GCE

Mathematics (MEI)

Advanced GCE A2 7895-8

Advanced Subsidiary GCE AS 3895-8

OCR Report to Centres June 2014

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This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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OCR REPORT TO CENTRES

4751 Introduction to Advanced Mathematics (C1)

General Comments:

Candidates in the main were confident with much of the content of this paper, being usually well-practised in using routine skills. Examiners were pleased to see that many candidates were able to make a good attempt at most of the questions. However there remained a couple of question parts which stretched the most able in applying their knowledge, in particular the last question.

Candidates did not always appreciate the difference between an equation and an expression, which led to loss of marks in 11(iii) and/or 12(i). Candidates did not always understand that, when asked to 'Show that...' evidence has to be provided, and so the full method must be shown. The worst confusion of language was seen in question 8, where many candidates did not distinguish correctly between a factor and a root, so that answers such as '*x* + 1 is a root' were common.

Arithmetic errors were seen frequently in question 7, where many attempted long multiplication and made errors, rather than using simpler methods using the powers of 2 and 5. In question 12, having substituted correctly in the quadratic formula, errors in simplifying the discriminant were common. Lack of facility in working with fractions caused errors in question 10iv.

Comments on Individual Questions:

Section A

Question No. 1
Most candidates knew what to do and handled the indices well. Errors such as $\sqrt[3]{27}=9$ were seen occasionally in the first part. In the second, the most frequent errors came from failing to cube the 4 or the a^2 correctly.

Question No. 2
Many obtained three marks here without any difficulty, with many candidates choosing to use the quick substitution of midpoint method to prove that the point was on the line. A minority failed to state a clear conclusion once this step had been performed. Longer methods were seen occasionally but were rarely completed successfully, with the equation of AB sometimes being found simply because the candidate did not know what to do.

Question No. 3
Both parts of this question were done correctly by a high proportion of candidates. In the first part very occasionally a horizontal translation was seen or a translation upwards. In the second part there were more errors, with a translation to the left instead of the right being given.

Question No. 4
Most candidates gained at least one mark in the first part for $-28\sqrt{3}$. Those who failed to reach the correct final answer often incorrectly expanded the last terms of the brackets, obtaining $\pm4\sqrt{3}$, 6 or 12 rather than +12. For most candidates the second part was more challenging than the first part. Errors tended to be introduced when rationalising the denominator, with many choosing to multiply by $\sqrt{50}$ or $-\sqrt{50}$. Those that did rationalise were then unsure how to simplify the numerator, often obtaining large roots which they were unable to simplify accurately. Those that had the most success in this question expressed the $\sqrt{50}$ in the denominator as $5\sqrt{2}$ and were then comfortable dividing surds and cancelling fractions.

Question No. 5
Rearranging the formula was usually done well. Those who found this difficult generally attempted to isolate just one *a* term and hence scored only the first mark. Other errors seen occasionally included sign errors and a final spoiling of the answer by invalidly 'cancelling' 3 into -12.

Question No. 6
In solving the quadratic inequality, most candidates were able to factorise the quadratic expression correctly, though a few produced incorrect factors. A small number resorted to using the formula to determine the end points, often failing to do so correctly. It was very clear that those candidates who drew a sketch to help them were generally successful in identifying the two different regions. But without a diagram many either just gave the single region between the end points, or having written down two correct inequalities tried incorrectly to combine them. Another error often seen was to believe that since $(3x + 1)(x + 3) > 0$, then $(3x + 1) > 0$ and/or $(x + 3) > 0$.

Question No. 7
Many candidates were able to establish the desired product of $35 \times 5^3 \times 2^4$ in finding the binomial coefficient. There were fewer failing to cope correctly with $(2x)^4$ than in similar past questions on this topic. However, few candidates were confident enough with their number bonds, or quick mental methods such as repeated doubling, to realise that $5^3 \times 2^4$ or 125×16 could be easily evaluated as 2000, often unsuccessfully attempting 35x125 or similar.

Question No. 8
Most candidates successfully used the remainder theorem to set up an equation to solve for *k*. They understood how to write down the expression for f(2) and equated it to 42. The correct solution was usually found though a few errors were made in evaluating 4×2^3. An occasional error seen was to put f(-2) =42. A number of candidates chose to ignore the guidance, and proceeded with long algebraic division with some degree of success, but with no x^2 term there was plenty of opportunity to go wrong. There were a few attempts at using synthetic division.

Confusion between factors and roots continues to be in evidence with many candidates. Some clearly stated the root was $(x + 1)$, whilst others gave both $(x + 1)$ and $x = -1$ without making it clear which one was the root. Some gave the answer as $f(-1) = 0$.

Question No. 9
The straightforward algebra in the first part was done correctly by most candidates. The most common errors were to write $(n + 1)^2$ as $n^2 + 1$, or sometimes $n^2 + 2n + 2$ and $(n + 2)^2$ as $n^2 + 4$.

There was some encouraging work in the proof part with a number of slightly different methods being demonstrated. The majority considered the three terms that they had found in (i) but others went further and expressed the quadratic function as $3n(n + 2) + 5$ or $3(n + 1)^2 + 2$. As these were the more capable candidates they were then often argued the case elegantly. Some candidates returned to the original function successfully with a few replacing *n* by $2m + 1$ and expanding to find a factor of 2. The candidates who fared badly were those who failed to draw any conclusion at all, those who attempted to use an incorrect expression from (i) or those who just tried to show it with some numerical values.

Section B

Question No. 10
(i) This was usually correct, with most candidates appreciating the symmetry about $x = 4$.

(ii) The correct method for finding the radius was usually seen; however, some candidates were let down by their poor arithmetic, for instance, 9 + 4 = 11. Most knew the form for the circle equation, although some failed to square the root 13 (or their number), or made some mistake in the left-hand side of the equation, such as sign errors or omitting the squared signs.

(ii) Most candidates found the coordinates of D correctly using a step/vector method. A few tried to find the intersection of AD with the circle, but these were usually unsuccessful.

(iii) Attempts at this part were variable. Whilst many recognised that they had to find the gradient of AD first, quite a few made errors in doing this. Most recognised that the gradient of the tangent was the negative reciprocal of this number and substituted this into either $y - y_1 = m(x - x_1)$ or $y = mx + c$. A few used their gradient of AD in the equation of the line. Quite a number of errors were seen in reaching the final answer, most of these associated with dealing with fractions. A few candidates did not give their answer in the required form.

Question No. 11

(i) The minimum point was generally well found, although some just gave the *y* coordinate. The question said, "Write down …", which should suggest to candidates that differentiating and putting the differential equal to zero was not needed. The line of symmetry was also usually well done, but some gave $x = -4$ or $y = 4$.

(ii) The *y* intercept was usually correct. For the *x* intercept, many went the long way round: expanding brackets and then using the quadratic formula rather than using the completing the square method.

(iii) Some candidates lost a mark as they forgot that an equation has 2 sides and omitted the 'y=', only giving an expression. Most candidates realised that they should replace *x* with $(x – 2)$. A minority expanded brackets before replacing *x* with $(x – 2)$ which was a less efficient method.

(iii) Since both equations were given, those were the ones which had to be used, and most candidates did so successfully. Some candidates did not realise that obtaining $(x – 5)^2 = 0$ led to sufficient evidence of a repeated root and also showed that the discriminant was zero. Some, of course, did not attempt to factorise anyway but opted for using the formula. The main error was in the very last mark, where some candidates substituted their *x* value back into the quadratic that they had just solved to find $y = 0$, rather than using the line or the curve to give $y = -2$.

Question No. 12

(i) Candidates struggled with this question. Often they managed to produce the product of binomial factors $(x+2)(x+5)(x-1.5)$ and failed to put it equal to *y* or put it equal to 0. Those who did have the correct product still very often had an expression only or equated to 0. Many candidates thought that the information about the *y*-intercept indicated that they should perform a vertical translation and an answer of $y=(x+2)(x+5)(x-1.5)-30$ was fairly common. Some candidates had an epiphany in part (ii) when they realised that their coefficients should be twice the size and sensibly went back to this part and corrected their error.

(ii) Many scored only one mark in this part, for correctly expanding a pair of their binomial factors, even after making an error in part (i). As said previously, the light dawned for many in this question and it was good to see that some of these made corrections to part (i). However, many did not and very often there would be a multiplication by 2 done at the end – with or without some attempt at justification for it.

(iii) Candidates found this straightforward on the whole, with many scoring full marks for this part. Nearly all drew an accurate line of sufficient length to intersect the curve in three places.

