Thursday 6 June 2013 – Morning

FSMQ INTERMEDIATE LEVEL

6989/01 Foundations of Advanced Mathematics (MEI)

Candidates answer on the Answer Sheet.

OCR supplied materials:
• Answer Sheet (MS4)

Other materials required:
• Eraser
• Scientific calculator
• Soft pencil
• Ruler

INSTRUCTIONS TO CANDIDATES

• Write your name clearly in capital letters, your centre number and candidate number on the Answer Sheet in the spaces provided unless this has already been done for you.
• Read each question carefully. Make sure you know what you have to do before starting your answer.
• Do not write in the bar codes.
• There are forty questions in this paper. Attempt as many questions as possible. For each question there are four possible answers, A, B, C and D. Choose the one you consider correct and record your choice in soft pencil on the separate Answer Sheet.
• Read very carefully the instructions on the Answer Sheet.

INFORMATION FOR CANDIDATES

• Each correct answer will score one mark. A mark will not be deducted for a wrong answer.
• This document consists of 24 pages. Any blank pages are indicated.
Area of trapezium = $\frac{1}{2} (a + b)h$

Volume of prism = (area of cross-section) × length

In any triangle $ABC$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

Volume of sphere = $\frac{4}{3}\pi r^3$
Surface area of sphere = $4\pi r^2$

Volume of cone = $\frac{1}{3}\pi r^2 h$
Curved surface area of cone = $\pi rl$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$
1  Look at this list of numbers.

\[
\begin{array}{ccccccc}
4 & 5 & 29 & 35 & 63 & 81 & 84 \\
\end{array}
\]

Three of the following statements are true and one is false. Which one is false?

A  There are exactly 2 prime numbers in the list.
B  There are exactly 2 square numbers in the list.
C  There are exactly 2 multiples of 7 in the list.
D  There are exactly 2 factors of 20 in the list.

2  Three of the following statements are true and one is false. Which one is false?

A  \((-7) \times (+2) = -14\)
B  \(\frac{-20}{-4} = -5\)
C  \(3 + 4 \times 5 = 23\)
D  \(8 - (2 - 3) = 9\)

3  Three of the following statements are true and one is false. Which one is false?

A  \(2^7 \times 3^7 = 6^7\)
B  \(5^9 \div 5^6 = 5^3\)
C  \(9^3 = 3^9\)
D  \(\sqrt[3]{32} = 2\)
4 Which one of the following expressions has the greatest value?

A $7 \frac{1}{10} - 1 \frac{3}{10}$
B $2 \frac{1}{2} + 3 \frac{1}{2}$
C $2 \frac{1}{2} \times 2 \frac{1}{4}$
D $18 ÷ 3 \frac{1}{3}$

5 In a group of 60 students 40 are female. One quarter of the students are aged under 18; 12 students are over 25.

Three of the following statements are true and one is false. Which one is false?

A The ratio of female students : male students is 2:1.
B $\frac{1}{3}$ of the students are over 25.
C There are 27 students aged between 18 and 25.
D 25% of the students are under 18.
6 Three of the following statements are true and one is false. Which one is false?

A \[0.005 = 5 \times 10^{-3}\]

B \[500000 = 5 \times 10^5\]

C \[5 \times 10^5 + 6 \times 10^5 = 1.1 \times 10^5\]

D \[(6 \times 10^3)^2 = 3.6 \times 10^7\]

7 Three of the following statements involve sensible units and one does not. In which statement are the units not sensible?

A The distance between London and Edinburgh is measured in kilometres.

B The amount of sugar given in a recipe for a cake is measured in grams.

C The amount of water in a spoon is measured in litres.

D The length of a pencil is measured in centimetres.

8 A runner completes a half marathon in 58 minutes 23 seconds. The distance is \(13 \frac{7}{64}\) miles.

Which one of the following is the correct average speed in miles per hour for this runner, correct to 4 decimal places?

A 13.0083

B 13.4718

C 13.4724

D 13.5079
Three of the following statements are true and one is false. Which one is false?

A 1.234 can be written 1.2, correct to 3 significant figures.
B 56.49 can be written 56, correct to the nearest whole number.
C 0.7654 can be written 0.765, correct to 4 significant figures.
D 34.651 01 can be written 34.65, correct to 2 decimal places.

Philippa makes the following statements about three sequences.

