod AB of length $4a$ is freely pivoted at A and can rotate in the vertical plane containing PQ , with B below the level of PQ . A particle of mass m is attached to the rod at B . A light elastic string, of natural length $2a$ and modulus of elasticity λ , passes round the pegs P and Q and its two ends are attached to the rod at the point X , where $AX = a$. The angle between the rod and the downward vertical is θ , where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ (see diagram). You are given that the elastic energy stored in the string is $\lambda a(1 + \cos \theta)$.

- (i) Show that $\theta = 0$ is a position of equilibrium, and show that the equilibrium is stable if $\lambda < 4mg$. [6]
- (ii) Given that $\lambda = 3mg$, show that $\ddot{\theta} = -k\frac{g}{a}\sin\theta$, stating the value of the constant k . Hence find the approximate period of small oscillations of the system about the equilibrium position $\theta = 0$. [6]



A uniform circular disc with centre C has mass m and radius a . The disc is free to rotate in a vertical plane about a fixed horizontal axis passing through a point A on the disc, where $AC = \frac{1}{2}a$. The disc is slightly disturbed from rest in the position with C vertically above A . When AC makes an angle θ with the upward vertical the force exerted by the axis on the disc has components R parallel to AC and S perpendicular to AC (see diagram).

- (i) Show that the angular speed of the disc is $\sqrt{\frac{4g(1 - \cos \theta)}{3a}}$. [4]
- (ii) Find the angular acceleration of the disc, in terms of a , g and θ . [2]
- (iii) Find R and S , in terms of m , g and θ . [6]
- (iv) Find the magnitude of the force exerted by the axis on the disc at an instant when $R = 0$. [3]

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