Occasionally some read the scale incorrectly when finding the negative solution or were careless with signs, omitting the negative when writing it down.

(iv) There were many attempts to substitute their answers from part (iii) into the given quadratic. Many candidates did not know how to obtain this quadratic, although most eventually went on to attempt to solve it using the formula, sometimes making arithmetic errors in so doing. Of those who did attempt to derive the quadratic, there were several attempts at equating the wrong pair of equations. Some who started correctly expected to see the given answer immediately and stopped at the simplified cubic they had obtained, sometimes having an erroneous -20. Relatively few were able to show that the quadratic factor was the required one, by long division or by showing multiplying out. A very few candidates used an elegant method of equating the line and cubic and using the factorised form of each to cancel a factor of $x + 2$ on both sides before simplifying.

4752 Concepts for Advanced Mathematics (C2)

General Comments:

The paper was accessible to a large majority of the candidates, and most candidates seemed well-prepared. A significant minority of candidates demonstrated a fair degree of understanding of Core 2 syllabus material, but failed to do themselves justice in the examination because of poor (GCSE level) algebra and careless arithmetical slips. Premature approximation followed by over-specification of final answers also cost some candidates easy marks.

Most candidates presented their work neatly and clearly, but in some cases work was very difficult to follow, and candidates should understand the importance of presenting a clear mathematical argument, especially when there is a "show that" request in the question.
It is disappointing to see some candidates misquoting formulae that are given to them in the booklet.
Centres are advised that using a graphical calculator to evaluate a definite integral or to solve an equation (eg question 10) will earn no credit unless the relevant working is presented.

Comments on Individual Questions:

Question No. 1
This was done well. A large number of candidates omitted " $+ c$ " and lost an easy mark, and a few candidates went astray when simplifying $7 \div \frac{7}{2}$, or didn't simplify it at all. A very few differentiated instead of integrating.

Question 2
Part (i)
This was done very well. A few candidates didn't appreciate the meaning of Σ and merely listed the terms. Similarly, a small number of candidates simply added the first and the last terms. Very few resorted to AP or GP formulae.

Part (ii)
Most recognised the arithmetic progression, but some were uncomfortable with a non-numerical *a* and made a spurious attempt to find its value. For a significant number of candidates, the tenth term was either left as $a + 9\times5$ or simplified thus: $a + 45 = 45a$. In both cases an easy mark was lost. Many started again to find the sum of the first ten terms, and did so successfully. There was no credit for those candidates who left their answers in terms of *a* and *d*. A number of candidates wasted time by trying to find the numerical value of *a*.

Question 3
Many candidates scored full marks here. A few switched the values in the numerator round to obtain + 6 and lost both marks. A small minority found the reciprocal of the gradient, which didn't score, and a tiny minority wrote down the correct calculation, but obtained an incorrect answer. Some candidates needlessly went on to obtain the equation of the chord, and then differentiated it just to convince themselves that the gradient really was – 6, and a few went straight to finding the equation of the chord, and then left the answer embedded, which cost the accuracy mark.

Question 4
Both parts were generally very well done, with many candidates scoring full marks. The most common errors were (6, − 6) and (12, − 3) in part (i) and (18, − 3) and (6, − 1) in part (ii). Some candidates applied the scale factor to both values.

Question 5

The cosine rule was very well understood and most candidates scored full marks. A small number left the calculator in radian mode and lost the final mark; a very small number tried to use Pythagoras or lost their way after earning the first method mark.

Question 6

This was very well done. By and large the correct formulae were used and the entire solution was worked in radians, nearly always resulting in full marks. Some candidates worked in degrees and then worked with rounded numbers, often following on to over specify their answer and lose the final mark. A significant minority did not use $\frac{1}{2} \times r^2 \times \sin\theta$, but used a variety of methods in order to arrive at ½×base×height for the area of the triangle. Often this went astray, resulting in a loss of three marks.

Question 7

Most candidates wrote down the required equations, and most went on to eliminate one of the variables correctly. What followed often proved too difficult, and no further marks were earned. A number of candidates obtained negative answers for both *a* and *r*, but never suspected anything was amiss.

Question 8

Most candidates recognised one of the trigonometric identities required, and then made no further progress. Of those who spotted both relationships, a good proportion made a mess of simplifying the fraction, often resulting in a final answer of $\frac{1}{cos\theta}$ .A surprising number tried squaring top and bottom, or concocted an equation which they attempted to solve.

Question 9

Most candidates started correctly, a few doubled 71.6 instead of halving it, but most successfully obtained 35.8°. 215.8° was frequently found, but the other two values were often missed. Some candidates rounded off their calculator value, and then over-specified their final values (215.79 etc was common), thus losing the second A mark. A common error was arctan(1.5) to start, and some candidates unwittingly worked in radians and went on to add multiples of 90°.

Question 10

Most candidates understood the initial step, but many omitted the brackets and never recovered. Many of those who did earn the first mark often made errors in manipulating the equation, and scored no further marks. The best candidates usually went on to score 4/4.

Question 11

Part (i)

Most knew what to do here, but $8x^{-3}$ and $1 - 8x^{-3}$ were often seen. Only a few candidates failed to show sufficient detail of their working to earn the third mark following a fully correct $\frac{dy}{dx}$.

Part (ii)

There were many correct solutions, although some candidates neglected to find the corresponding value of *y*, or evaluated the second derivative as + 1.5, and concluded the stationary value must be a local maximum. A few obtained $x = 2$ following correct differentiation, but never looked at the graph to realise that this must be wrong. It was surprising just how many candidates solved $8x^{-3} = 0$ to obtain $x = 2$, without realising that something must have gone wrong.

Part (iii)

This was generally well done. Only a small minority of candidates did not understand how to obtain the gradient of the normal, and many obtained follow through marks, at least. Some candidates slipped up finding the value of *y*, and a few made sign errors when finishing off.

Question 12
Part (i)
This was very well done, with many candidates scoring full marks. The most common error was the omission of the outer brackets; occasionally h = 5 was seen, and occasionally y- values were misplaced.

Part (ii) A
Many correctly substituted x = 12, and showed their working so that even if arithmetic went astray, a method mark was still earned. A common error was to omit the minus sign from the first term. Strangely many candidates stopped there, or subtracted 10.872 from 12 instead of 11.

Part (ii) B
This was generally very well done. Occasionally candidates made a sign error or inserted an extra zero in one or both of the first two terms. Some left “9” untouched or used an upper limit of 12 instead of 15.

Question 13
Part (i)
Wrong working often spoiled a correct final answer in this question.

Part (ii)
This was very well done. A few candidates made errors in the table – usually the first or the penultimate value. A tiny minority gave all values to a different degree of accuracy to the one requested, thus losing two easy marks – although credit was still available for the plots and the line. Most plotted the points adequately and drew a single ruled line of best fit across the whole range of x-values to earn two marks.

Part (iii)
Those candidates who used their graph to find the gradient and the intercept often went on to score full marks in this part. Those who adopted other methods such as simultaneous equations often went astray, and obtaining a positive value for the Y- intercept or a large value for the gradient evidently did not cause concern. It would seem that a significant minority did not connect this part with earlier parts of the question.

Part (iv)
t = 70, 10 and 55 were all seen, but many candidates used t = 60 successfully with their model, and then subtracted either 15.9 or f(50) to earn both marks. Unfortunately a few candidates stopped at f(60) and lost both marks.

Part (v)
Many candidates wrote sensible responses to this question. Unfortunately, many of them failed to score, in spite of their likely truth, as they were vague or missed the point. Candidates were expected to comment on the model continuing to predict an ever increasing rate of reduction in the thickness of the ice, in spite of the fact that at some point all the ice will have melted.

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments:

This paper appeared to be broadly in line with recent papers in the level of difficulty of the questions and variability of responses. All candidates appeared to have time to complete the paper. Section A was often very well done, with plenty of straightforward tests of understanding. The two Section B questions had some more demanding parts which tested more able candidates. In general most candidates seem well prepared, and there were no obvious areas of the syllabus which were weaker than others.

The standard of presentation was as usual extremely variable, ranging from very well argued, well written solutions to others which were difficult to follow. Candidates should be advised that presenting logical mathematical arguments, rather than disjointed train-of-thought attempts, is likely to encourage accurate work. Many candidates required additional sheets to answer questions, usually caused by multiple attempts. Sometimes, their final attempt gained fewer marks than preceding solutions – candidates should be advised that their final solution, rather than the best, will be marked.