• The \(n\)th term of the sequence 3, 5, 7, 9, 11, .... is \(2n + 1\).
• The \(n\)th term of the sequence 3, 6, 12, 24, 48, .... is \(3 \times 2^{n-1}\).
• The \(n\)th term of the sequence 3, 7, 13, 21, 31, .... is \(2n^2 - 2n + 3\).

How many of the statements are true?

A 0  B 1  C 2  D 3

A curve has equation \(y = x^3 + x^2 - 3x + 4\).

Three of the following points lie on the curve and one does not. Which one does not?

A \((-3, -5)\)  B \((-1, 5)\)  C \((1, 3)\)  D \((3, 31)\)
12 Three of the following statements are true and one is false. Which one is false?

A \( \frac{1}{2x^2} = 2x^{-2} \)
B \( \frac{6x^3}{2x^2} = 3x \)
C \( \frac{1}{2}x^3 \times 4x^{-3} = 2 \)
D \( (2x^2)^3 = 8x^6 \)

13 Three of the following quadratic expressions factorise in the form \((x - 2)(x + a)\) where \(a\) is an integer, and one does not. Which one does not?

A \( x^2 + x - 6 \)
B \( x^2 - 5x + 6 \)
C \( x^2 + 3x - 10 \)
D \( x^2 - x - 6 \)
14  Three of the following statements are true and one is false. Which one is false?

A  The solution of the equation \(5x - 8 = 9\) is \(x = 3.4\).

B  The solution of the equation \(\frac{2}{3x} = 1\) is \(x = \frac{2}{3}\).

C  The solution of the equation \(2(x + 3) = 3(2x - 1)\) is \(x = 2\frac{1}{7}\).

D  The solution of the equation \(x = 1 - 2(x + 1)\) is \(x = -\frac{1}{2}\).

15  Which one of the following is the correct solution of this pair of simultaneous equations?

\[
\begin{align*}
5x - y &= 1 \\
3x + 2y &= 11
\end{align*}
\]

A  \(x = 2, \ y = 9\)

B  \(x = \frac{9}{7}, \ y = \frac{38}{7}\)

C  \(x = 1, \ y = 4\)

D  \(x = \frac{59}{45}, \ y = \frac{52}{7}\)

16  Which one of the following is the correct solution of the inequality \(2x - 3 > 4x + 7\)?

A  \(x > 5\)  

B  \(x < 2\)  

C  \(x > -2\)  

D  \(x < -5\)
17 Eric and Fatima are doing some algebra.

Eric multiplies out two brackets as follows.

\[(x + 1)(y - 2) = x(y - 2) + 1(y - 2) = xy - 2x + y - 2\]

Fatima factorises an expression as follows.

\[2xy + y + 6x + 3 = y(2x + 1) + 3(2x + 1) = (2x + 1)(y + 3)\]

Which one of the following statements is true?

A Eric and Fatima are both correct.
B Eric is incorrect, but Fatima is correct.
C Eric is correct, but Fatima is incorrect.
D Eric and Fatima are both incorrect.

18 The cooking instructions for a joint of meat are as follows.

*Cook for \(\frac{1}{2}\) an hour per kilogram plus 20 minutes.*

In the formulae below, \(T\) is the cooking time in minutes and \(m\) is the mass of the meat in kilograms.

Which one of the following is a correct formula for \(T\)?

A \(T = 30m + 20\)
B \(T = 30(m + 20)\)
C \(T = \frac{1}{2}(m + 20)\)
D \(T = \frac{m + 40}{2}\)
This map, on a two centimetre square grid, shows a rectangular village green. The church, pub, shop and bus stop are located at the four corners of the village green, as shown.

The scale of the map is such that 2 cm represents 40 m.

Three of the following statements are true and one is false. Which one is false?

A  The scale is 1 : 200.

B  The area of the village green is $19200 \text{ m}^2$.

C  The distance from the pub to the bus stop is 200 m.

D  The actual perimeter of the green is 560 m.
20 Two ordinary fair dice are rolled and the score is found by adding the two numbers showing.

Three of the following statements are true and one is false. Which one is false?

A There are eleven different possible scores.

B The probability that the score is more than 6 is $\frac{1}{2}$.

C The probability that the score is 11 is the same as the probability that the score is 3.

D The probability that the score is 9 is $\frac{1}{3}$.

21 Betty is using the formula $t = \frac{v - u}{a}$.

She is given the values $u = 7.9$, $v = 22.6$ and $a = 1.9$.

Three of the following statements are true and one is false. Which one is false?

