OCR
RECOGNISING ACHIEVEMENT

Level 3 Certificate

# Mathematics for Engineering

OCR Level 3 Certificate

**H860/02** Paper 2

# Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

| Question | | | Answer | Mark | Guidance |
|---|---|---|---|---|---|
| 1 | (a) | | $P_p = 2rfg = 4 \times 100 \times 9.8 = 3920$ W | 1<br>**[1]** | Accept answers between 3900 and 4000 |
| 1 | (b) | (i) | mass $= \frac{\Delta\theta}{\omega} f$<br>$\Delta\theta = \frac{2\pi}{n} = \frac{2\pi}{18} = \frac{\pi}{9}$<br>$\omega = \frac{5 \times 2\pi}{60}$ rad s$^{-1}$ $= \frac{\pi}{6}$<br>mass $= \frac{\pi/9}{\pi/6} \times 100 = \frac{2}{3} \times 100 \approx 67$ kg | <br>1<br>1<br>1<br>**[3]** | <br>Allow 0.3491<br>Allow 0.5236<br>Allow $\frac{0.3491}{0.5236} \times 100$ with ECF |
| 1 | (b) | (ii) | $\mathrm{X}(\theta) = 1$ for $0 \le \theta \le \frac{5\pi}{9}$<br>$\mathrm{X}(\theta) = 0$ for $\theta > \frac{5\pi}{9}$ | 1<br>**[1]** | |
| 1 | (b) | (iii) | $\tau \approx \frac{fgr}{\omega} \sum_{j=1}^{n/2} \mathrm{X}(\frac{2\pi}{n} j) \sin(\frac{2\pi}{n} j) \Delta\theta$<br>$\frac{fgr\Delta\theta}{\omega} \sum_{j=1}^{5} \sin(\frac{2\pi}{18} j)$<br>$P = fgr\Delta\theta \sum_{j=1}^{5} \sin(\frac{2\pi}{18} j)$<br>$P = 100 \times 9.8 \times 2 \times \frac{2\pi}{18} (\sin\left(\frac{\pi}{9}\right) + \sin\left(\frac{2\pi}{9}\right) + \sin\left(\frac{3\pi}{9}\right) + \sin\left(\frac{4\pi}{9}\right) + \sin\left(\frac{5\pi}{9}\right))$<br>$P = 684.169 \times 3.8205 \approx 2614$ W | <br>1<br>1<br>1<br>2<br>**[5]** | <br><br>Allow FT from **b ii** |

| Question | | | Answer | Mark | Guidance |
|---|---|---|---|---|---|
| 2 | (a) | | $P \approx fgr\int_0^{\pi} \mathrm{X}(\theta)\sin(\theta)\ \mathrm{d}\theta$<br>$\mathrm{X}(\theta) = 1$ for $0 \le \theta \le \theta_A$<br>$\mathrm{X}(\theta) = 0$ for $\theta > \theta_A$<br>$P \approx fgr\int_0^{\theta_A} \sin(\theta)\ \mathrm{d}\theta$<br>$P = fgr\left[-\cos(\theta)\right]_0^{\theta_A}$<br>$P = fgr\left(-\cos(\theta_A) - (-1)\right) = fgr\left(1-\cos(\theta_A)\right)$ | <br><br><br>1<br>1<br>1<br>**[3]** | <br><br><br><br>Allow 1 for $\int \sin\theta = -\cos\theta$ seen |
| 2 | (b) | | $P = fgr\left\{\int_0^{\frac{\pi}{2}} \sin\theta\ \mathrm{d}\theta + \int_{\frac{\pi}{2}}^{\pi} 2(1-\frac{\theta}{\pi})\sin\theta\ \mathrm{d}\theta\right\}$<br>$P = fgr\left\{\left[-\cos\theta\right]_0^{\pi/2} + 2\left[-\cos\theta\right]_{\pi/2}^{\pi} - \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi}\theta\sin\theta\ \mathrm{d}\theta\right\}$<br>$\int\theta\sin\theta \mathrm{d}\theta = -\theta\cos\theta + \int\cos\theta \mathrm{d}\theta = -\theta\cos\theta + \sin\theta$<br>$P = fgr\left\{1 + 2 - \frac{2}{\pi}\left[-\theta\cos\theta + \sin\theta\right]_{\pi/2}^{\pi}\right\}$<br>$P = fgr\{3 - \frac{2}{\pi}\left((\pi+0) - (-0+1)\right) = fgr(1+\frac{2}{\pi})$<br>$P = 100 \times 9.8 \times 2 \times (1+\frac{2}{\pi}) \approx 3208\ \mathrm{W}$ | 1<br>1<br>1<br>1<br><br>1<br>**[5]** | Solution must show two integrals with correct limits<br><br><br>Solution must demonstrate integration by parts |

