

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Express $\frac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions. [5]

2 Find $\int x^8 \ln(3x) dx$. [5]

3 Determine whether the lines whose equations are

$$\mathbf{r} = (1 + 2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3 + 5\lambda)\mathbf{k} \quad \text{and} \quad \mathbf{r} = (\mu - 1)\mathbf{i} + (5 - \mu)\mathbf{j} + (2 - 5\mu)\mathbf{k}$$

are parallel, intersect or are skew. [6]

4 The equation of a curve is $y = \cos 2x + 2 \sin x$. Find $\frac{dy}{dx}$ and hence find the coordinates of the stationary points on the curve for $0 < x < \pi$. [6]

5 (i) Show that $\frac{1}{1 - \tan x} - \frac{1}{1 + \tan x} \equiv \tan 2x$. [2]

(ii) Hence evaluate $\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1 - \tan x} - \frac{1}{1 + \tan x} \right) dx$, giving your answer in the form $a \ln b$. [5]

6 Use the substitution $u = 1 + \ln x$ to find $\int \frac{\ln x}{x(1 + \ln x)^2} dx$. [6]

7 Points $A(2, 2, 5)$, $B(1, -1, -4)$, $C(3, 3, 10)$ and $D(8, 6, 3)$ are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC , and the angle BAC . [4]

(ii) Show that \overrightarrow{AD} is perpendicular to both \overrightarrow{AB} and \overrightarrow{AC} . [3]

(iii) Calculate the volume of the pyramid $ABCD$. [3]

[The volume of the pyramid is $V = \frac{1}{3} \times \text{base area} \times \text{perpendicular height}$.]

8 At time t seconds, the radius of a spherical balloon is r cm. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at 1.08 cm s^{-1} .

(i) Write down a differential equation to model this situation, and solve it to express r in terms of t . [7]

(ii) How much air is in the balloon initially? [2]

[The volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

- 9 A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.
- (i) Find $\frac{dy}{dx}$ in terms of t , simplifying your answer. [3]
- (ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. [4]
- (iii) Find a cartesian equation of the curve. [2]
- 10 (i) Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x . [2]
- (ii) Use this result, together with a suitable value of x , to obtain a decimal estimate of the value of $\frac{100}{729}$. [2]
- (iii) Show that $\frac{x}{(1-x)^3} = -\frac{1}{x^2} \left(1 - \frac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of $\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$. [4]
- (iv) Comment on the suitability of substituting the same value of x as used in part (ii) in the expansion in part (iii) to estimate the value of $\frac{100}{729}$. [1]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.