

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 By using the substitution $t = \tan \frac{1}{2}\theta$, find $\int_0^{\frac{1}{2}\pi} \frac{1}{1 + \cos \theta} d\theta$. [5]

2 (i) Using the definitions for $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x} , show that $\cosh^2 x - \sinh^2 x \equiv 1$. [3]

(ii) Hence solve the equation $\sinh^2 x = 5 \cosh x - 7$, giving your answers in logarithmic form. [5]

3 It is given that $f(x) = \tanh^{-1}\left(\frac{1-x}{3+x}\right)$ for $x > -1$.

(i) Show that $f''(x) = \frac{1}{2(x+1)^2}$. [6]

(ii) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2 . [4]

4 It is given that $I_n = \int_0^{\frac{1}{2}\pi} \cos^n x dx$ for $n \geq 0$.

(i) Show that $I_n = \frac{n-1}{n} I_{n-2}$ for $n \geq 2$. [5]

(ii) Hence find I_{11} as an exact fraction. [3]

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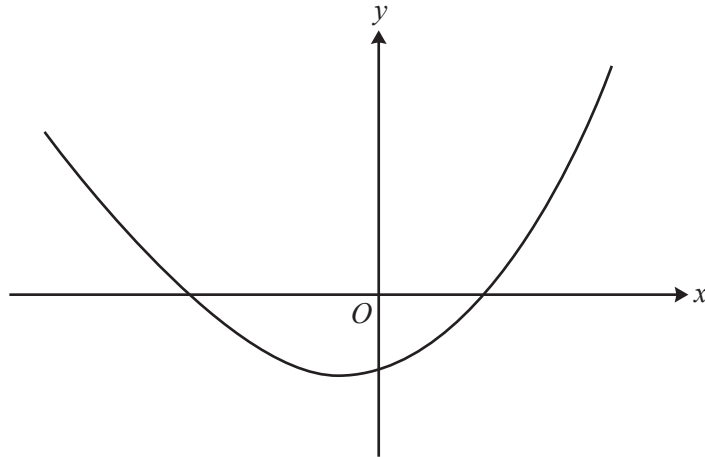
5 You are given that the equation $x^3 + 4x^2 + x - 1 = 0$ has a root, α , where $-1 < \alpha < 0$.

(i) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n + 1}. \quad [3]$$

(ii) Using the initial value $x_1 = -0.7$, find x_2 and x_3 and find α correct to 5 decimal places. [3]

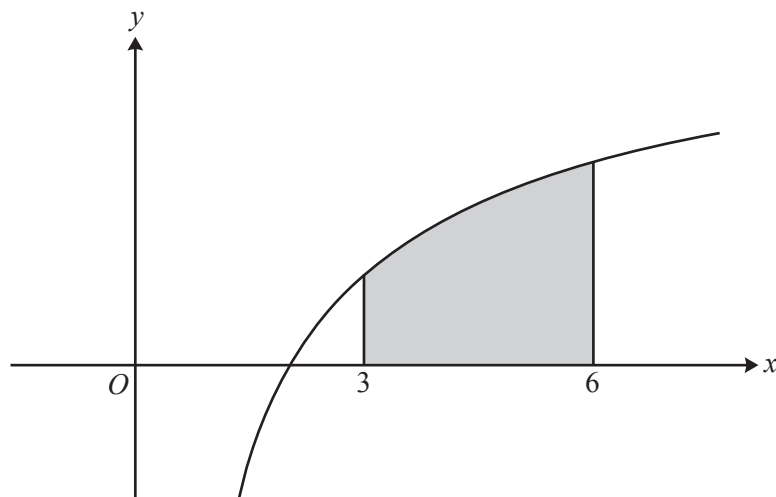
(iii) The diagram shows a sketch of the curve $y = x^3 + 4x^2 + x - 1$ for $-1.5 \leq x \leq 1$.



Using the copy of the diagram in your answer book, explain why the initial value $x_1 = 0$ will fail to find α . [2]

[Questions 6, 7 and 8 are printed overleaf.]

6



The diagram shows part of the curve $y = \ln(\ln(x))$. The region between the curve and the x -axis for $3 \leq x \leq 6$ is shaded.

(i) By considering n rectangles of equal width, show that a lower bound, L , for the area of the shaded region is $\frac{3}{n} \sum_{r=0}^{n-1} \ln\left(\ln\left(3 + \frac{3r}{n}\right)\right)$. [3]

(ii) By considering another set of n rectangles of equal width, find a similar expression for an upper bound, U , for the area of the shaded region. [1]

(iii) Find the least value of n for which $U - L < 0.001$. [4]

7 The equation of a curve is $y = \frac{x^2 + 1}{(x + 1)(x - 7)}$.

(i) Write down the equations of the asymptotes. [3]

(ii) Find the coordinates of the stationary points on the curve. [5]

(iii) Find the coordinates of the point where the curve meets one of its asymptotes. [3]

(iv) Sketch the curve. [3]

8 The equation of a curve is $x^2 + y^2 - x = \sqrt{x^2 + y^2}$.

(i) Find the polar equation of this curve in the form $r = f(\theta)$. [3]

(ii) Sketch the curve. [2]

(iii) The line $x + 2y = 2$ divides the region enclosed by the curve into two parts. Find the ratio of the two areas. [6]