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LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/02 Paper 2

INSERT

Duration: 1 hour 30 minutes

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Electrical power from an overshot waterwheel

Introduction

In an old mill house an overshot waterwheel, whose operation is described below, originally powered the milling equipment. Today the waterwheel is still in place and operational, but the milling mechanism and the old wooden gearing have been removed. The owner of the mill has installed new gearing and other equipment designed to provide heating and lighting for a henhouse. A schematic diagram of the equipment is shown in Fig. 1.

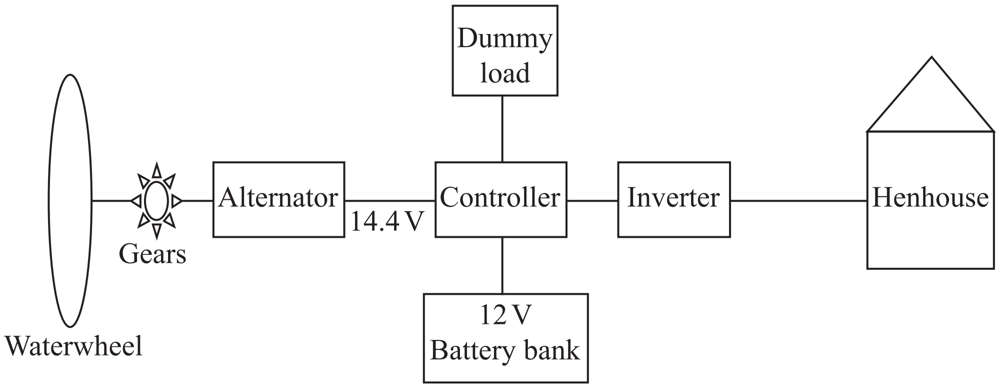
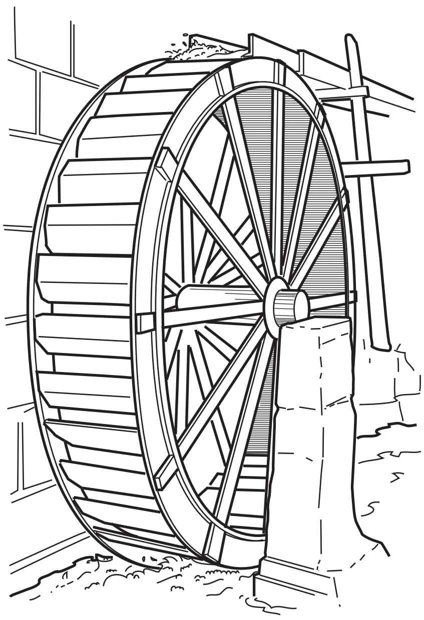


Fig. 1



The axle of the waterwheel is connected to new gearing, which causes the armature of the alternator to rotate. As the armature rotates it produces an output voltage of 14.4 V. The battery bank is used to store power for later use. The controller ensures that the battery bank is never overcharged and is never drained completely. Connected to the controller is a dummy load that dissipates any excess power not required for charging the battery bank or required by the equipment in the henhouse. The inverter converts the output of the alternator and battery bank to an AC supply to power the equipment in the henhouse.

At certain periods in the day there is hardly any load requirement from the henhouse in which case most of the alternator output power can be used to charge the battery bank. At other periods in the day the henhouse may require all the alternator output power to satisfy its requirement. In addition, power may be drawn at the same time from the battery bank.

Operation of an overshot waterwheel

Fig. 2 is a cross-sectional diagram of an overshot waterwheel rotating clockwise about its axle. Around the waterwheel there are a number of specially shaped buckets, which are used to collect water as it falls from a chute directly above. As each bucket is filled, the water collected creates a downward force causing the waterwheel to turn about its axle. As the waterwheel turns, water spills over the lips of the buckets until the buckets reach a position where all the water has escaped. While the waterwheel keeps turning, the empty buckets on the rising side of the waterwheel eventually return to the top and collect further water before their next downward journey.

Large waterwheels are several metres in diameter and rotate at typical angular speeds of between four and eight revolutions per minute depending on the volume of water flowing, the size of the waterwheel and the resistance to its motion.

