RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for June 2013

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$y = -0.5x + 3$ oe www isw	3 **[3]**	B2 for $2y = -x + 6$ oe or M1 for gradient = $-\frac{1}{2}$ oe seen or used and M1 for $y - 1 =$ *their* m $(x - 4)$	for 3 marks must be in form $y = ax + b$ or M1 for $y =$ *their* $mx + c$ and (4, 1) substituted
2			substitution to eliminate one variable simplification to $ax = b$ or $ax - b = 0$ form, or equivalent for y (0.7, 0.1) oe or $x = 0.7$, $y = 0.1$ oe isw	M1 M1 A2 **[4]**	or multiplication to make one pair of coefficients the same; condone one error in either method or appropriate subtraction / addition; condone one error in either method A1 each	independent of first M1
3	(i)		25	2 **[2]**	M1 for $\left(\frac{10}{2}\right)^2$ or $\left(\frac{1}{0.2}\right)^2$ oe soi or for $\frac{1}{0.04}$ oe	ie M1 for one of the two powers used correctly M0 for just $\frac{1}{0.4}$ with no other working
3	(ii)		$8a^9$	3 **[3]**	B2 for 8 or M1 for $16^{\frac{1}{4}} = 2$ soi and B1 for a^9	ignore ± eg M1 for 2^3; M0 for just 2

4			$r = \sqrt{\dfrac{3V}{\pi(a+b)}}$ oe www as final answer	3 **[3]**	M1 for dealing correctly with 3 and M1 for dealing correctly with $\pi(a + b)$, ft and M1 for correctly finding square root, ft *their* 'r^2 ='; square root symbol must extend below the fraction line	M0 if triple-decker fraction, at the stage where it happens, then ft; condone missing bracket at rh end M0 if $\pm$... or $r >$ ... for M3, final answer must be correct
5			f(2) = 18 seen or used $32 + 2k - 20 = 18$ oe [k =] 3	M1 A1 A1 **[3]**	or long division oe as far as obtaining a remainder (ie not involving x) and equating that remainder to 18 (there may be errors along the way) after long division: $2(k + 16) - 20 = 18$ oe	 A0 for just 2^5 instead of 32 unless 32 implied by further work

6			-2560 www	4 **[4]**	B3 for 2560 from correct term (NB coefficient of x^4 is 2560) or B3 for neg answer following $10 \times 4 \times -64$ and then an error in multiplication or M2 for $10 \times 2^2 \times (-4)^3$ oe; must have multn signs or be followed by a clear attempt at multn; or M1 for $2^2 \times (-4)^3$ oe (condone missing brackets) or for 10 used or for 1 5 10 10 5 1 seen for those who find the coefft of x^2 instead: allow M1 for 10 used or for 1 5 10 10 5 1 seen ; and a further SC1 if they get 1280, similarly for finding coefficient of x^4 as 2560	ignore terms for other powers; condone x^3 included; but eg $10 \times 4 \times -64 = 40 - 64 = -24$ gets M2 only condone missing brackets eg allow M2 for $10 \times 2^2 \times -4x^3$ ${}^5C_3$ or factorial notation is not sufficient but accept $\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1}$ oe 10 may be unsimplified, as above M1 only for eg 10, 2^2 and $-4x^3$ seen in table with no multn signs or evidence of attempt at multn [lack of neg sign in the x^2 or x^4 terms means that these are easier and so not eligible for just a 1 mark MR penalty]
7	**(i)**		$5^{3.5}$ oe or $k = 7/2$ oe	2 **[2]**	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ soi	M0 for just answer of 5^3 with no reference to 125

7	**(ii)**		attempting to multiply numerator and denominator of fraction by $1+2\sqrt{5}$	M1		some cands are incorporating the $10+7\sqrt{5}$ into the fraction. The M1s are available even if this is done wrongly or if $10+7\sqrt{5}$ is also multiplied by $1+2\sqrt{5}$
			denominator = –19 soi	M1	must be obtained correctly, but independent of first M1	eg M1 for denominator of 19 with a minus sign in front of whole expression or with attempt to change signs in numerator
			$8+3\sqrt{5}$	A1 **[3]**		
8			$3(x-2)^2 - 7$ isw or $a = 3$, $b = 2$ $c = 7$ www	4	B1 each for $a = 3$, $b = 2$ oe and B2 for $c = 7$ oe or M1 for $[-]\frac{7}{3}$ or for $5 - their\ a(their\ b)^2$ or for $\frac{5}{3} - (their\ b)^2$ soi	condone omission of square symbol; ignore '= 0' may be implied by their answer
			–7 or ft	B1 **[5]**	B0 for (2, –7)	may be obtained by starting again eg with calculus
9	**(i)**		$3n$ isw	1 **[1]**	accept equivalent general explanation	