One issue worth highlighting from this paper is that of exact answers. Often the use of calculators encourages inexact solutions, but if the question asks for an exact answer (as was the case quite often in this paper) it should not be approximated. Sometimes we can ignore subsequent working after an exact answer is seen.

Comments on Individual Questions:

1 This question proved to be a straightforward starter for 3 marks. Sign errors in integrating sin 3x occurred occasionally, as did $-3\cos 3x$ instead of $-1/3 \cos 3x$.

2 Plenty of candidates scored 5 marks here with little difficulty. Some missed out the derivative of $1-\cos 2x$, and some wrote $1/2\sin 2x$ instead of $1/(1-\cos 2x)$. The substitution of $\pi/6$ into the correct derivative was usually done correctly. Some approximation of $2\sqrt{3}$ was found, but could usually be condoned by ignoring subsequent working.

3 Although plenty of candidates scored full marks with apparent ease, there were all sorts of errors as well. Some clearly do not understand the modulus function; many duplicate work by solving 4 equations from $\pm (3 - 2x) = \pm 4x$, and in the process produced additional solutions due to poor algebra. A surprisingly common error was to write $3 = 6x \Rightarrow x = 2$! Some even discounted the solution $x = -3/2$ on the grounds that answers to a modulus question need to be positive! Squaring both sides was seen occasionally, and although this method is somewhat long-winded, it does avoid conceptual errors such as $|3 - 2x| = 3 + 2x$.

4 (i) For some, this was a write-down for 2 marks. Some used transformation arguments, other substituted in particular coordinates. The most common errors were $a = 3$ and $b = 2$ or ¼ .

4 (ii) We allowed plenty of follow-through marks for the first two method marks here, which were almost always gained. Fully correct inverse functions were common. As for domain and range, most seem to know that the these are the reverse of the domain and range for f. However, the 'A' mark here demanded accurate use of notation, with '*x*' used for domain and '*y*' for range, and this mark was often lost.

5 This question proved to be accessible to the overwhelming majority of candidates, and there were many fully correct solutions. Even those who failed to get full marks usually picked up an M1 for a correctly stated chain rule, B1 for $dV/dr = 4\pi r^2$, and a B1 for $dV/dt = 10$. Approximate answers are perhaps preferable in a contextual question, but exact answers were also allowed.

6 Exponential growth and decay is well understood by most candidates, and this was a high scoring question.

6 (i) This was a very accessible two marks, provided candidates answered question – the loss in value rounded to the nearest £100.

6 (ii) Again, this was very well answered. Occasionally the final 'A1' was missed by skipping straight from $-k = \ln(13/15)$ to $k = 0.143$: as this is a given answer, some additional working was required. Occasionally, the result was verified by substituting $k = 0.143$ and evaluating $15000e^{-0.143}$. This was treated as a special case and got 1 mark only. It is important candidates know the difference between 'show' and 'verify'.

6 (iii) This was a little more demanding, requiring candidates to combine the $e^{-0.2}$ and $e^{-0.143}$ terms. This defeated quite a few candidates. Some listed the values of each car for $t = 1$, 2, 3, etc years, and if successful picked out $t = 5$ as when they were closest. This was judged to be worth 1 mark only.

7 This proved to be quite an effective test of understanding of proof and disproof, albeit for only 4 marks.

7 (i) Most candidates stated this was false and looked for a counter-example, usually 25 and 27. We did require them to show their counter-examples were composite for a second B mark.

7 (ii) This was less successful. Most candidates could see it was true, but then failed to come up with a coherent argument. Some wrote $2n(2n+2)= 4n^2 + 4n$ or equivalent, but then failed to explain why this is then divisible by 8 (rather than just 4, which got 1 out of 2). Most successful candidates got the idea that alternate even numbers are divisible by 4 and hence the product of this with another even number is divisible by 8.

8 (i) Most candidates stated that for an odd function $f(-x) = -f(x)$ or equivalent. It is important when writing $f(-x)$ that brackets are placed round the $-x$ terms: if these were missing, the 'A' mark was lost. The structure of this 'show' was often a bit 'muddy': $f(-x) = \ldots = \ldots = -f(x)$ is clear, but writing $f(-x) = -f(x)$ and then writing expressions for each side of this equation below and showing they are equal is less so, as the direction of the argument, or implications, is not clear. The geometrical description of an odd function required three elements: 'rotational', 'order 2' and 'centre O' or equivalent; reflection in Ox followed by Oy was also allowed.

8 (ii) The difficulty with this sort of product or quotient rule question lies in factorising and hence simplifying the expression, and this was the case here. Many wrote down correct expressions, but then failed to 'show' the printed answer. This difficulty often encouraged multiple attempts, sometimes using a quotient rule, followed by a product rule, etc. A surprising number of candidates muddled up their 'u' and 'v' and quotient and product rule, for example using $v = (2+x^2)^{-1/2}$ in their quotient rule. Often the final answer failed to score because we insisted on this being simplified to $1/\sqrt{2}$ or equivalent.

8 (iii) A substantial minority of candidates thought this integral should be done by parts, and therefore scored nothing after the first B1. Those who tried substituting often got muddled before arriving at $\int 1/2\sqrt{u}\, du$, and some then integrated this incorrectly, e.g as $\ln\sqrt{u}$.

8 (iv) This simple piece of algebra was often over-complicated by round-the-houses methods. An all- too-commonly seen mistake was $x^2/(2+x^2) = x^2/2 + 1$.

8 (v) The implicit differentiation was usually correct, as was the algebra to arrive at the printed result. The exact logic behind why $x = 0$ and $y = 0$ could not be substituted into the result expression was often faulty (for example many stated the result would be zero or infinite); we condoned this provided they stated the idea that division by zero is undefined or not possible.

9 (i) This was well answered by the majority of candidates.

9 (ii) The product rule here was generally well done, followed by substituting $x = -\frac{1}{2}\ln m$, where some sign errors occurred. Some left the $e^{\ln m}$ terms unresolved, which was condoned here. The main error was to get a derivative of $-2xe^{-2x}$.

9 (iii) The first two marks here were the least successfully answered, because most candidates were not familiar with the fact that lines equally inclined to the *x*-axis have gradients *m* and –*m*. Only the best candidates found the result successfully. However, many recovered to find the coordinates of P correctly.

9 (iv) This question tested the more able candidates. The integration by parts required careful control of negative signs and accurate work; the area of the triangle (or integral of the line) were quite often discernable from the working, which was often scrambled and incoherent.

4754 Applications of Advanced Mathematics (C4)

General Comments:

This paper was of a similar standard to previous papers-or perhaps slightly more straightforward. The questions were accessible to all candidates but there were sufficient questions for the more able candidates to show their skills. The standard of work was quite high.

The comprehension was well understood and good marks were scored in this Paper.
It was pleasing to see that candidates have improved in some areas particularly when:-

- Remembering to put a constant when integrating
- Reading questions carefully
- Giving more detail when trying to establish a given result
- Using brackets
- Giving exact answers when required

Some candidates still use rounded answers when going on to use their answers in subsequent calculations. Also, too many candidates used values to 3dp when calculating an answer to 3dp in the trapezium rule.

The trigonometry question 4 involving 'Show' caused problems for many. Some felt that they could divide one side of an equation by something, usually $\cos\alpha\cos\beta$, without doing so on both sides. The concept of 'equals' was not understood. There was a general lack of rigour in this question.

Candidates should be encouraged to change constants when multiplying through, say, by a value.

Some candidates still assume that showing that a vector is perpendicular to one vector in the plane is sufficient to show that it is a normal vector.

Quite a number of candidates failed to attempt some parts.

There did not appear to be a shortage of time for either Paper.

Comments on Individual Questions:

Paper A

Question 1
Most candidates understood the method of expressing the fraction in partial fractions. Many were completely successful and most errors were arithmetic. A few incorrectly used
$\frac{A}{(2-x)}+\frac{B}{(4+x^2)}$.

Question 2
Much here depended upon the candidate's ability to factorise correctly. On too many occasions the factor was found to be 4 or ¼ instead of 8. The general method for expanding the binomial expansion was understood and the binomial coefficients were usually correct. Some who had factorised correctly then forgot to include the 8 at the final stage. The validity was often correct but was sometimes omitted and a variety of incorrect responses were also seen including $-1/4<x<1/4$.

Question 3
(i) Many errors were seen here. In a number of cases the candidates were in degree mode. For others *h* was given incorrectly. Many others used the wrong formula and some substituted *x* values in the formula or omitted 0 from the formula. However, probably the most common error was giving the *y* values to 3dp and then using these to give a final answer correct to 3dp.
(ii) This was a good discriminator as it really tested whether candidates understood how the trapezium rule estimates area. Some believed that it always underestimated or always overestimated.