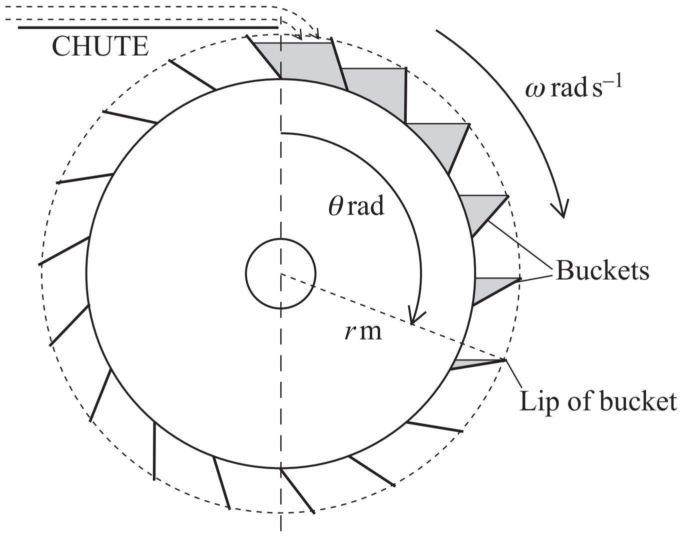


Fig. 2

Calculating the power of an overshot waterwheel

The power that would be available due to the potential energy of the water falling a vertical distance of $2r$ from the top to the bottom of the waterwheel is approximately P_P W where

$$P_P = 2rfg \qquad \textbf{(1)}$$

and where

r m is the radius of the waterwheel measured from the axle of the waterwheel to the lip of a bucket,
f kg s^{-1} is the rate at which water is delivered at the top of the waterwheel,
g m s^{-2} is the acceleration due to gravity.

In practice this power can never be fully utilised because water escapes from the buckets as the waterwheel turns.

A better way of estimating the power generated by a waterwheel is to consider the torque created by the waterwheel as it turns about its axle. Referring to Fig. 2 assume that there are n buckets around the periphery of the waterwheel, all of which have a sufficient size to hold all water flowing from the chute. Assume also that the angular speed of the waterwheel is $\omega \, \mathrm{rad\,s^{-1}}$. The time, in seconds, that each bucket will be near the top of the waterwheel and in a position to collect water is

$$\frac{2\pi}{\omega n} = \frac{\Delta\theta}{\omega}$$

where

$$\Delta\theta = \frac{2\pi}{n}. \qquad \textbf{(2)}$$

$\Delta\theta$ is the angle through which the waterwheel turns while a single bucket receives water from the chute.

The mass of water collected by each bucket is therefore

$$\frac{\Delta\theta}{\omega} f.$$

It is given that the torque, τ_{B} N m, about the axle of the waterwheel due to the water in each bucket as it descends is approximately

$$\tau_{\mathrm{B}} = \left(\mathrm{X}(\theta)\frac{\Delta\theta}{\omega} f\right) gr\sin(\theta) = \frac{fgr}{\omega}\mathrm{X}(\theta)\sin(\theta)\Delta\theta \qquad \textbf{(3)}$$

where θ is the angle, measured clockwise, between the vertical centre line of the waterwheel and the lip of the bucket (see Fig. 2),

$\mathrm{X}(\theta)$ is a value between 1 and 0 that represents the proportion of water that remains in the bucket for angle θ.

In order to simplify the calculations, the distance between the centre of the waterwheel and the centre of mass of the water in each bucket has been taken to be r as defined in equation (1). In practice, this distance depends upon the distribution of water in each bucket and therefore the value of θ.

The total driving torque, τ_{D}, about the axle of the waterwheel is the sum of the torques created by the water in each bucket on their downward travel and is given by

$$\tau_{\mathrm{D}} \approx \frac{fgr}{\omega}\sum_{j=1}^{\frac{1}{2}n}\mathrm{X}\left(\frac{2\pi}{n}j\right)\sin\left(\frac{2\pi}{n}j\right)\Delta\theta. \qquad \textbf{(4)}$$

Since *power* = (*angular velocity*) × (*torque*),
the power, P, generated by the water in the waterwheel is

$$P \approx fgr\sum_{j=1}^{\frac{1}{2}n}\mathrm{X}\left(\frac{2\pi}{n}j\right)\sin\left(\frac{2\pi}{n}j\right)\Delta\theta. \qquad \textbf{(5)}$$

As long as n is not too small this sum may be reasonably approximated by an integral, giving

$$P \approx fgr\int_0^{\pi} \mathrm{X}(\theta)\sin(\theta)\,\mathrm{d}\theta. \qquad \textbf{(6)}$$

The efficiency, η, of this waterwheel is

$$\eta = \frac{P_{\mathrm{O}}}{P_{\mathrm{P}}}$$

where P_{O} is the output power of the alternator and P_{P} is the potential power of the descending water, as defined by equation (1). The efficiency takes account of the losses within the waterwheel itself, the losses due to friction in the bearings and gearing and the mechanical and electrical losses in the alternator. Further losses will also occur in the inverter. Normally, efficiency is expressed as a percentage.