9	**(ii)**		at least one of $(n-1)^2$ and $(n+1)^2$ correctly expanded $3n^2 + 2$ comment eg $3n^2$ is always a multiple of 3 so remainder after dividing by 3 is always 2	M1 B1 B1 **[3]**	must be seen dep on previous B1 B0 for just saying that 2 is not divisible by 3 – must comment on $3n^2$ term as well allow B1 for $\frac{3n^2+2}{3} = n^2 + \frac{2}{3}$	M0 for just $n^2 + 1 + n^2 + n^2 + 1$ accept even if no expansions / wrong expansions seen SC: n, $n+1$, $n+2$ used similarly can obtain first M1, and allow final B1 for similar comment on $3n^2 + 6n + 5$
10	**(i)**		[radius =] $\sqrt{20}$ or $2\sqrt{5}$ isw [centre =] (3, 2)	B1 B1 **[2]**	B0 for $\pm\sqrt{20}$ oe	 condone lack of brackets with coordinates, here and in other questions

10	**(ii)**		substitution of $x = 0$ or $y = 0$ into circle equation	M1	or use of Pythagoras with radius and a coordinate of the centre eg $20 - 2^2$ or $h^2 + 3^2 = 20$ ft their centre and/or radius	equation may be expanded first, and may include an error bod intent allow M1 for $(x-3)^2 = 20$ and/or $(y-2)^2 = 20$
			$(x-7)(x+1)$ [=0]	M1	no ft from wrong quadratic; for factors giving two terms correct, or formula or completing square used with at most one error	completing square attempt must reach at least $(x-a)^2 = b$ following use of Pythagoras allow M1 for attempt to add 3 to [±]4
			(7, 0) and (−1, 0) isw	A1	accept $x = 7$ or -1 (both required)	
			$[y =]\dfrac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times (-7)}}{2}$ oe	M1	no ft from wrong quadratic; for formula or completing square used with at most one error	completing square attempt must reach at least $(y-a)^2 = b$ following use of Pythagoras allow M1 for attempt to add 2 to [±]$\sqrt{11}$
			$(0, 2 \pm \sqrt{11})$ or $\left(0, \dfrac{4 \pm \sqrt{44}}{2}\right)$ isw	A1 **[5]**	accept $y = \dfrac{4 \pm \sqrt{44}}{2}$ oe isw	annotation is required if part marks are earned in this part: putting a tick for each mark earned is sufficient

10	**(iii)**		show both A and B are on circle	B1	explicit substitution in circle equation and at least one stage of interim working required oe	or clear use of Pythagoras to show AC and BC each = $\sqrt{20}$
			(4, 5)	B2	B1 each or M1 for $\left(\frac{7+1}{2}, \frac{6+4}{2}\right)$	
			$\sqrt{10}$	B2 **[5]**	from correct midpoint and centre used; B1 for $\pm\sqrt{10}$ M1 for $(4-3)^2+(5-2)^2$ or 1^2+3^2 or ft their centre and/or midpoint, or for the square root of this	may be a longer method finding length of ½ AB and using Pythag. with radius; no ft if one coord of midpoint is same as that of centre so that distance formula/Pythag is not required eg centre correct and midpt (3, −1) annotation is required if part marks are earned in this part: putting a tick for each mark earned is sufficient

11	**(i)**		sketch of cubic the right way up, with two tps and clearly crossing the x axis in 3 places	B1		no section to be ruled; no curving back; condone slight 'flicking out' at ends but not approaching another turning point; condone some doubling (eg erased curves may continue to show); accept min tp on y-axis or in 3rd or 4th quadrant; curve must clearly extend beyond the x axis at both 'ends'
			crossing/reaching the x-axis at –4, –2 and 1.5	B1	intersections must be shown correctly labelled or worked out nearby; mark intent	accept curve crossing axis halfway between 1 and 2 if 3/2 not marked
			intersection of y-axis at –24	B1 **[3]**		NB to find –24 some are expanding f(x) here, which gains M1 in iiiA. If this is done, put a yellow line here and by (iii)A to alert you; this image appears again there
11	**(ii)**		–2, 0 and 7/2 oe isw or ft their intersections	2 **[2]**	B1 for 2 correct or ft or for (–2, 0) (0, 0) and (3.5, 0) or M1 for $(x + 2)\,x\,(2x – 7)$ oe or SC1 for –6, –4 and –1/2 oe	

11	**(iii)**	(*A*)	correct expansion of product of 2 brackets of f(x)	M1	need not be simplified; condone lack of brackets for M1 or allow M1 for expansion of all 3 brackets, showing all terms, with at most one error: $2x^3 + 4x^2 + 8x^2 - 3x^2 + 16x - 12x - 6x - 24$	eg $2x^2 + 5x - 12$ or $2x^2 + x - 6$ or $x^2 + 6x + 8$ may be seen in (i) – allow the M1; the part (i) work appears at the foot of the image for (iii)A, so mark this rather than in (i)
			correct expansion of quadratic and linear and completion to given answer	A1 **[2]**	for correct completion if all 3 brackets already expanded, with some reference to show why –24 changes to –9	condone lack of brackets if they have gone on to expand correctly; condone '+15' appearing at some stage NB answer given; mark the whole process

