OCR
RECOGNISING ACHIEVEMENT

Monday 24 June 2013 – Afternoon

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A camshaft inside an engine is rotating with angular speed $42\,\text{rad s}^{-1}$. When the throttle is opened the camshaft speeds up with constant angular acceleration, and 8 seconds after the throttle was opened the angular speed is $76\,\text{rad s}^{-1}$.

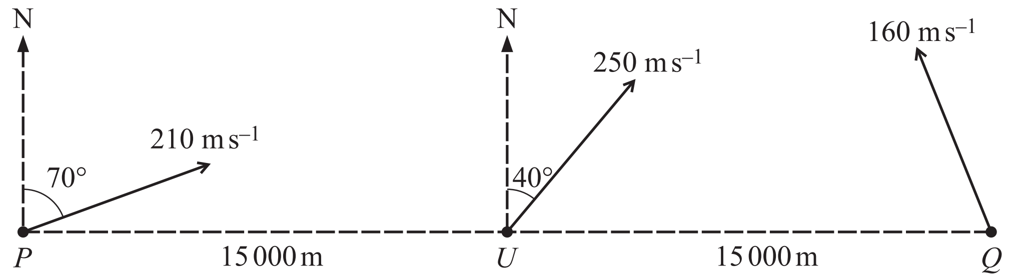
(i) Find the angular acceleration of the camshaft. **[2]**

(ii) Find the time taken for the camshaft to turn through 810 radians from the moment that the throttle was opened. **[3]**

2 A straight rod AB has length a. The rod has variable density, and at a distance x from A its mass per unit length is given by $k\left(4-\sqrt{\frac{x}{a}}\right)$, where k is a constant. Find the distance from A of the centre of mass of the rod. **[7]**

3 The region R is bounded by the x-axis, the y-axis, the curve $y = ae^{\frac{x}{a}}$ and the line $x = a\ln 2$ (where a is a positive constant). A uniform solid of revolution, of mass M, is formed by rotating R through 2π radians about the x-axis. Find, in terms of M and a, the moment of inertia about the x-axis of this solid of revolution. **[8]**

4



An unidentified aircraft U is flying horizontally with constant velocity $250\,\text{m s}^{-1}$ in the direction with bearing 040°. Two spotter planes P and Q are flying horizontally at the same height as U, and at one instant P is 15 000 m due west of U, and Q is 15 000 m due east of U (see diagram).

(i) Plane P is flying with constant velocity $210\,\text{m s}^{-1}$ in the direction with bearing 070°.

(a) Find the magnitude and bearing of the velocity of U relative to P. **[4]**

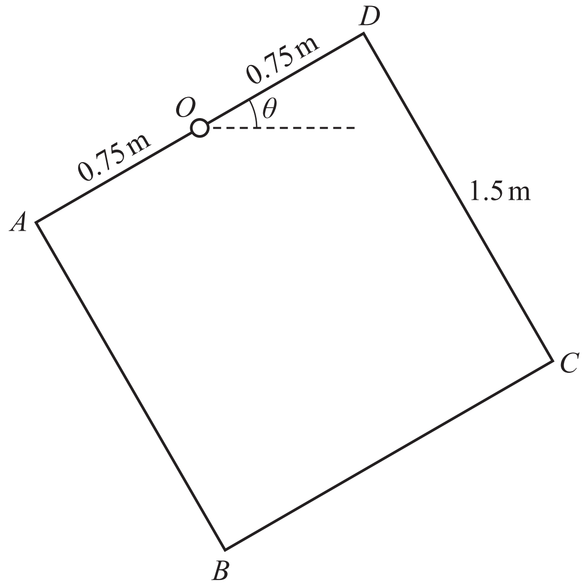
(b) Find the shortest distance between P and U in the subsequent motion. **[2]**

(ii) Plane Q is flying with constant velocity $160\,\text{m s}^{-1}$ in the direction which brings it as close as possible to U.

