

GCSE (9–1)

Teacher Guide

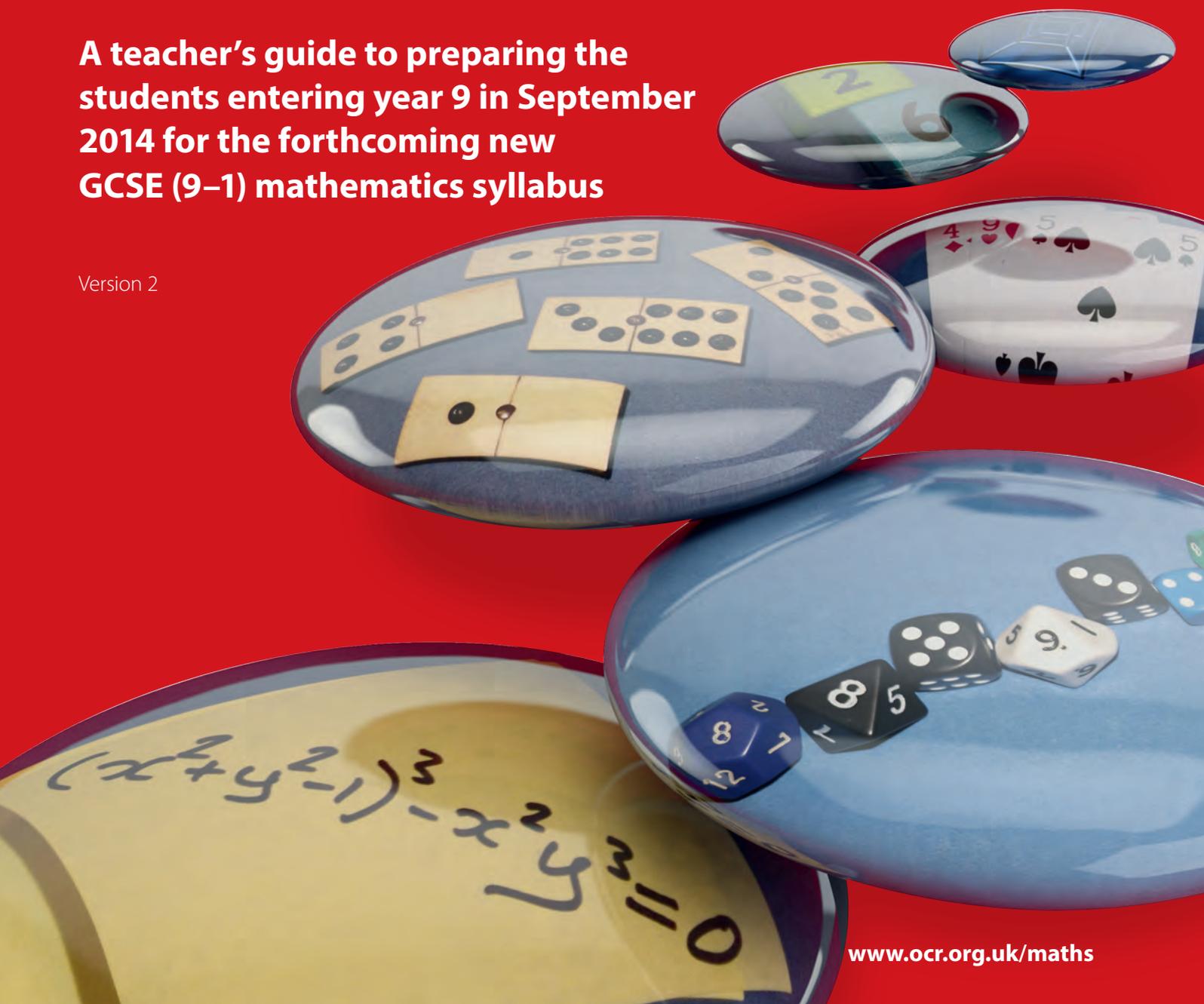
MATHEMATICS

J560

For first teaching in 2015

**A teacher's guide to preparing the
students entering year 9 in September
2014 for the forthcoming new
GCSE (9–1) mathematics syllabus**

Version 2



A TEACHER'S GUIDE TO PREPARING THE STUDENTS ENTERING YEAR 9 IN SEPTEMBER 2014 FOR THE FORTHCOMING NEW GCSE (9–1) MATHEMATICS SYLLABUS

INTRODUCTION

How can we best prepare students entering year 9 in September 2014 for the new GCSE mathematics (first teaching from September 2015), given that they will not be following the new key stage 3 programme of study? How do we need to change our teaching to accommodate the changes in key stages 3 and 4?

The new national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Levels of attainment have disappeared altogether and there is now a list of content that needs to be covered over key stage 3. Progression across the key stage will now be decided not only by the understanding of the concepts studied, but by the ability to solve rich and sophisticated problems.

There are some key changes to the structure of the new GCSE mathematics that will affect this year group and these should be taken into consideration for these students from September 2014, ahead of September 2015.

- The amount of content has increased, particularly for Foundation tier.
- The assessment objectives have changed, putting an increased emphasis on reasoning and problem solving skills. At Foundation, AO1 (covering procedural skill and recall) is weighted at 50% and for Higher at 40%.

- Students will now be expected to know many more common mathematical formulae; only a small number can be given to them in the exam.
- The expectation is now that a larger proportion of the content will be common to both tiers and will be examined at both tiers. Some topics that were previously only Higher tier content are now to be set at the Foundation tier as well and there are also new topics in the Higher tier content.
- Much of the key stage 3 programme of study now appears explicitly as part of the GCSE content, so that all students can be assessed on this content at an appropriate level of demand.