Question 4
(i) There were some very good solutions here when showing the two trigonometric expressions were equal. However, the majority were not successful. The most common overall error was not treating both sides of an equation equally. Too often only one side was changed.
A common starting point was
$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta}=1-\tan\alpha\tan\beta$.
This was then followed by a confused attempt at dividing by $\sec\alpha\sec\beta$.
Candidates need to multiply 'top and bottom' by the same thing. Questions that involve 'Showing' need more rigour.
(ii) This part was more successful provided that candidates wrote down the identity for $\sec^2\alpha$. There were, however, some long and confused attempts.
(iii) Most candidates scored the first two marks here. Many failed to give the second solution of 150º.

Question 5
(i) Scores here tended to be 4 or 1. Much depended upon whether the candidates used the product rule when finding d*y*/d*t.* The very common error was to state that $dy/dt=2te^{2t}$. The majority of candidates did understand the general method of using d*y*/d*t* ÷ d*x*/d*t*. A number also failed to cancel their final answer, often leaving it as $\frac{e^2}{e^3}$.
(ii) Most candidates found *t* correctly in terms of *x* and then substituted it into *y*. The simplification was not always then complete.

Question 6
Many successfully scored full marks when finding this volume of revolution. Common errors included failing to rearrange correctly for *x* (or x^2) and using the lower limit as 0. A few tried to use *x* limits in a *y* function, or tried to find $\int \pi y^2 \, dx$.

Question7
(i) Most candidates scored well in this part. Most understood how to find the lengths of the edges and the angle CAB. Some candidates used the cosine rule in this part but most used the scalar product. Finding the area of the triangle was less successful. Often, ½ base X height was used with lengths AB and AC, or occasionally ½ *a*b cos*C*.

(ii) (A) Usually two marks were scored in this part, but some candidates only found the scalar product with one vector in the plane.
(iii) (B) The equation of the plane $4x-3y+10z+12=0$ was usually found correctly. Some had i,j,k in their answer instead of x,y,z.
(iv) Candidates usually gave the correct equation of the plane although sometimes '*r*=' was omitted. The method for finding the point of intersection was usually understood but there were numerical errors. Many obtained full marks.
(v) This was one of the least well answered questions on the paper. This was because candidates thought that the perpendicular height of the tetrahedron was 5, equal to the height CD.

Question 8
(i) The method of separating the variables and integrating was more popular than verification, and was more successful. Those that verified usually forgot to use the initial conditions. When integrating there was sometimes confused work when the arbitrary constant was changed but continued to be used as *c*.
(ii) Good marks were scored in this part by all candidates. Some made the question more difficult when finding *A* by using a quadratic equation. The most common error was in using
$\sqrt{0.5}$ as 0.25 when finding *t.*
(iii) A pleasing number of candidates scored full marks here. Most separated the variables correctly and successfully integrated the RHS, including the inclusion of +*c.* Those candidates who realised to expand the bracket and divide often were able to score all the remaining marks. A few used the approach from integration by parts but usually did not reach the end.
(iv) Many good scores were achieved here when substituting to find *B* and *t*. There were a lot of numerical errors from others.

Paper B

Question 1
(i) Most candidates correctly completed the table.
(ii) £24000/150=£160 was usually correct. Some candidates multiplied or divided by additional values, usually 18 or 9.
(iii) This was often correct but often long methods were seen including £24000x1.5=£36000, then 18.75% of £36000=£6750 instead of the more direct approach of £4500 x 1.5=£6750. Some candidates stopped at £6750.

Question 2
This was not particularly well answered. Candidates seemed almost equally to find correctly that k=1/0.35= 2.86 or to use 1/0.65. A number of candidates used 1-0.65 as 0.45.

Question 3
This was a good discriminator on Paper B. There were many solutions scoring full marks. Others often included the wrong number of 0.65s in their total or sometimes incorrectly used 0.7+0.6+0.5+0.4…..and failed to subtract.
Some others had no idea and tried to find say 65% of £3875.

Question 4
(i) The graph was generally good. The commonest error was that it often stopped at x=44 instead of continuing to at least x=45. In other cases the graph looked as though it would cut through the x axis when produced.

(ii) Few candidates were able to give a complete solution that as $x \to \infty, be^{-k(x-17)} \to 0$ and so as $y \to a, a = 1$ or equivalent. Many referred to the general shape of the curve or asymptotes or said that at x=45, y=1. Generally the responses were unclear or incomplete.

(iii) This was well answered although some candidates forgot to compare the result of their calculation with the given point. Most compared 6 with 6.60… but some used 23. A few tried to establish the given constants b and k.

Question 5

Most candidates were successful here. A few did not round a money answer to an appropriate accuracy.

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments:

Most of the candidates in this examination showed that they were well prepared and familiar with the content of the specification. It did not appear that the majority of candidates ran short of time. There were many very good scripts submitted in which the quality of communication was also extremely good, however, a good number of candidates' scripts were much poorer in this respect. Many candidates who were less successful often demonstrated inadequate algebraic skills. It is appreciated that candidates have to work under pressure of time but attention to the placement of brackets, completing expressions and using correct notation could help to avoid errors.

Comments on Individual Questions:

Question 1
This gave most candidates a safe start to the paper and it was well answered by the great majority. The series was successfully split in nearly every case, and the sum of squares could be obtained from the MF2 booklet, for an easy mark. Surprisingly for this level, the second sum was not always correct. It is the wise course to learn the result by heart. Some candidates forgot the multiplier 2 and some thought that the sum $\sum r$ was n. The factorisation of the resulting expression was not always efficiently carried out. Careless use of brackets, or lack of them, could result in a sign error. The final result expected was for linear factors with integer coefficients and a numerical factor of 1/6.

Question 2
Again, many good answers were seen in all parts of this question. There were however some fairly common misconceptions, where in parts (i) and (iii) candidates confused a transformation matrix with the 2x4 matrix of position vectors for the quadrilateral. The 2x2 matrix given in part (ii) was not always correct and most candidates answered part (iii) by carrying out matrix multiplication, not always in the correct sequence. The easiest method, looking for the images of $\overrightarrow{OA}$ and $\overrightarrow{OC}$ after reflection, did not seem to be used. Maybe candidates were uncertain whether simply writing down the result would involve a penalty.

Question 3
This question was again usually well answered, and the best responses left the reader in no doubt of the values of the roots found. In many cases however the quality of communication was poor. For example, the complex conjugate was written down but rarely identified as a root. Solutions to a quadratic equation were obtained, but again it was often not stated that these were indeed the roots required. The most efficient solutions to this problem found the quadratic factor arising from the complex roots and then the second quadratic factor, either by long division or by equating coefficients. It was good to be informed when the process was carried out 'by inspection'. The two real roots then followed. Many solutions seen used the root relationships, and sometimes there was a mixture of the methods. The two real roots could be quite easily found by less sophisticated methods. These were given credit provided there was adequate justification.

Question 4
This was another question for which candidates were well prepared, but in which the unwary did encounter some hazards. A full solution was expected here, displaying how the method of differences worked. Most candidates did this well, showing clearly the cancellation pattern of

terms in the expanded series. Some erred in forgetting that the sum of n terms was required and left off at the r th term. Some failed finally to obtain a single fraction. Some either failed to incorporate the necessary factor of ½, or multiplied by 2.

Question 5
Algebraic mistakes were seen quite often in the solutions to this question. In general the substitution method was the most successful method used, very few candidates used the wrong one, and nearly all dealt with the cubic expansion required. There were some sign errors, usually from lack of brackets, when dealing with the squared term. Another common mistake was to forget the constant term when clearing fractions. Candidates who had a strategy for carefully setting out their work fared best. The method of using the root relationships was popular, and usually successful when careful attention was paid to the signs required in the coefficients of the new equation. Some candidates lost a mark by leaving an expression, not an equation.

Question 6
It was good to see many extremely well argued proofs. Very many candidates showed that they appreciated the need for clarity and logical statements, but the three explanation marks were not always earned. Proof by induction has a standard format:

prove for $n = 1$;
assume the conjecture is true for $n = k$ and hence show it is true for $n = k + 1$;
state "if it is true for $n = k$ then it is true for $n = k + 1$; since it is true for $n = 1$ …" etc.

