GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

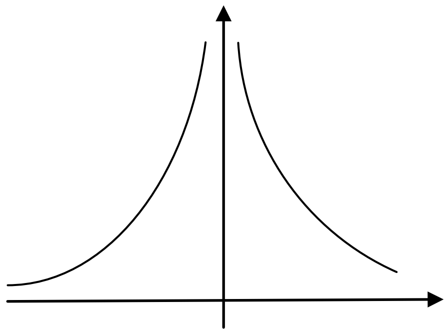
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

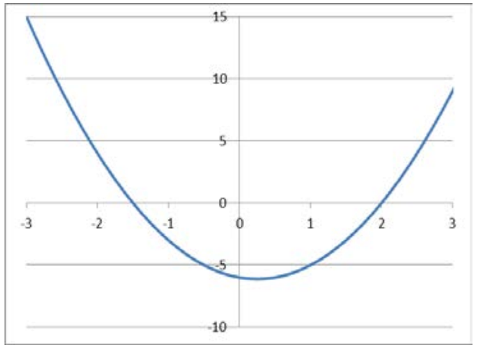
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$4\sqrt{45}$ $=12\sqrt{5}$	M1 A1 **[2]**	or $4\sqrt{5}\sqrt{3}\times\sqrt{3}$ (not just $4\sqrt{5\times3}\times\sqrt{3}$) or $\sqrt{720}$ or $\sqrt{240}\times\sqrt{3}$ or better Correctly simplified answer	For method mark, makes a correct start to manipulate the expression i.e. at least combines two parts correctly or splits one part correctly
1	(ii)		$\frac{20\sqrt{5}}{5}=4\sqrt{5}$	B1 **[1]**	**cao**, do not allow unsimplified, do not allow if clearly from wrong working	
1	(iii)		$5\sqrt{5}$	B1 **[1]**	**cao www**, do not allow unsimplified, do not allow if clearly from wrong working	
2			$k = x^3$ $8k^2 + 7k - 1 = 0$ $(8k - 1)(k + 1) = 0$ $k = \frac{1}{8},\ k = -1$ $x = \frac{1}{2},\ x = -1$	M1* DM1 * A1 M1 A1 **[5]**	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^3 Correct method to solve a quadratic Both values of k correct Attempt to cube root at least one value to obtain x Both values of x correct and no other values	**No marks** if whole equation cube rooted etc. **No marks** if straight to formula with no evidence of substitution at start and no cube rooting/cubing at end. **Spotted solutions:** If **M0 DMO** or **M1 DM0** **SC B1** $x = -1$ **www** **SC B1** $x = \frac{1}{2}$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)

Question			Answer	Marks	Guidance	
3	**(i)**		$f(x) = 6x^{-2} + 2x$ $f'(x) = -12x^{-3} + 2$	M1 A1 B1 **[3]**	kx^{-3} obtained by differentiation $-12x^{-3}$ $2x$ correctly differentiated to 2	**ISW** incorrect simplification after correct expression
3	**(ii)**		$f''(x) = 36x^{-4}$	M1 A1 **[2]**	Attempt to differentiate **their (i)** i.e. at least one term "correct" Fully correct **cao** No follow through for **A** mark	Allow constant differentiated to zero **ISW** incorrect simplification after correct expression
4	**(i)**		$3(x^2 + 3x) + 10$ $= 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4} + 10$ $= 3\left(x + \frac{3}{2}\right)^2 + \frac{13}{4}$	B1 M1 A1 **[3]**	$\left(x + \frac{3}{2}\right)^2$ $10 - 3p^2$ or $\frac{10}{3} - p^2$ Allow $p = \frac{3}{2}$, $q = \frac{13}{4}$ **A1 www**	If p, q found correctly, then **ISW** slips in format. $3(x + 1.5)^2 - 3.25$ **B1 M0 A0** $3(x + 1.5) + 3.25$ **B1 M1 A1 (BOD)** $3(x + 1.5x)^2 + 3.25$ **B0 M1 A0** $3(x^2 + 1.5)^2 + 3.25$ **B0 M1 A0** $3(x - 1.5)^2 + 3.25$ **B0 M1 A1 (BOD)** $3x(x + 1.5)^2 + 3.25$ **B0M1A0**
4	**(ii)**		$\left(-\frac{3}{2}, \frac{13}{4}\right)$	B1 B1 **[2]**	FT i.e. – their p FT i.e. their q	If restarted e.g. by differentiation: Correct method to find x value of minimum point **M1** Fully correct answer **www A1**
4	**(iii)**		$9^2 - (4 \times 3 \times 10)$ $= -39$	M1 A1 **[2]**	Uses $b^2 - 4ac$ Ignore >0, <0 etc. **ISW** comments about number of roots	Use of $\sqrt{b^2 - 4ac}$ is **M0** unless recovered

