OCR
RECOGNISING ACHIEVEMENT

Monday 24 June 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4777/01 Numerical Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** The equation $f(x) = 0$ has a root α. The equation is rearranged to $x = g(x)$, and the corresponding iterative formula gives the sequence $x_0, x_1, x_2, \ldots$.

Given that $x_{r+1} - \alpha \approx k(x_r - \alpha)$ for some constant k, where $k \neq 1$, show that

$$\alpha \approx \frac{kx_0 - x_1}{k - 1}, \qquad \text{where} \qquad k \approx \frac{x_2 - x_1}{x_1 - x_0}.$$ **[3]**

(ii) The equation

$$1 + mx = e^x,$$

where m is a constant and $m > 1$, is to be solved for $x > 0$.

Show, graphically or otherwise, that the equation has exactly one positive root. **[3]**

(iii) For the case $m = 1.5$, show that the iteration

$$x_{r+1} = \ln(1 + mx_r) \qquad (*)$$

converges slowly.

Show also that the iteration

$$x_{r+1} = \frac{e^{x_r} - 1}{m} \qquad (**)$$

diverges from the required root. **[6]**

(iv) Use the method of part **(i)** to speed up the convergence of (*).

Investigate whether or not the method of part **(i)** makes (**) converge to the required root. **[9]**

(v) Modify your spreadsheet to find, correct to 3 decimal places, the value of m in the interval $[1, 2]$ for which $\alpha = 0.5m$. **[3]**

2 In the table, the values of x are exact and the values of y are correct to 2 decimal places.

x	0.5	1.0	2.0	4.0	6.0	8.0
y	2.23	1.43	1.04	1.22	6.96	40.53

(i) Estimated values of y are required for various values of x. Explain briefly the merits of using Newton's divided difference formula here in preference to other methods of interpolation. **[3]**

(ii) Use a spreadsheet to obtain a sketch of the data. **[2]**

(iii) Use divided differences to produce a sequence of estimates, linear, quadratic, cubic and quartic, of y when $x = 2.9$.

Discuss briefly the likely accuracy of these estimates. Give the estimate of y to the accuracy that is justified. **[14]**

(iv) Modify the spreadsheet in part **(iii)** so that it will estimate y for user-specified values of x between 4 and 5. Hence determine, correct to 2 decimal places, the value of x in this range for which $y = 2$. **[5]**

3 **(i)** Apply the standard Runge-Kutta order 4 method, with $h = 0.5$ and $0 \leqslant x \leqslant 2$, to the differential equations

$$\frac{dy}{dx} = x^3, \text{ with } y = 1 \text{ when } x = 0,$$

$$\text{and } \frac{dy}{dx} = x^4, \text{ with } y = 1 \text{ when } x = 0.$$

Compare the numerical solutions with the exact solutions and comment. **[11]**

(ii) Use the standard Runge-Kutta order 4 method to find a numerical solution for $0 \leqslant x \leqslant 2$ to the differential equation

$$\frac{dy}{dx} = e^x \sin y - e^y \sin x, \text{ with } y = 1 \text{ when } x = 0.$$

Draw a graph of the solution.

Reducing h as necessary, determine, correct to 3 decimal places, the coordinates of the local maximum on the solution curve. **[13]**

4 This question concerns the system of linear equations with the following augmented matrix.

$$\left(\begin{array}{cccc|c} a & 3 & 5 & 1 & 1 \\ b & 4 & 3 & 4 & 2 \\ 2 & 6 & 2 & 5 & 3 \\ 4 & 1 & 9 & 5 & 4 \end{array}\right)$$

(i) For the case $a = 4$ and $b = 2$, solve the equations using Gaussian elimination with partial pivoting. Provide a check that your solutions fit the equations.

Use the numbers generated in the Gaussian elimination process to find the magnitude of the determinant of the coefficient matrix. Show your method clearly. **[16]**

(ii) Now consider small changes in coefficients as follows.

(A) $a = 4.01$, $b = 2$,

(B) $a = 4$, $b = 2.01$.

In each case, find the percentage changes in the solutions and in the determinant. Comment. **[8]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.