The mark schemes of past years set out the argument clearly but there are still those who believe that "$n = k$" is sufficient to tell the reader that a result is being assumed to be true, and that "true for "$n = 1$, $n = k$ and $n = k+1$ is adequate to replace "if…then…since…".
This particular series should not have been difficult to deal with algebraically. Candidates could help themselves by choosing the simplest denominator when combining fractions. When this was not done and a cubic obtained by expanding the numerator, it was not always convincingly re-factorised. Some candidates made the mistake of adding $\frac{1}{2k(2k+2)}$ as the $(k+1)$th term.
Some lost a mark because, in their work, the last term of the series was equated to the sum of the terms.

Question 7
Part (i) This part was well answered. Some candidates lost marks by giving a line equation rather than co-ordinates. Each point requires two line equations, not one. Wrapping the two numbers up in brackets is much quicker!
Part (ii) was also well answered. A common error was to omit the horizontal asymptote $y = 0$. The majority of candidates knew how to find both intercepts and asymptotes.

Part (iii) was less well done. It was quite easy to forget that $x = \frac{1}{2}$ was also an asymptote and to produce a curve with three branches instead of four. There was uncertainty about the behaviour of the right hand branch above the x - axis, where a turning point was not shown. A graphical calculator is obviously a valuable tool but there is hardly time for detailed examination of the curve, and an understanding of how the asymptotes are approached is essential. Adequate labelling is also important, for both intercepts and asymptotes.

In the final part (iv), many candidates answered well, using their graphs and not delving into algebra. Some candidates showed that they didn't understand the idea of an asymptote by inserting ≤ or ≥ in their inequalities.

Question 8
Nearly all candidates answered this well. A few forgot to insert j in their final answer. There were some incorrect values for the modulus, $|w| = \sqrt{10}$ being most frequently seen. Some candidates thought it sufficient to find the modulus and argument and to stop there, although the question asks for w expressed in modulus-argument form.
Part (ii) proved to be more difficult. The marks relating to the Argand diagram were on the whole easily obtained, and a convincing sector indicated. It was necessary to give some clear indication of the radius of the circle. It was surprisingly common to see the circle centred at w. It was also not uncommon to see $\theta = -\frac{\pi}{2}$ drawn in the wrong place, usually looking more like $\theta = -\frac{\pi}{4}$. Both the boundary half-lines should have had labels, but some benefit of the doubt was given if they were shown in appropriate positions. Many candidates failed to understand the connection between part (i) and the position of w in their diagram.
The last three marks were very rarely earned. The most popular choice for the requested z was diametrically opposite w, and therefore not in the region. Other positions were also popular, for example on the real axis. Many candidates omitted any attempt at this. When the correct location was found, it was good to see a few candidates using purely geometric calculations of the required distance, showing an ability to think outside the "tunnel" of complex numbers, and providing a very neat exact solution.

Question 9
After the last question candidates found this one straightforward and many earned full marks. In part (i) some candidates used the wrong row and column, (1)(8) + (3)(1) + (-1)(-5) was seen a number of times. There was also a bit of fiddling to get zero, "$-3\alpha + 1 + 5\alpha - 2\alpha + 1 = 0$" was seen quite often.
Part (ii) proved very straightforward.
In part (iii) not all candidates were able to make the link between (i) and (ii). Many evaluated B using $\alpha = 2$ but omitted 1/15 from their answer. It is likely that many candidates overlooked the second request in part (iii) where there was no answer or work shown relating to it.
Part (iv) was again straightforward and very few candidates failed to use a matrix method to solve the equations. A few placed the inverse of A in the wrong position, thereby raising the suspicion that their equations had been solved effortlessly on a calculator and without the understanding of how the matrix algebra worked.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments:

The overall performance of candidates was comparable with previous series. The vast majority of candidates displayed sound knowledge of standard results and techniques; very few scored 20 marks or fewer, and about one-third scored 60 marks or more. Question 3 (matrices) was the best-done question, followed by Question 2 (complex numbers) and then Question 1 (calculus and polar co-ordinates) and Question 4 (hyperbolic functions).

Presentation was generally good and most scripts were easy to follow, although a few made extensive use of supplementary sheets. Two graphs were required and many candidates drew these carefully, to an appropriate size, and with appropriately-labelled axes. There was very little evidence of time trouble.

Candidates could have done even better if they had:

- resolved the ambiguities of sign in Q1(a)(ii) (especially) and Q4(i);
- been more precise with their use of language, for example, in Q1(b)(i), “gradient is positive” is not true for all values of θ;
- used simpler methods if possible, for example, in Q1(b)(ii);
- understood better what an eigenvector is, rather than just how to find one (Q3(b)(ii)).

Comments on Individual Questions:

Question No.1 (calculus and polar co-ordinates).

Part (a) was about the inverse cosine function. In (i), the graph of $y = \arccos x$ was required. Most candidates had a good idea of the general shape but only a small proportion gained both marks. Candidates were expected to label the axes at -1, 1, $\pi/2$ and π and many omitted at least one of these; many also found difficulty in representing the undefined gradient at ± 1. There were many multiple-valued functions, which were given some credit as long as critical points were labelled.

In (ii) candidates were required to differentiate the function. The vast majority could score 2/3 very efficiently, although a few omitted the trigonometric identity required to score the second mark. Very few observed that the gradient was negative, which resolved the ambiguity of sign.

Then in (iii) a Maclaurin series for $\arccos x$ was required. One possible method was to expand the answer to (ii) by the Binomial Theorem and integrate the first few terms, but few chose that route and instead differentiated the expression in (ii) to obtain the second, and then the third, derivative. A substantial number managed this perfectly well, but others lost the minus sign from (ii), and for many it proved too difficult to manage the combination of the product and chain rules required to obtain the third derivative. Most candidates knew how to obtain a Maclaurin series from their results, although some gave the first term as 90, rather than $\pi/2$.

Part (b) was about the polar curve $r = \theta + \sin\theta$. In (i), the vast majority were able to differentiate r with respect to θ, although on a few scripts θ disappeared on differentiation. The explanation of why r increases as θ increases was often insufficiently precise, as mentioned above. The graph was well done, with many scoring both marks.

In (ii), we expected candidates to substitute θ for $\sin\theta$, and then integrate a multiple of θ^2. Many candidates did this, scoring all three marks very efficiently, but a substantial number either attempted to integrate $\theta^2 + 2\theta\sin\theta + \sin^2\theta$ by parts and via double angle formulae, and then either gave up, or substituted θ for $\sin\theta$ at some later stage, or substituted $\sin\theta$ for θ at the beginning. Both methods were rarely completely successful: if candidates ended up with a term in $\cos\theta$ in the result of their integration, they generally did not know how to replace it by a valid small angle approximation.

Answers:

(a)(i) graph, (ii) given answer, (iii) $\frac{\pi}{2} - x - \frac{x^3}{6}$; (b)(i) $\frac{\mathrm{d}r}{\mathrm{d}\theta} = 1 + \cos\theta$ which is never negative; graph (ii) $\frac{2}{3}\alpha^3$.

Question No.2 (complex numbers)

Part (a) was well done. Most candidates were able to form a geometric series and sum it correctly; a few produced the sum to n terms, rather than the sum to infinity. The big hurdle, that of "realising" the denominator, was crossed by a large number of candidates, many of which were able to pursue their solution to a fully successful conclusion. A few multiplied $ae^{j\theta}$ by $ae^{-j\theta}$ and obtained 1, and there were other minor errors, but this question was done rather better than similar questions in previous series.

Part (b) then explored some complex number geometry, and was an excellent source of marks for many candidates. Most managed (i) very efficiently, usually finding the other vertices of the hexagon by expressing $\sqrt{3} + j$ in exponential form or equivalent, and then repeatedly adding $\frac{\pi}{3}$ to the argument, or by using symmetry. Partial credit was given to candidates who did not express their final answers in the required form. A few were determined to find the sixth roots of $\sqrt{3} + j$.

In (ii) they had to square these complex numbers to form a new figure, and find its area. Most managed this well but some errors crept in; a few squared the real and imaginary parts separately, while others made slips which produced more than three distinct complex numbers and ended up finding areas of irregular pentagons and other shapes, rather than the correct isosceles triangle.

Answers:

(a) S given, $C = \frac{a\cos\theta - a^2}{1 - 2a\cos\theta + a^2}$; (b)(i) $2j, -\sqrt{3} + j, -\sqrt{3} - j, -2j, \sqrt{3} - j$; (ii) $2 \pm 2\sqrt{3}j, -4; 12\sqrt{3}$.