A If the given values are exact, $t = 18.4$, correct to 3 significant figures.

B If Betty rounds the given values to the nearest whole number, then the calculation gives $t = 7.5$.

C If the given values have been rounded to 1 decimal place, then the value of $t$ cannot be greater than 8.

D A rearrangement of the formula is $v = u + at$. 
A robot is negotiating a series of routes as shown in the diagram. At each of the five junctions the probability that it takes the left fork is \( \frac{2}{3} \). Otherwise it takes the right fork.

Three of the following statements are true and one is false. Which one is false?

A  The probability that the robot arrives at P is \( \frac{8}{27} \).
B  The probability that the robot arrives at Q is \( \frac{8}{27} \).
C  The probability that the robot arrives at R is \( \frac{8}{27} \).
D  The probability that the robot arrives at S is \( \frac{8}{27} \).
23 Which **one** of the graphs shown below is the graph of \( y = 5 \sin x + 2 \)?

A

B

C

D
John and Paula are asked to give an approximate value for \( n \) where
\[
    n = \frac{89.2 \times 11.1}{0.84 \times 0.79}.
\]
In order to do this, John makes the following approximations:
89.2 to 100, 11.1 to 10, 0.84 to 1 and 0.79 to 1.

Paula makes the following approximations:
89.2 to 90, 11.1 to 10, 0.84 to 0.9 and 0.79 to 0.8.

Three of the following statements are true and one is false. Which one is false?

A. John has rounded the numbers to one significant figure correctly.
B. John’s calculation gives the value of \( n \) correct to one significant figure.
C. Paula’s calculation gives the value of \( n \) as 1250.
D. Paula’s value of \( n \) is closer than John’s to the exact answer.

Asif measures the length of his bookshelf to be 110 cm, correct to the nearest cm. He has a set of identical books, each of which has thickness 25 mm, correct to the nearest mm.

Three of the following statements are true and one is false. Which one is false?

A. The thickness of each book is no more than 25.5 mm.
B. The length of the shelf is at least 1095 mm.
C. It may be possible to fit 45 books on the shelf.
D. It is definitely possible to fit 43 books on the shelf.
26 The triangle PQR shown represents the cross-section of a roof of a house.

[Diagram of triangle PQR with dimensions and labels]

S is the foot of the perpendicular from the top of the roof, P.
The height of the roof, PS, is 3 metres.
The length of PR is 8 metres.
The length of QS is 4 metres.

Three of the following statements are true and one is false. Which one is false?

A  PQ = 5 m
B  \( \tan PQS = 0.75 \)
C  \( \cos PRS = 0.375 \)
D  Angle RPS = 68°, correct to the nearest degree.

27 The quadratic equation \( x^2 - 3x - 7 = 0 \) has two roots.

Which one of the following is the correct description of the roots?

A  Both roots are positive.
B  Both roots are whole numbers.
C  The roots are equal.
D  One root is positive and one root is negative.
In the triangle shown, $BC = 5\, \text{cm}$, Angle $C = 40^\circ$ and angle $A = 55^\circ$.

Which one of the following is the correct length for AB?

A. $6.081\, \text{cm}$, correct to 3 decimal places.
B. $3.923\, \text{cm}$, correct to 3 decimal places.
C. $4.195\, \text{cm}$, correct to 3 decimal places.
D. $6.372\, \text{cm}$, correct to 3 decimal places.

Which one of the following is a correct simplification of $\frac{2x + 1}{8} - \frac{3x - 1}{12}$?

A. $\frac{2 - x}{4}$  
B. $\frac{1}{24}$  
C. $\frac{1 - x}{4}$  
D. $\frac{5}{24}$
A pyramid $VABCD$ has its vertex, $V$, 10 cm directly above the centre of a rectangular base $ABCD$, as shown. 
$AB = CD = 8$ cm. $BC = DA = 5$ cm.

Three of the following statements are true and one is false. Which one is false?

A  $VA = 12.02$ cm, correct to 2 decimal places.

B  The angle between $VA$ and the base is $64.7^\circ$, correct to 1 decimal place.

C  The angles that the edges $VA$, $VB$, $VC$ and $VD$ make with the base are all the same.

D  Angle $AVC = 50.5^\circ$, correct to 1 decimal place.
31 Natalie and Philip decide to take a sample of students from their year group in order to carry out an investigation.

Natalie writes each name on a separate sheet of paper, puts them in a hat and draws out 10. She claims that she has produced a random sample.