<u>Changing torque</u>

The equation of motion for a rotating wheel is

$$J\alpha = J\frac{\mathrm{d}\omega}{\mathrm{d}t} = \tau_{\mathrm{T}} = \tau_{\mathrm{D}} - \tau_{\mathrm{L}}$$

where

$J\,\mathrm{kg\,m^2}$ is the moment of inertia of the wheel,

$\alpha\,\mathrm{rad\,s^{-2}}$ is the angular acceleration of the wheel,

$\omega\,\mathrm{rad\,s^{-1}}$ is the angular speed of the wheel,

$t\,\mathrm{s}$ is time,

$\tau_{\mathrm{T}}\,\mathrm{N\,m}$ is the net torque applied about the axle of the wheel,

$\tau_{\mathrm{D}}\,\mathrm{N\,m}$ is the driving torque,

$\tau_{\mathrm{L}}\,\mathrm{N\,m}$ is the load (retarding) torque created by friction and the alternator.

For constant angular speed

$$\tau_{\mathrm{D}} = \tau_{\mathrm{L}}.$$

If the rate at which the water is flowing changes there will be a corresponding change in the driving torque causing a change in the waterwheel's angular speed. The change in angular speed will also have an influence on the load torque. Assume that the initial angular speed is $\omega_1\,\mathrm{rad\,s^{-1}}$ and that after $t\,\mathrm{s}$ the angular speed has changed to $\omega_2\,\mathrm{rad\,s^{-1}}$. For constant angular acceleration, $\alpha\,\mathrm{rad\,s^{-2}}$, the following standard equations for rotational motion apply

$$\omega_2 = \omega_1 + \alpha t$$

$$\theta_t = \left(\frac{\omega_1 + \omega_2}{2}\right)t$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta_t$$

where θ_t rad is the angular displacement that has occurred during time $t\,\mathrm{s}$.

Battery charging and discharging

While the alternator is rotating it produces an output voltage of 14.4 V. The alternator provides power for the henhouse when needed. In addition it charges the battery bank maintaining 14.4 V across its terminals. When the combined power required by the henhouse and the battery bank is less than that provided by the alternator, excess power is drained by the dummy load.

Manufacturers of batteries specify battery capacity in terms of ampere hours (Ah). A capacity of C_H is the total number of ampere hours available when discharging completely with a constant current over a period H hours. The value of H is known as the rating; typically $H = 20$. A capacity, C_{20}, of 100 Ah, for example, means that the battery would be discharged completely with a constant current of 5 A in 20 hours. However, when the same battery is discharged with a different constant current its capacity is no longer 100 Ah. It is convenient to know the time taken, T hours, to discharge a battery completely with a constant current of I A. This can be approximated using the known value of C_H in Peukert's equation

$$T = H\left(\frac{C_H}{HI}\right)^n$$

where n is a value dependent upon the particular battery. A typical value for n is 1.3.

Conclusion

Although the benefits of generating electricity from a waterwheel may appear attractive, the amount of power that can be practically realised should not be overestimated. The overall efficiency will typically be little more than about 40% because of losses in the waterwheel itself, the gearing and the other components. Moreover, the maintenance of the system described here will incur costs. Batteries used to store the power do not last indefinitely and will possibly need replacing after about four years depending upon how deeply they are discharged. Other components will also need inspection and servicing, adding to the operating costs.

Refer to the appendix for a list of symbols used in this document together with their meaning.

Appendix

List of symbols and units of measurement.

Symbol	Units	Meaning
I	A	Current
P	W	Power
r	m	Radius of the waterwheel
f	$kg\,s^{-1}$	Water flow
g	$m\,s^{-2}$	Acceleration due to gravity
n		Number of buckets
$\Delta\theta$	rad	Angle occupied by one bucket around the waterwheel
θ	rad	Displacement angle as shown in Fig. 2
ω	$rad\,s^{-1}$	Angular speed
ω_1	$rad\,s^{-1}$	Initial Angular speed
ω_2	$rad\,s^{-1}$	Final Angular speed
τ_B	N m	Torque created by water in one bucket
τ_D	N m	Driving torque
τ_L	N m	Load torque
τ_T	N m	Total torque
η	%	Efficiency
J	$kg\,m^2$	Moment of inertia
α	$rad\,s^{-2}$	Angular acceleration
t	s	Time
C_R	A h	Battery capacity
T	h	Battery discharge time
H	h	Battery rating