(a) Find the bearing of the direction in which Q is flying. **[4]**

(b) Find the shortest distance between Q and U in the subsequent motion. **[2]**

5



A square frame $ABCD$ consists of four uniform rods AB, BC, CD, DA, rigidly joined at A, B, C, D. Each rod has mass 0.6 kg and length 1.5 m. The frame rotates freely in a vertical plane about a fixed horizontal axis passing through the mid-point O of AD. At time t seconds the angle between AD and the horizontal, measured anticlockwise, is θ radians (see diagram).

(i) Show that the moment of inertia of the frame about the axis through O is $3.15\,\mathrm{kg\,m^2}$. **[4]**

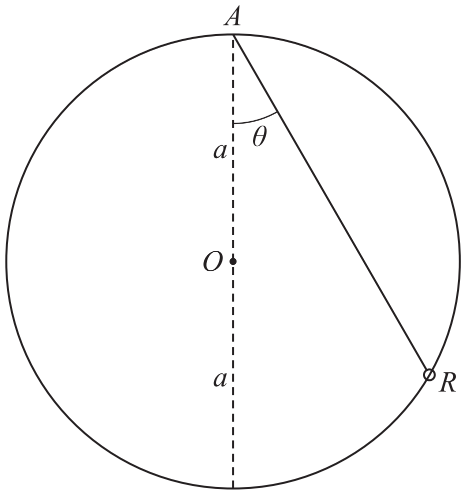
(ii) Show that $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -5.6\sin\theta$. **[3]**

(iii) Deduce that the frame can make small oscillations which are approximately simple harmonic, and find the period of these oscillations. **[3]**

The frame is at rest with AD horizontal. A couple of constant moment 25 N m about the axis is then applied to the frame.

(iv) Find the angular speed of the frame when it has rotated through 1.2 radians. **[4]**

6



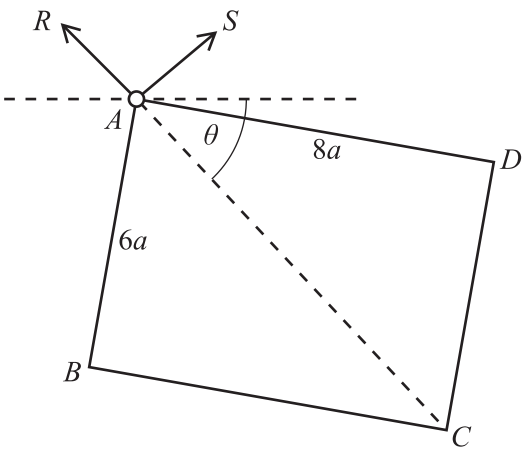
A smooth wire forms a circle with centre O and radius a, and is fixed in a vertical plane. The highest point on the wire is A. A small ring R of mass m moves along the wire. A light elastic string, with natural length $\frac{1}{2}a$ and modulus of elasticity $2mg$, has one end attached to A and the other end attached to R. The string AR makes an angle θ (measured anticlockwise) with the downward vertical (see diagram), and you may assume that the string does not become slack.

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy of the system is $mga(6\cos^2\theta - 4\cos\theta + \frac{1}{2})$. **[4]**

(ii) Show that there are two positions of equilibrium for which $0 \leqslant \theta < \frac{1}{2}\pi$. **[4]**

(iii) For each of these positions of equilibrium, determine whether it is stable or unstable. **[4]**

7



ABCD is a uniform rectangular lamina with mass m and sides $AB = 6a$ and $AD = 8a$. The lamina rotates freely in a vertical plane about a fixed horizontal axis passing through A, and it is released from rest in the position with D vertically above A. When the diagonal AC makes an angle θ below the horizontal, the force acting on the lamina at A has components R parallel to CA and S perpendicular to CA (see diagram).

(i) Find the moment of inertia of the lamina about the axis through A, in terms of m and a. **[3]**

(ii) Show that the angular speed of the lamina is $\sqrt{\frac{3g(4 + 5\sin\theta)}{50a}}$. **[3]**

(iii) Find the angular acceleration of the lamina, in terms of a, g and θ. **[2]**

(iv) Find R and S, in terms of m, g and θ. **[6]**

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