Question No.3 (matrices)

Part (a)(i), which required candidates to find eigenvalues and eigenvectors for a 2 × 2 matrix, was extremely well done, with most candidates achieving full marks. Errors, where they occurred, included making slips in forming or solving the characteristic equation (often leading to 6 and −1 as eigenvalues) and giving an eigenvector as $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ when faced with the equation $4x - 3y = 0$. Part (ii) was as well done as part (i), although "eigenvectors" of $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ were not followed through as they give a singular matrix for **P**.

In part (b), (i) was very well handled, with, again, most candidates achieving full marks. There were very few errors in showing that $\lambda = 5$ was a root of the cubic characteristic equation, and the quadratic factor was obtained very efficiently. Showing that the associated quadratic equation had no real roots was usually accomplished via the discriminant; we condoned this being referred to as the "determinant" (among other terms).

Part (ii) was less well done. This part required candidates to consider what eigenvectors do, and, although there were many efficient solutions, many candidates attempted to find the elements of the matrix B, or omitted the part altogether: about a quarter of candidates failed to score at all. A substantial number of otherwise successful candidates misread the second instruction and tried to find $\mathbf{B}^2\begin{pmatrix}-2\\1\\4\end{pmatrix}$.

By contrast, part (iii), using the Cayley-Hamilton theorem, was extremely well done, with a very high proportion of well-expressed and fully correct solutions.

Answers:

(a)(i) eigenvalues 2 and 3; corresponding eigenvectors $\begin{pmatrix}3\\4\end{pmatrix}$ and $\begin{pmatrix}1\\1\end{pmatrix}$; (ii) $\mathbf{P}=\begin{pmatrix}1&3\\1&4\end{pmatrix}$ and $\mathbf{D}=\begin{pmatrix}3&0\\0&2\end{pmatrix}$; (b)(i) given answers; (ii) $\begin{pmatrix}-10\\5\\20\end{pmatrix}$, $\begin{pmatrix}100\\-50\\-200\end{pmatrix}$; x = −4, y = 2 and z = 8; (iii) given answer.

Question No.4 (hyperbolic functions)

Part (i) required candidates to obtain the logarithmic form of arsinh, and then differentiate it. The vast majority knew the method required for the first item and did it efficiently, although as in Q1(a)(ii), not many were able to resolve the ambiguity of sign and there were many spurious arguments involving the sum or the product of the "roots" (one of which did not exist as a real number). Then many were able to differentiate the given expression, but far fewer were able to show convincingly that their result was equivalent to the required $\frac{1}{\sqrt{1+x^2}}$. Some candidates ignored the instruction to "differentiate (*)" and went back to the hyperbolic functions, often because they could not perform the last step and show that their derivative was equivalent to the given result; if candidates had deleted otherwise good work to do this, some credit was given if the work could be read.

Part (ii) was very well done. A variety of (correct) logarithmic forms were given, although some left their answer as an arsinh and others omitted the $\frac{1}{2}$.

Part (iii) was a challenging final part which produced a good spread of marks. Most candidates used the given substitution although a few preferred their own. One or two thought that $\sqrt{25+4x^2}=5+2x$ and then went ahead and used the hyperbolic substitution anyway. A few performed the change of variable from dx to du "upside down", so that cosh u ended up in the denominator (and they had a constant to integrate), but many were able to reach an expression involving $\cosh^2 u$ and many of those used a double "angle" formula to produce an expression they could integrate; fewer converted to exponential form. Having obtained an expression of the form $\frac{k}{4}\sinh 2u+\frac{ku}{2}$, many could not make further progress and write their answer in terms of x; they often just copied the given answer. But there were a substantial number of candidates who could make this last step and obtained full, or nearly full, marks. One or two candidates attempted to reintroduce x by using the exponential form of sinh $2u$: this often filled pages and was never fully successful.

Answers:

(i) given answers; (ii) $\frac{1}{2}\ln\left(x+\sqrt{x^2+\frac{25}{4}}\right)$; (iii) given answer.

4757 Further Applications of Advanced Mathematics (FP3)

General Comments:

The candidates for this paper exhibited a wide range of ability; there were a few with very low marks, and several with full marks. Most candidates were well prepared and were able to demonstrate their knowledge and technical competence within their chosen topics. Q.1 and Q.2 were considerably more popular than Q.3, Q.4 and Q.5. Almost all candidates appeared to have sufficient time to make complete attempts at three questions, and only a few offered attempts at more than the required three questions.

Comments on Individual Questions:

Question No. 1 (Vectors)

Most candidates attempted this question and showed good knowledge of the relevant techniques.

In part (i) the vector product was usually calculated accurately and almost all candidates knew how to find the equation of the plane. Some candidates 'simplified' the vector product (for example by dividing it by 21) and this was penalised if the correct value did not appear anywhere.

In part (ii) the perpendicular distance from a point to a plane was usually found efficiently and correctly. The most common error was an incorrect sign for the constant term (32), and some candidates used the equation of the plane *P* instead of *Q*.

In part (iii) most candidates could find the line of intersection of two planes.

In part (iv) the perpendicular distance from a point to a line was usually found confidently as the magnitude of a vector product, with some candidates using alternative methods.

In part (v) most candidates wrote down a suitable scalar triple product and showed that they knew how to evaluate such a product. One of the vectors needed was AD and very many candidates had difficulty finding this, often confusing it with the position vector OD.

Question No. 2 (Multi-variable calculus)

This question was also attempted by most candidates, and all parts except (v) were generally well answered.

The partial differentiation in part (i) and finding the normal line in part (ii) were done correctly by almost all the candidates.

In part (iii) most candidates used $g_x\delta x + g_y\delta y + g_z\delta z$ to obtain an approximate linear relationship between h and the parameter in the normal line, but many did not go on to find the distance.

In part (iv) most candidates used $g_x = g_z = 0$ to obtain $x : y : z = 1 : -1 : -1$. It was then necessary to substitute into the equation of S; many candidates thought that $g_y = 1$ and used this instead.

In part (v) the first step is to show that $g_x : g_y : g_z = 10 : -1 : 2$ leads to $x : y : z = 1 : 3 : -5$. Then substituting into the equation of the tangent plane gives the required point $(-2, -6, 10)$. If the equation of S is used two possible points are obtained and it is then necessary to check which of these lies on the given tangent plane. A very large number of candidates thought that $g_x = 10$, $g_y = -1$ and $g_z = 2$, giving the point $(-1/4, -3/4, 5/4)$. This earned just 1 mark out of 8.

Question No. 3 (Differential geometry)

Candidates who attempted this question usually showed good understanding of the relevant techniques.

In part (a)(i) the equation was usually rearranged correctly and there were some good sketches of the curve. The sketch needed to show a zero gradient at P, the curve passing through O with a positive and increasing gradient, and Q marked in the first quadrant. Many candidates did not attempt the sketch at all.

In part (a)(ii) the arc length was very often found correctly. Some candidates just evaluated s at Q which gives the arc length PQ instead of the required OQ.

In part (a)(iii) most candidates indicated that they needed to find $ds/d\psi$ (or $d\psi/ds$) but many were unable to differentiate accurately. Those who first wrote s in the form $2\ln\pi - 2\ln(\pi - 3\psi)$ were more successful than the others.

In part (a)(iv) almost all candidates understood how to use their radius of curvature to find the required centre of curvature.
Part (b)(i) was well answered, with many candidates finding the curved surface area correctly. Some used the formula for rotation about the x-axis instead of the y-axis.

In part (b)(ii) the method for finding the envelope was very well understood.

Question No. 4 (Groups)

Most of the candidates who attempted this question were able to demonstrate a sound understanding of the topics tested.

Part (i) was generally well answered. As well as showing that $(a^2b)^6 = e$ it was necessary to show that no lower power was the identity, and not all candidates did this.

Part (ii) was very well done, usually by writing out the composition table for the subgroup.

In part (iii) there are three cyclic subgroups of order 2, one of order 3 and three of order 6. It was common for some of these to be omitted (notably those of order 6), and some candidates included the subgroup from part (ii) or G (which are non-cyclic) in their list.

Parts (iv) and (v) were almost always answered correctly.

In part (vi) most candidates identified $a = 11$ and $b = 19$ and often obtained the correct non-cyclic subgroups.

Question No. 5 (Markov chains)

The candidates who attempted this question usually answered it well. They were very competent at using their calculators, and powers of matrices were almost invariably evaluated correctly.

In part (i) the transition matrix **P** was almost always correct.

In part (ii) the probabilities were usually obtained correctly. Some candidates used $\mathbf{P}^{10}$ instead of $\mathbf{P}^{9}$.