Philip chooses the first 10 names from the alphabetical list of the year group. He claims that he has produced a random sample.

Which one of the following is a correct statement?

A Natalie and Philip are both correct.

B Natalie is correct, but Philip is incorrect.

C Natalie is incorrect, but Philip is correct.

D Natalie and Philip are both incorrect.

32 Three of the following statements are true and one is false. Which one is false?

A The line $3x + 4y = 5$ has a gradient of $-0.75$.

B The line $y = 5x + 6$ has a $y$-intercept of 6.

C The lines $2y = x + 4$ and $4y - 2x = 5$ are parallel.

D The line $\frac{x}{3} + \frac{y}{4} = 1$ passes through the origin.
The graph of \( y = x^3 + 3x^2 + x - 1 \) is drawn on the grid below.

Three of the following statements are true and one is false. Which one is false?

A The equation \( x^3 + 3x^2 + x - 1 = 0 \) has two negative roots, one of which is an integer.

B There are two points on the curve \( y = x^3 + 3x^2 + x - 1 \) at which the gradient is zero.

C The equation \( x^3 + 3x^2 + x - 1 = 6 \) has only one root.

D The gradient of the curve is negative for \( x > 1 \).
The graph below shows the speed of a train when travelling from one station to the next.

Three of the following statements are true and one is false. Which one is false?

A. Between 50 and 150 seconds the train is travelling at constant speed.

B. During the first 50 seconds the train is accelerating at 0.4 m s\(^{-2}\).

C. The distance between the two stations is 3.5 km.

D. The average speed is 10 m s\(^{-1}\).
Imran is flying a small aircraft at a constant speed of 120 km h\(^{-1}\). There is a constant wind of speed 50 km h\(^{-1}\) from the West.

Three of the following statements are true and one is false. Which one is false?

A If Imran steers due North, the aircraft actually travels on a bearing of 023°, correct to the nearest degree.

B If Imran steers due North, the speed over the ground of the aircraft is 130 km h\(^{-1}\).

C In order to fly due North, Imran must steer on a bearing of 337°, correct to the nearest degree.

D If Imran steers a course to fly due North, he will take more than 2 hours to travel 240 km.

Two vectors are given by \( \mathbf{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \) and \( \mathbf{b} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \).

Which one of the following is vector \( 2\mathbf{a} - \mathbf{b} \)?

A \( \begin{pmatrix} 6 \\ 8 \end{pmatrix} \)  
B \( \begin{pmatrix} 6 \\ 6 \end{pmatrix} \)  
C \( \begin{pmatrix} 6 \\ -6 \end{pmatrix} \)  
D \( \begin{pmatrix} 6 \\ -5 \end{pmatrix} \)
37 In a certain shop, all prices of goods for sale were increased by 20% on June 1st. A particular item was on sale on June 1st for £180.

Which one of the following was the correct price of this item for sale on May 31st?

A £144  B £150  C £171  D £176.47

38 The table shows the money earned each week by a group of students.

<table>
<thead>
<tr>
<th>Money earned (£x)</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ≤ x &lt; 10</td>
<td>20</td>
</tr>
<tr>
<td>10 ≤ x &lt; 20</td>
<td>17</td>
</tr>
<tr>
<td>20 ≤ x &lt; 30</td>
<td>10</td>
</tr>
<tr>
<td>30 ≤ x &lt; 40</td>
<td>7</td>
</tr>
<tr>
<td>40 ≤ x &lt; 50</td>
<td>3</td>
</tr>
</tbody>
</table>

Kylie draws this histogram to represent the data.

The first block, representing 0 ≤ x < 10, has been drawn correctly. Some of the others have been drawn incorrectly.

How many of the others have been drawn incorrectly?

A 1  B 2  C 3  D 4
Four students are asked to solve the following simultaneous equations graphically.

\[ y = 2x + 1 \]
\[ x + 2y = 12 \]

The four students draw different diagrams, labelled A, B, C and D as shown below.

Which one of the following diagrams can be used to find the correct solution of the simultaneous equations?
Liam travels to work each day by car. He records the length of time it takes him over a period of days and displays his results on a cumulative frequency graph, as shown below.

Three of the following statements are true and one is false. Which one is false?

A  Liam records 50 journeys.
B  The range of times is 55 minutes.
C  The median for these data is approximately 39 minutes.
D  The interquartile range for these data is approximately 6 minutes.