

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 Express $\frac{5x}{(x-1)(x^2+4)}$ in partial fractions. [5]
- 2 The equation of a curve is $y = \frac{x^2-3}{x-1}$.
- (i) Find the equations of the asymptotes of the curve. [3]
- (ii) Write down the coordinates of the points where the curve cuts the axes. [1]
- (iii) Show that the curve has no stationary points. [3]
- (iv) Sketch the curve and the asymptotes. [3]
- 3 By first expressing $\cosh x$ and $\sinh x$ in terms of exponentials, solve the equation
- $$3 \cosh x - 4 \sinh x = 7,$$
- giving your answer in an exact logarithmic form. [6]
- 4 You are given that $I_n = \int_0^1 x^n e^{2x} dx$ for $n \geq 0$.
- (i) Show that $I_n = \frac{1}{2}e^2 - \frac{1}{2}nI_{n-1}$ for $n \geq 1$. [4]
- (ii) Find I_3 in terms of e. [4]
- 5 You are given that $f(x) = e^{-x} \sin x$.
- (i) Find $f(0)$ and $f'(0)$. [3]
- (ii) Show that $f''(x) = -2f'(x) - 2f(x)$ and hence, or otherwise, find $f''(0)$. [4]
- (iii) Find a similar expression for $f'''(x)$ and hence, or otherwise, find $f'''(0)$. [2]
- (iv) Find the Maclaurin series for $f(x)$ up to and including the term in x^3 . [2]

6 By first completing the square, find $\int_0^1 \frac{1}{\sqrt{x^2 + 4x + 8}} dx$, giving your answer in an exact logarithmic form. [6]

7 A curve has polar equation $r = 5 \sin 2\theta$ for $0 \leq \theta \leq \frac{1}{2}\pi$.

(i) Sketch the curve, indicating the line of symmetry and stating the polar coordinates of the point P on the curve which is furthest away from the pole. [4]

(ii) Calculate the area enclosed by the curve. [3]

(iii) Find the cartesian equation of the tangent to the curve at P . [3]

(iv) Show that a cartesian equation of the curve is $(x^2 + y^2)^3 = (10xy)^2$. [3]

8 It is required to solve the equation $\ln(x - 1) - x + 3 = 0$.

You are given that there are two roots, α and β , where $1.1 < \alpha < 1.2$ and $4.1 < \beta < 4.2$.

(i) The root β can be found using the iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3.$$

(a) Using this iterative formula with $x_1 = 4.15$, find β correct to 3 decimal places. Show all your working. [2]

(b) Explain with the aid of a sketch why this iterative formula will not converge to α whatever initial value is taken. [3]

(ii) (a) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{3 - 2x_n - (x_n - 1)\ln(x_n - 1)}{2 - x_n}. \quad [5]$$

(b) Use this formula with $x_1 = 1.2$ to find α correct to 3 decimal places. [3]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



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