Question			Answer	Marks	Guidance	
5	**(i)**			B1 B1 **[2]**	Excellent curve for $y=\frac{2}{x^2}$ in either quadrant Excellent curve for $y=\frac{2}{x^2}$ in other quadrant and no more. **SC B1** Reasonably correct curves in 1st and 2nd quadrants and no more	**N.B. Ignore 'feathering' now that answers are scanned.** For Excellent: Correct shape, not touching axes, asymptotes clearly the axes. Allow slight movement away from asymptote at one end but not more. Not finite. For **SC B1**, graph must not touch axes more than twice.
5	**(ii)**		$y=\frac{2}{(x+5)^2}$	M1 A1 **[2]**	$\frac{2}{(x+5)^2}$ or $\frac{2}{(x-5)^2}$ seen Fully correct, must include "y =" or "f(x) ="	
5	**(iii)**		Stretch scale factor $\frac{1}{2}$ parallel to y-axis	B1 B1 **[2]**	Or "stretched" etc; **do not accept** squashed, compressed etc. **oe** e.g. scale factor $1/\sqrt{2}$ parallel to x-axis	**0/2** if more than one type of transformation mentioned **ISW** non-contradictory statements For "parallel to the y-axis" allow "vertically", "up", "in the (positive) y direction". **Do not accept** "in/on/ across/up/along/to/towards the y-axis"
6	**(i)**		Centre (0, −4) $x^2 + (y + 4)^2 - 16 - 24 = 0$ Radius = $\sqrt{40}$	B1 M1 A1 **[3]**	 $(y \pm 4)^2 - 4^2$ seen (or implied by correct answer) Do not allow **A** mark from $(y - 4)^2$	 Or attempt at $r^2 = f^2 + g^2 - c$ **A0** for $\pm\sqrt{40}$
6	**(ii)**		(−2, −10)	B1FT B1FT **[2]**	FT through centre given in **(i)** FT through centre given in **(i)**	i.e. (their $2x - 2$, their $2y - 2$) Apply same scheme if equation of diameter found and attempt to solve simultaneously; no marks until a correct value of x/y found.

Question			Answer	Marks	Guidance	
7	(i)		$8x < -1$	B1	soi, allow $-8x > 1$ but not just $8x + 1 < 0$	Allow ≤ or ≥ for first mark Do not **ISW** if contradictory Do not allow ≤ or ≥
			$x < -\frac{1}{8}$	B1	Correct working only, allow $-\frac{1}{8} > x$ Do not allow $\frac{1}{-8}$	
				[2]		
7	(ii)		$2x^2 - 10x \le 0$	M1*	Expand brackets and rearrange to collect all terms on one side	No more than one incorrect term
			$2x(x-5) \le 0$	DM1*	Correct method to find roots of resulting quadratic	Allow $(2x + 0)(x - 5)$ **Do not allow (2x – 4)(x – 3),** this is the original expression.
				A1	0, 5 seen as roots – could be on sketch graph	
				DM1*	Chooses "inside region" for their roots of their **resulting** quadratic (not the original)	Dependent on first M1 only
			$0 \le x \le 5$	A1	Do **not** accept strict inequalities for final mark	Allow "$x \ge 0$, $x \le 5$" , "$x \ge 0$ **and** $x \le 5$" but do not allow "$x \ge 0$ **or** $x \le 5$"
				[5]		
8			Midpoint of AB is $\left(\frac{-2+3}{2}, \frac{6+-8}{2}\right)$	M1	Correct method to find midpoint – can be implied by one correct value	**NB – "correct" answer can be found with wrong mid-pt. Check working thoroughly.**
			$\left(\frac{1}{2}, -1\right)$	A1		
			Gradient of given line = $\frac{1}{3}$	B1	Must be stated or used – just rearranging the equation is not sufficient	
			Gradient of $l = -3$	B1FT	Use of $m_1m_2 = -1$ (may be implied), allow for any initial non-zero numerical gradient	
			$y + 1 = -3\left(x - \frac{1}{2}\right)$	M1	Correct equation for line, any non-zero numerical gradient, through their $\left(\frac{1}{2}, -1\right)$	
				A1	Correct equation in any three-term form	
			$6x + 2y - 1 = 0$	A1 **[7]**	$k(6x + 2y - 1) = 0$ for integer k **www**	Must include "= 0"

Question			Answer	Marks	Guidance	
9	(i)		$(2x+3)(x-2)=0$	M1	Correct method to find roots	
			$x=-\frac{3}{2},\ x=2$	A1	Correct roots	
				B1	Reasonably symmetrical positive quadratic curve, must cross x axis	
				B1	y intercept $(0, -6)$ only	Indicated on graph or clearly stated, but there must be a curve
				B1	Good curve, with correct roots indicated and min point in 4th quadrant (not on axis)	Only allow final B1 if curve is clearly intended to be a quadratic symmetrical about min point in 4th quadrant
				[5]		
9	(ii)		$\frac{dy}{dx}=4x-1=0$	M1	Attempt to find x coordinate of vertex by differentiating and equating/comparing to zero, completing the square, finding the mid-point of their roots oe	**SC** Award **B1 (FT)** for $x<0$ if clearly from their graph
			Vertex when $x=\frac{1}{4}$	A1	cao	**NB Look for solution to 9ii done in the space below 9i graph**
			$x<\frac{1}{4}$	A1 FT	$x<$ their vertex, allow $\le$	
				[3]		
9	(iii)		$2x^2-x-6=4$	M1	Set quadratic expression equal to 4	
			$2x^2-x-10=0$ $(2x-5)(x+2)=0$	M1	Correct method to solve **resulting** three term quadratic	**Not** $2x^2-x-6=0$ with no use of 4
			$x=\frac{5}{2},\ x=-2$	A1	Must have both solutions – no mark for one spotted root	
			Distance $PQ=4½$	B1FT	FT from their x values found from their resulting quadratic, provided $y=4$	Allow $\frac{9}{2}$ oe, but do **not** accept unsimplified expressions like $\sqrt{\frac{81}{4}}$
				[4]		