There are similar key changes to the key stage 3 programme of study. Many topics that were previously part of the level 8 and exceptional performance criteria have become part of the core of key stage 3, along with some new elements.

- Distinguish between exact representations of roots and their decimal approximations.
- Interpret and compare numbers in standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or zero.
- Use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a \leq x < b$.
- Appreciate the infinite nature of the sets of integers, real and rational numbers.
- Model situations or procedures by translating them into algebraic expressions or formulae and by using graphs.
- Quadratic graphs.
- Find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs.
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics.
- Solve problems involving direct and inverse proportion, including graphical and algebraic representations.
- Use compound units such as speed, unit pricing and density to solve problems.
- Identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids.



- Use Pythagoras' theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles.
- Use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D.
- Enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams.

This key stage 3 content is included in the new GCSE content, and so the expectation is that if it has been taught well and the students have a good understanding there will be no need to cover this again, rather that the GCSE outcomes may build upon the existing subject knowledge. This will allow for increased use of problem solving, reasoning and proof across GCSE and for the new Higher tier content for GCSE. Understandably, there will be a case for weaker students to revisit much of the key stage 3 content in further detail to ensure that they have a solid knowledge base on which to build the key stage 4 content.

An overview of GCSE Content for **all students**:

- Calculations with pi
- Use of standard form
- Rounding to a given number of significant figures
- Expansion of pairs of brackets
- Factorising simple quadratics
- Sketch simple quadratic, cubic and reciprocal graphs
- Derive simultaneous equations from given situations
- Solve simultaneous equations
- Calculate with mass, density and volume
- Percentage change and reverse percentage change
- Use direct and inverse proportion
- Compound interest
- Similar shapes and side lengths
- Congruence of triangles
- Surface area and volume of spheres, cones, pyramids
- Arc length and sector area
- Trigonometry
- Tree diagrams for probability
- Properties of population from sampling
- Find the equation of a line through 2 points
- Write a ratio as a linear function
- Growth and decay problems
- Write inequalities for error intervals
- Derive the sum of angles in a triangle
- Know exact sin, cos, tan values for key angles

- Use Venn diagrams
- Solve geometrical problems using vectors
- Understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors.

Some of these topics will obviously need careful handling for students targeting grades 1-3 (of the new GCSE grading system), and it would be sensible for teachers to cover these topics using only the simplest of examples to make them accessible for these students.

New GCSE content for Higher tier students only includes:

- Equations of circles
- Equations of tangents to circles
- Iteration
- Quadratic inequalities
- Proof of standard circle theorems
- Interpret areas under graphs and gradients of graphs in real life contexts
- Using vectors in geometric proof
- Recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number greater than 0 or a surd) and other sequences
- Deduce expressions to calculate the n th term of quadratic sequences
- Interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- Interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts.

There are mapped resources to the different areas of the curriculum and the different key stages available at [kangaroo maths](#), [NRICH](#) and [NCETM](#).

Relevant issues for teaching

The key issue must be to ensure that all students have access to any areas of the curriculum that may be new to key stage 3 for them, in order to prepare for the increased content of GCSE and the new way of working. Teachers need to prepare students appropriately for the new aims and should include the beginnings of proof and increased use of problem solving and reasoning as early as possible within the next year.



There are many ways of including these within the normal lesson structure and a variety of pedagogical approaches are appropriate here.

It would make sense to begin by encouraging students to justify their answers, particularly when considering geometrical areas of the curriculum. This enables the students to reason their approach using key facts (e.g. the angles on a straight line) and ad hoc arguments, rather than starting with anything more formal. If this can become part of routine classroom practice at key stage 3, this aspect of the GCSE will become a fairly straightforward continuation.

Other approaches could include:

- giving students examples with answers and asking them to explain the approach taken
- discussions of different approaches that could be taken, with further discussion as to why there are different approaches to a problem
- considering examples with incorrect or incomplete answers for critique and improvement, particularly where there is an opportunity to explore misconceptions. It would also be useful to look at examples where students have arrived at the correct answer, but have used a completely incorrect method.

Proof at key stage 3

There are many topics at key stage 3 that are suitable for exploring the simple beginnings of mathematical proof, but it is important to ensure that students are aware of the need for proof in mathematics and the meaning of mathematical proof as opposed to other forms.

Starting with concrete, physical demonstrations can be useful for setting the context in which a general argument or proof is going to happen. Simple geometric examples include: angles in a triangle (through drawing, cutting and sticking onto a straight line for example) and Pythagoras' theorem ([Perigal's dissection](#) or a [2 minute proof](#)). Numerical examples include the proof that the sum of 3 consecutive even numbers is always a multiple of 6, where an exploration of examples might lead to a discussion about why a general proof is needed. There are many useful examples to be found on [Suffolk maths](#).

There is also a very useful selection of materials from the [MEP](#); this [PDE](#) has a nice explanation of the use of proof and plenty of examples of word based proof for students to follow.

Problem Solving and Reasoning at key stage 3

There are many excellent resources available for problem solving and reasoning at key stage 3, such as at [NRICH](#). With the changes to the programme of study and the new curriculum aims, it is important that this now becomes embedded within day to day teaching.

The GCSE content requires students to be guided towards a wider application with increased sophistication. There is potentially a substantial leap in demand for weaker students who may at key stage 3 have only worked with routine problems, so it is important to introduce some of these higher order skills slowly and in familiar contexts.





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