In part (iii) most candidates used the probabilities for the 12th vehicle, but some combined these with the probabilities for the 13th vehicle instead of the diagonal elements from **P**.

There was a lot of good work in part (iv) with candidates repeating the work in part (iii) for different values of *n*. A common error was, after doing all the calculations correctly, to give the answer as $n = 7$ instead of $n = 8$.

In part (v) the expected number was often given as 9 instead of 10.

Part (vi) was well answered. Some candidates only gave the equilibrium probabilities and did not exhibit the limiting matrix as required by the question.

In part (vii) many candidates calculated 0.7225^3 or 0.7225×0.9^3 instead of 0.7225×0.9^2.

In part (viii) most candidates knew how to use the equilibrium probabilities to find the new transition matrix, and many completed this accurately.

4758 Differential Equations (Written Examination)

General Comments:

The standard of the responses on this paper were of a pleasingly high standard, and many candidates scored full marks on some or all of the questions. The methods required to solve the second order differential equations in Questions 1 and 4 were known by almost all candidates and these two questions were attempted by the majority of the candidates. Question 2 was more popular than Question 3, with part (b) of Question 3 proving to be the most challenging to the candidates. It appeared that the syllabus item of tangent fields is less well-known by the candidates.

Comments on Individual Questions:

Question No. 1

Second order linear differential equations

(i) All candidates were familiar with the method of solution required in this part. Any marks lost were because of arithmetical errors in solving either the simultaneous equations to find the particular integral or the quadratic equation to find the roots of the auxiliary equation.

(ii) All candidates applied initial conditions to their general solution from part (i). A minority of candidates used the incorrect initial condition $\frac{dx}{dt}=0$ instead of the given condition $\frac{dx}{dt}=10$ a few candidates did not use the product rule when differentiating.

(iii) The majority of candidates considered the behaviour of their particular solution from part (ii) for large values of t and obtained an expression of the form $x\text{->}p\sin 2t+q\cos 2t$. A significant number of candidates, however, did not seem to know how to find the amplitude of these oscillations, with p or q or $p + q$ being common incorrect answers.

(iv) Almost all candidates scored both marks.

(v) The majority of candidates scored full marks in this part. The most common error was to omit the constant term in the general solution that arose from the zero root of the auxiliary equation. Some candidates used incorrect initial conditions or did not use the product rule when differentiating.

(vi) The first mark in this part was awarded for a sketch in which the solution curve had a positive gradient (given in the question as 10) at the origin. The second mark was awarded for a curve that tended, for large values of t, to the constant value in the particular solution from part (v). The majority of candidates earned both marks.

Question No. 2

First order differential equations

(i) Almost all candidates were able to obtain the correct general solution to this differential equation, but a significant minority did not apply the initial condition. To gain the final mark, comments on the suitability of the model needed to include the idea that it predicted infinite growth for large values of t. About half of the candidates earned this final mark, with many of the other candidates not making a comment.

(ii) This part was answered well by almost all of the candidates. The most common mistake was a sign error when calculating the particular integral.

(iii) A significant number of candidates were unable to give a meaningful Interpretation of the suitability of the solution obtained in part (ii). Many candidates offered the observation that it was more suitable than that in part (i), with no further comment. Others suggested that it was more suitable because it took environmental factors into account. Since this model was introduced in the question as a refined model taking account of environmental factors, neither of these comments earned any marks.

(iv) Attempts at this part of the question tended to be polarised into those that were totally correct and those that gained only one or two marks. A significant number of candidates appeared to struggle to make any real progress and this often led them to abandon their attempts at Question 2 and instead try Question 3. It was very pleasing, however, to see a considerable number of excellent solutions, with candidates showing confidence in the method of solution and an ability to apply it accurately.

(v) The single mark in this part was only available to those candidates who had made progress in part (v) and it was earned by the vast majority of them.

(vi) Again, those candidates who had made progress in part (iv) usually scored both marks in this part.

Question No. 3

First order linear differential equation and tangent fields

This was by far the least popular question on the paper, with a significant number of attempts resulting in little progress.

(a)

(i) Only a small minority of candidates were able to complete the integration involved in finding the integrating factor. The stumbling block was in trying to find $\int \frac{-x}{x+1}\,\mathrm{d}x$. Most, however, used the given result to attempt the general solution for y. A common error was to omit the $(x+1)$ when multiplying through by the integrating factor.

(ii) The method mark was earned by almost all candidates, but accuracy errors abounded, either because of an incorrect solution in part (i) or because of an arithmetical slip.

(b)

(i) Almost all candidates who attempted this part earned both marks.

(ii) Although there were a few excellent solutions to this part, the majority of candidates seemed confused by what was required. It was surprising that having been told, and indeed having shown in part (i), that one of the isoclines was a circle, many candidates did not then sketch this circle. Some hinted at it by drawing a few direction indicators roughly around what would have been the circumference of the circle. Attempts at drawing the tangent field, by sketching direction indicators, were generally weak. Many candidates simply put indicators, in seemingly random directions, at the corners of the squares of the grid.

(iii) Many of the candidates who had made a good attempt at the previous part also sketched an appropriate solution curve. Most candidates, however, made no progress.

(iv) Almost invariably, solutions to this request were very good.

Question No. 4

Simultaneous second order linear differential equations

(i) There were many excellent responses to this part and the majority of candidates scored full marks. The most common errors were made in obtaining the second order differential equation for x, resulting in an incorrect coefficient of e^{-2t} on the right hand side. This usually led to the loss of two accuracy marks.

(ii) Almost all candidates gained the three method marks.

(iii) All candidates made a good attempt at this part. For many candidates, application of the correct method failed to lead to success because of accuracy errors. Some of these followed on from errors in the earlier parts of the question, but many arose when solving the pair of simultaneous equations produced by applying the initial conditions.

(iv) Most candidates made good progress applying the correct method.

4761 Mechanics 1

General Comments:

This paper produced a satisfactory mark distribution. Candidates of all abilities were able to show what they could do but there were places where even the most able were challenged.

A common weakness shown by many candidates was a reluctance to draw good diagrams to guide their work.

Comments on Individual Questions:

Section A

1) This question, about interpreting a velocity-time graph, was well answered. It ended with a request to draw the equivalent distance-time graph, parts of which were curves; many candidates did not realise this and so lost the final mark.

2) This question was about vectors defined using **i**, **j** notation. It was well answered with many candidates obtaining full marks. Most of the errors that did occur were in part (iii) where candidates were required to interpret the vectors as forces on a particle in equilibrium, with its weight now introduced. Some did not interpret the equilibrium conditions sufficiently rigorously, failing to recognise that both the horizontal and vertical components of the resultant had to be zero. The question ended with a request for the mass of the particle and some candidates confused this with its weight.

3) This was a statics question involving three forces. In the first part candidates were asked to draw a triangle of forces and many dropped some of the marks for this; lack of arrows, incorrect or missing labels and errors in the directions were all quite common. A number of candidates drew a force diagram instead and at most only one of the three marks was available for this.

   The final part of the question required candidates to draw graphs of $W\sin\alpha$ and $W\cos\alpha$; many candidates found this difficult and a significant number made no attempt at an answer.

4) This question was about projectiles and was well answered with many candidates gaining all the marks. Virtually all candidates knew what they were trying to do but many made sign errors in the vertical motion equation.

   The most straightforward approach to this question involved treating the motion in a single stage. A few candidates considered it in two, or even three, stages; this increased the scope for errors and consequently most such responses were less than perfect.

   The question ended by asking candidates to comment on the effect of taking a different value for *g*. This produced a pleasing number of highly articulate responses.

5) This question involved motion with non-uniform acceleration. It was set in the context of a science fiction flight to the moon. It was well answered with many candidates obtaining full marks. Only a few candidates made the mistake of attempting to use constant acceleration formulae. There were some mistakes in the integration to go from speed to distance; an extraneous *t* appeared in some scripts.

Section B

6) This question was about motion in three dimensions described using vectors. It was also about modelling. The context was a sky-dive.

In part (i), candidates were asked to use the given (vector) initial velocity to find the initial speed and direction. Nearly all candidates found the speed but there were many mistakes with the direction, particularly finding the wrong angle or using the initial displacement instead of the initial velocity.

In part (ii), candidates were asked to account for the three forces acting on the sky-diver. This involved interpreting the information given in the question. Many did this correctly but there were also candidates who dropped marks here.

In part (iii), candidates were asked to find the acceleration of the sky-diver as a vector and to verify its magnitude. While this was on the whole well answered, a number of candidates omitted to find the vector form and so lost a mark.