Question			Answer	Marks	Guidance	
10	(i)		$y = -x^3 - 3x^2 + 4x - kx + k$	M1 A1	Attempt to multiply out brackets Can be unsimplified	Must have $\pm x^3$ and 5 or 6 terms
			$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$	M1 A1	Attempt to differentiate **their** expansion (**M0** if signs have changed throughout)	**<u>If using product rule:</u>** Clear attempt at correct rule **M1*** Differentiates both parts correctly **A1** Expand brackets of both parts ***DM1**
			When $x = -3$, $\frac{dy}{dx} = 0$	M1* DM1*	Sets $\frac{dy}{dx} = 0$ Substitutes $x = -3$ into their $\frac{dy}{dx} = 0$	
			$-27 + 18 + 4 - k = 0$ $k = -5$	A1 **[7]**	**www**	Then as main scheme
10	(ii)		$\frac{d^2y}{dx^2} = -6x - 6$	M1	Evaluates second derivative at $x = -3$ or other fully correct method	**Alternate valid methods include:** 1) Evaluating gradient at either side of -3 2) Evaluating y at either side of -3 3) Finding other turning point and stating "negative cubic so min before max"
			When $x = -3$, $\frac{d^2y}{dx^2}$ is positive so min point	A1 **[2]**	No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 12. (Ignore errors in k value)	
10	(iii)		$-3x^2 - 6x + 9 = 9$	M1	Sets their gradient function from (i) (or from a restart) to 9	Allow first **M** even if k not found but look out for correct answer from wrong working.
			$3x(x + 2) = 0$ $x = 0$ or $x = -2$	A1	Correct x-values	**<u>SEE NEXT PAGE FOR ALTERNATIVE METHODS</u>**
			When $x = 0$, $y = -9$ for line $y = -5$ for curve	M1	One of their x-values substituted into both curve and line/substituted into one and verified to be on the other	**Note:** Putting a value into $x^3 + 3x^2 - 4 = 0$ (where the line and curve meet) is equivalent
			When $x = -2$, $y = -27$ for line $y = -27$ for curve	M1	Conclusion that $x = -2$ is the correct value **<u>or</u>** Second x-value substituted into both curve and line/verified as above	<u>If curve equated to line before differentiating:</u>
			$x = -2$, $y = -27$	A1 **[5]**	$x = -2$, $y = -27$ **www *(Check k correct)***	**M0 A0**, can get **M1M1** but **A0 ww** Maximum mark **2/5**

Question			Answer	Marks	Guidance
10	(iii)		Alternative method Attempt to solve equations of curve and tangent simultaneously **and** uses valid method to establish at least one root of the resulting cubic ($x^3 + 3x^2 - 4 = 0$ oe) **M1** All roots found **A1** Either 1) States $x = -2$ is repeated root so tangent **M2** (If double root found but not explicitly stated that repeated root implies tangent then **M0** but **B1** if (−2, −27) found) Or 2) Substitutes one x value into their gradient function to determine if equal to gradient of the line **M1** Substitutes other x value into their gradient function to determine if equal to gradient of the line **or** conclusion that −2 is the correct one **M1** $x = -2$, $y = -27$ **A1 www** **SC** Trial and Improvement Finds at least one value at which the gradient of the curve is 9 **B1** Verifies on both line and curve **B1 2/5**		

APPENDIX 1

<u>Allocation of method mark for solving a quadratic</u>

e.g. $2x^2 - x - 6 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x - 3)(x + 2)$	**M1**	$2x^2$ and -6 obtained from expansion
$(2x - 3)(x + 1)$	**M1**	$2x^2$ and $-x$ obtained from expansion
$(2x + 3)(x + 2)$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times 2}$ earns **M1** (minus sign incorrect at start of formula)

$\dfrac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$ earns **M1** (6 for c instead of −6)

$\dfrac{-1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2}$ **M0** (2 sign errors: initial sign and c incorrect)

$\dfrac{1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times -6}$ **M0** ($2c$ on the denominator)

Notes – for equations such as $2x^2 - x - 6 = 0$, then $b^2 = 1^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - x - 6 = 0$$

$$2\left(x^2 - \frac{1}{2}x\right) - 6 = 0$$

$$2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6 = 0$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{49}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{1}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt

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