

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 Find $\int x \cos 3x \, dx$. [4]
- 2 Find the first three terms in the expansion of $(9 - 16x)^{\frac{3}{2}}$ in ascending powers of x , and state the set of values for which this expansion is valid. [5]
- 3 The equation of a curve is $xy^2 = x^2 + 1$. Find $\frac{dy}{dx}$ in terms of x and y , and hence find the coordinates of the stationary points on the curve. [7]
- 4 The equations of two lines are
- $$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}).$$
- (i) Show that these lines meet, and find the coordinates of the point of intersection. [5]
- (ii) Find the acute angle between these lines. [3]
- 5 The parametric equations of a curve are
- $$x = 2 + 3 \sin \theta \quad \text{and} \quad y = 1 - 2 \cos \theta \quad \text{for} \quad 0 \leq \theta \leq \frac{1}{2}\pi.$$
- (i) Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$. [5]
- (ii) Find the cartesian equation of the curve. [2]
- 6 Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} \, dx$. [7]
- 7 (i) Given that $y = \ln(1 + \sin x) - \ln(\cos x)$, show that $\frac{dy}{dx} = \frac{1}{\cos x}$. [4]
- (ii) Using this result, evaluate $\int_0^{\frac{1}{3}\pi} \sec x \, dx$, giving your answer as a single logarithm. [3]
- 8 The points $A(3, 2, 1)$, $B(5, 4, -3)$, $C(3, 17, -4)$ and $D(1, 6, 3)$ form a quadrilateral $ABCD$.
- (i) Show that $AB = AD$. [2]
- (ii) Find a vector equation of the line through A and the mid-point of BD . [3]
- (iii) Show that C lies on the line found in part (ii). [1]
- (iv) What type of quadrilateral is $ABCD$? [1]

- 9 The temperature of a freezer is -20°C . A container of a liquid is placed in the freezer. The rate at which the temperature, $\theta^{\circ}\text{C}$, of a liquid decreases is proportional to the difference in temperature between the liquid and its surroundings. The situation is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta + 20),$$

where time t is in minutes and k is a positive constant.

- (i) Express θ in terms of t , k and an arbitrary constant. [3]

Initially the temperature of the liquid in the container is 40°C and, at this instant, the liquid is cooling at a rate of 3°C per minute. The liquid freezes at 0°C .

- (ii) Find the value of k and find also the time it takes (to the nearest minute) for the liquid to freeze. [5]

The procedure is repeated on another occasion with a different liquid. The initial temperature of this liquid is 90°C . After 19 minutes its temperature is 0°C .

- (iii) Without any further calculation, explain what you can deduce about the value of k in this case. [1]

- 10 (i) Use algebraic division to express $\frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \frac{Cx + D}{x^2 - x - 6}$, where A , B , C and D are constants. [4]

- (ii) Hence find $\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6} dx$, giving your answer in the form $a + \ln b$. [7]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



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