| Question | | | Answer | Mark | Guidance |
|---|---|---|---|---|---|
| 2 | (c) | | $\eta = \dfrac{\text{Output power}}{\text{Power of decending water}} = \dfrac{fgr(1-\cos\theta_A)}{2frg} = \dfrac{(1-\cos\theta_A)}{2}$<br>Using $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ from the formula list provided<br>With $A = \theta_A$ and $B = 0$<br>$\cos\theta_a - 1 = -2\sin^2\dfrac{\theta_A}{2}$<br>$\dfrac{(1-\cos\theta_A)}{2} = \sin^2\dfrac{\theta_A}{2}$ | 1<br>1<br>1<br>**[3]** | Accept use of any standard formulae eg $\cos 2x = 1 - 2\sin^2 x$ |
| 3 | (a) | | Power at the output of the alternator<br>$P_{out} = VI = 14.4 \times 120 = 1728$ W<br>$P_{in} = 4 \times 100 \times 9.8 = 3920$ W<br>$\eta = \dfrac{1728}{3920} \approx 44\%$ | 1<br>1<br>**[2]** | Allow 1 mark for $\eta = \dfrac{P_{OUT}}{P_{IN}}$ seen<br>Accept 0.44 |
| 3 | (b) | | Power from inverter = 450 W<br>Power into inverter = 450/0.9 = 500 W<br>Current drawn by inverter = 500/14.4 ≈ 34.72<br>Current available to charge battery = 120 – 34.72 ≈ 85.28 A | 1<br>1<br>**[2]** | Allow 1 mark for $P = VI$ OE seen<br>Allow ECF |
| 3 | (c) | (i) | Power out of inverter = 2000 W<br>Power in = 2000/0.9 = 2222.22 W<br>Current reqd. = 2000/0.9 /14.4 = 154.32 A<br>Current from battery = 154.32 – 120 = 34.32 A | 1<br>1<br>**[2]** | |

| Question | | | Answer | Mark | Guidance |
|---|---|---|---|---|---|
| 3 | (c) | (ii) | Time to drain battery = $T \approx 20\times\left(\dfrac{C_{20}}{20I}\right)$<br>$= 20\times\left(\dfrac{200}{20\times 34.32}\right) \approx 4$ hours | 1<br>1<br>**[2]** | <br>Allow ECF for $I$ |
| 4 | (a) | (i) | $J\alpha = \tau_T$<br>$\alpha = \dfrac{\tau_T}{J} = \dfrac{50}{2000} = 0.025$<br>$\omega_2^2 = \omega_1^2 + 2\alpha\theta$<br>$\omega_1 = \dfrac{5\times 2\pi}{60} = 0.5236\text{ rad s}^{-1}$<br>$\omega_2 = \sqrt{\omega_1^2 + 2\alpha\theta} = \sqrt{0.5236^2 + 2\times 0.025\times 2\pi} = 0.767$ (7.234 revs per minute) | 1<br>1<br>1<br>**[3]** | |
| 4 | (a) | (ii) | $\theta = \left(\dfrac{\omega_1+\omega_2}{2}\right)t$<br>$t = \dfrac{2\times 2\times\pi}{0.5236+0.7670} = 9.737$ s<br>or<br>$\omega_2 = \omega_1 + \alpha t$<br>$t = \dfrac{(\omega_2-\omega_1)}{\alpha} = \dfrac{(0.767-0.5236)}{0.025} = 9.736$ s | 1<br>1<br>**[2]** | <br>Accept answers between 9.7 and 9.8 |

| Question | | | Answer | Mark | Guidance |
|---|---|---|---|---|---|
| 4 | (b) | | $J\alpha = \tau_T = \tau_D - \tau_L$ | | |
| | | | $J\dfrac{d\omega}{dt} = -200(1+\omega)$ | 1 | Assuming that $\tau_D = 0$ |
| | | | $\dfrac{d\omega}{(1+\omega)} = -\dfrac{200}{2000}dt$ | | |
| | | | $\ln(1+\omega) = -\dfrac{t}{10} + C$ | 1 | |
| | | | $1+\omega = Ae^{-\frac{t}{10}}$ | 1 | |
| | | | $\omega = Ae^{-\frac{t}{10}} - 1$ | | |
| | | | $\omega = \omega_0$ when $t = 0 \Rightarrow A = (\omega_0 + 1)$ | | |
| | | | $\omega = (\omega_0 + 1)e^{-\frac{t}{10}} - 1$ | 1 | |
| | | | When $\omega = 0$ $(\omega_0 + 1) = e^{\frac{t}{10}}$ | 1 | |
| | | | $\dfrac{t}{10} = \ln(\omega_0 + 1)$ | | |
| | | | $t = 10\ln(0.5236 + 1) = 4.2108$ s | 1 | |
| | | | | **[6]** | |

**OCR (Oxford Cambridge and RSA Examinations)**
**1 Hills Road**
**Cambridge**
**CB1 2EU**

**OCR Customer Contact Centre**

**Education and Learning**
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

**www.ocr.org.uk**

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

**Oxford Cambridge and RSA Examinations**
**is a Company Limited by Guarantee**
**Registered in England**
**Registered Office; 1 Hills Road, Cambridge, CB1 2EU**
**Registered Company Number: 3484466**
**OCR is an exempt Charity**

**OCR (Oxford Cambridge and RSA Examinations)**
**Head office**
**Telephone: 01223 552552**
**Facsimile: 01223 552553**