11	**(iii)**	(*B*)	$g(1) = 2 + 9 – 2 – 9$ [=0]	B1	allow this mark for $(x – 1)$ shown to be a factor and a statement that this means that $x = 1$ is a root [of $g(x) = 0$] oe	B0 for just $g(1) = 2(1)^3 + 9(1)^2 – 2(1) – 9$ [=0]
			attempt at division by $(x − 1)$ as far as $2x^3 − 2x^2$ in working	M1	or inspection with at least two terms of quadratic factor correct	M0 for division by $x + 1$ after $g(1) = 0$ unless further working such as $g(–1) = 0$ shown, but this can go on to gain last M1A1
			correctly obtaining $2x^2 + 11x + 9$	A1	allow B2 for another linear factor found by the factor theorem	NB mixture of methods may be seen in this part – mark equivalently eg three uses of factor theorem, or two uses plus inspection to get last factor;
			factorising a correct quadratic factor	M1	for factors giving two terms correct; eg allow M1 for factorising $2x^2 + 7x – 9$ after division by $x + 1$	allow M1 for $(x + 1)(x + 18/4)$ oe after $–1$ and $–18/4$ oe correctly found by formula
			$(2x + 9)(x + 1)(x – 1)$ isw	A1 **[5]**	allow $2(x + 9/2)(x + 1)(x – 1)$ oe; dependent on 2nd M1 only; condone omission of first factor found; ignore '= 0' seen	SC alternative method for last 4 marks: allow first M1A1 for $(2x + 9)(x^2 – 1)$ and then second M1A1 for full factorisation

12	(i)		$y = 2x + 3$ drawn accurately	M1	at least as far as intersecting curve twice	ruled straight line and within 2mm of (2, 7) and (–1, 1)
			(–1.6 to –1.7, –0.2 to –0.3)	B1	intersections may be in form $x = ..., y = ...$	
			(2.1 to 2.2, 7.2 to 7.4)	B1		
				[3]		if marking by parts and you see work relevant to (ii), put a yellow line here and in (ii) to alert you to look
12	(ii)		$\frac{1}{x-2} = 2x+3$	M1	or attempt at elimination of x by rearrangement and substitution	may be seen in (i) – allow marks; the part (i) work appears at the foot of the image for (ii) so show marks there rather than in (i)
			$1 = (2x + 3)(x - 2)$	M1	condone lack of brackets	implies first M1 if that step not seen
			$1 = 2x^2 - x - 6$ oe	A1	for correct expansion; need not be simplified; NB A0 for $2x^2 - x - 7 = 0$ without expansion seen [given answer]	implies second M1 if that step not seen after $\frac{1}{x-2} = 2x+3$ seen
			$\frac{1 \pm \sqrt{1^2 - 4 \times 2 \times -7}}{2 \times 2}$ oe	M1	use of formula or completing square on given equation, with at most one error	completing square attempt must reach at least $[2](x - a)^2 = b$ or $(2x - c)^2 = d$ stage oe with at most one error
			$\frac{1 \pm \sqrt{57}}{4}$ isw	A1	isw eg coordinates; after completing square, accept $\frac{1}{4} \pm \sqrt{\frac{57}{16}}$ or better	
				[5]		

12	**(iii)**		$\frac{1}{x-2}=-x+k$ and attempt at rearrangement	M1		
			$x^2-(k+2)x+2k+1[=0]$	M1	for simplifying and rearranging to zero; condone one error; collection of x terms with bracket not required	eg M1 bod for $x^2-(k+2)x+2k$ or M1 for $x^2-2kx+2k+1[=0]$
			$b^2-4ac=0$ oe seen or used	M1		= 0 may not be seen, but may be implied by their final values of k
			[k =] 0 or 4 as final answer, both required	A1 **[4]**	SC1 for 0 and 4 found if 3rd M1 not earned (may or may not have earned first two Ms)	eg obtained graphically or using calculus and/or final answer given as a range

Appendix: revised tolerances for modified papers for visually impaired candidates (graph in 12(i) with 6mm squares)

12	**(i)**		$y = 2x + 3$ drawn accurately	M1	at least as far as intersecting curve twice	ruled straight line and within **3** mm of (2, 7) and (–1, 1)
			(–1.6 to –1.**8**, –0.2 to –0.3)	B1	intersections may be in form $x = ..., y = ...$	
			(2.1 to 2.**3**, 7.**1** to 7.4)	B1		
				[3]		if marking by parts and you see work relevant to (ii), put a yellow line here and in (ii) to alert you to look

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