Part (iv) carried 6 marks and many candidates obtained all of them. Candidates were asked to find expressions for the velocity and position vector at time t, and then to show that at a given time the sky-diver was at the origin. The commonest mistake among those who otherwise answered this part well was to omit the initial displacement from the position vector. However, several candidates used g for the acceleration instead of the acceleration they should have found in part (iii).

Part (v) was not well answered. Candidates were asked to show that a consideration of the sky-diver's landing velocity showed the model to be unrealistic. This was part of the overall modelling process; they had considered the landing position in part (iv) and were now required to consider the velocity on landing. So they needed to calculate the velocity on landing, or at least its vertical component. Most candidates failed to do this and made comments that were not based on evidence obtained from the question.

7) This question was a good source of marks for many candidates. It was about the motion of a train and the forces in one of the couplings. Most candidates were able to answer the parts that involved considering the train as a whole. Fewer were successful when it came to working with part of the train to find the force in a particular coupling.

In part (i) candidates were asked find the acceleration of the whole train and most were successful. A number failed to convert the mass of the train from tonnes into kilograms. This was penalised here but follow-through was then applied for all the marks in the next two parts and for the first two marks in part (iv).

In part (ii) candidates were asked to find the tension in the coupling between the two trucks. Most candidates answered this correctly but some introduced extraneous forces.

Part (iii) carried 7 marks. The first four of these involved the motion of the train as a whole in a new situation and many candidates obtained all of these marks. The last three marks were for finding the new force in the coupling between the trucks. Most of those candidates who had been successful in part (ii) obtained these marks but others were unable to identify which forces were relevant and which were not.

Part (iv) involved a new situation in which the train was on a slope. Candidates were asked to find the angle of the slope. While there were plenty of correct answers, many of them were not very well explained. Good force diagrams were something of a rarity.

Part (v) provided the last two marks on the paper. It exemplified the interesting (and little known) result that the tensions between trucks when the train is going up a slope at constant speed are the same as those when it is accelerating on level ground, under the same driving force. In this part candidates were asked to do no more than find the same numerical answer as they had obtained in part (ii). Stronger candidates were successful on this but many others had failed to find the correct earlier results (the tension in part (ii) and the angle in part (iv)) that were needed here.

4762 Mechanics 2

General Comments:

The standard of the solutions presented by candidates was generally pleasing. There was the usual wide spread of marks, but most candidates were able to make a reasonable attempt at most parts of the paper. There was some evidence that candidates felt rushed towards the end of the paper and the final part of the last question proved to be a stumbling-block for many, both because it was unusual and because of the time pressure.

As always, candidates should be encouraged to draw clear and labelled diagrams when appropriate, particularly when dealing with forces or velocities. A lot of potentially very good work was marred by sign errors that perhaps could have been avoided by a clear diagram.

A particular issue revealed itself this session in the need for a clear use and understanding of notation. In Question 2(iv) many candidates used the letter F both to represent force in Newton's second law and to represent friction and almost invariably managed to confuse themselves. Similarly in Question 4(b), F was defined to be the resistance, but candidates chose also to use F as the total force in $F = ma$.

Comments on Individual Questions:

Question No. 1

Momentum and Impulse

(a)(i) Many candidates produced a diagram with the masses and velocities all marked clearly and these candidates usually scored full marks in the subsequent calculations. A significant minority of candidates, however, seemed to confuse themselves because of an unclear sign convention. Others made sign errors when writing down the conservation of linear momentum equation.

(a)(ii) Candidates had no problems in writing down the fact that the impulse was equal to the change of momentum, but relatively few scored both marks. A significant number of candidates calculated the impulse of Q on P, without realising that a sign change was required to obtain the requested impulse of P on Q.

(b)(i) Most candidates were able to form a pair of simultaneous equations based on motion parallel and perpendicular to the motion R. A very common error was a sign error in the second of these equations. Other errors resulted from mixing sine and cosine when resolving and from the omission of the masses in all or part of the conservation of momentum equations.

(b)(ii) Almost all candidates who had obtained values for u and v in part (i) were able to calculate the corresponding increase in kinetic energy.

Question No. 2

Centres of mass

(i) The vast majority of candidates were able to find the z-coordinate of the centre of mass of the box A. A few candidates chose to consider either a closed box or a bottomless box, even though the question gave two separate and clear indications that the box was open.

(ii) Many of the solutions to this part were of a high quality, with candidates presenting their work in a clear and concise way. Few candidates used symmetry to find the x-coordinate of the centre of mass of B, preferring instead to use a vector method to find all three coordinates.

(iii) Many candidates produced completely correct solutions using the geometry of the situation. A minority of candidates chose to take moments about the point of tipping, usually successfully.

(iv) A significant number of candidates confused the *F* in $F = ma$ with the frictional force *F* in $F = \mu R$ and this often led to the omission of any meaningful attempt at applying Newton's second law; it was also common to find *g* omitted from the reaction force *R*.

Question No. 3

Forces

(a) (i) This part was usually answered competently, with many candidates scoring the full four marks. The most common error was to ignore the fact that the length of BC was given in the question, as 2 metres. This omission led to some candidates either assuming the length of AB was 2 metres or guessing its value. In such cases, candidates were able to gain the remainder of the marks with accurate use of resolution of forces and/or taking moments.

(ii) Candidates were asked to show all of the forces acting on the pin-joints and most did so. Internal forces should be shown with a pair of arrows, either as a tension or a compression, and also with a label, for example, $T_{AB}, T_{BC},$ and $T_{AC},$ The external forces, *X*, *Y* and *R* needed to be shown, either as letters or with the values calculated in part (i).

(iii) Many candidates scored full marks in this part, offering accurate and concise solutions. Most errors that occurred were as a result of taking moments in directions other than horizontally and vertically. Less often, arithmetic and algebraic errors crept in when simplifying the moments equations.

(b) The two parts of this question were the least well done on the whole paper, with many errors in taking moments and also in understanding what was required in part (ii).

(i) Candidates needed to realise that B was the only sensible point about which to take moments. Almost all those who opted for taking moments about A, G or C omitted to include the force at the hinge at *B* and so made no creditable progress. Of those candidates who did take moments about *B*, a significant number made errors in identifying the correct distances for some of the moments and/or in confusing sine and cosine.

(ii) Very few candidates realised that $Q = P / \sin\theta$ and the majority simply repeated their work as in part (i), with all of its errors. Follow-through marks were awarded for Q, where possible. The majority of candidates did not attempt to find the force exerted on the rod by the hinge at B. Of those candidates who did make an attempt, most found only the component of the force at right angles to the rod. The simplest method of solution was to write down the horizontal and vertical components of this force ($(Q+102)$ N and *X* N and then use Pythagoras' theorem to find the magnitude of the total force.

Question No. 4

Work and Energy

(a) (i) The majority of candidates used an energy method, finding the kinetic energy at A (1378 J) and the potential energy at B (1372 J) and noting that the first exceeded the second. Other candidates calculated the speed of the object at B, noting that it was positive, or found the height at which the object would come to rest as 14.1 m, noting that it exceeded 14m.

(iii) Most candidates made a good attempt at calculating the work done, using the distance of 25 m correctly calculated by use of Pythagoras' theorem. They then wrote down a work-energy equation, enabling them to find a value for the required velocity of the object at D. Unfortunately, a significant number of candidates had wrong or missing terms in their equation, usually because of some confusion with their reference points. It was equally satisfactory to consider energy changes from A to C or from B to C, but not a mixture of the two.

(iv) Although some candidates grasped what was required in this part, and almost all of these candidates scored full marks, many other candidates seemed convinced that they had to use their answer to part (ii) in some way. The key point of understanding was to realise that the initial vertical velocity at D was zero. The speed with which the object reached the ground could then be found from a simple application of a suvat equation, with $u = 0$.

(b) Only a minority of candidates scored more than two of the six marks available in this part of the question. These two marks were awarded for the equation $P = 50F$. Most candidates struggled to find another equation involving P and F and many seemed confused about how to deal with the percentage reduction in the power. Candidates also confused the *F* in $F = ma$ with the resistance force *F* given in the question.
Having said this, some very neat and fully correct solutions were seen where candidates realised that 20% of the driving force was equal to the mass times the acceleration.

4763 Mechanics 3

General Comments:

Most of the candidates demonstrated a sound understanding of the topics involved. The extent to which they could apply these mechanical principles to the particular problems set varied considerably and resulted in a wide range of marks for the paper. The questions on dimensional analysis (Q.1(a)), simple harmonic motion (Q.1(b)) and circular motion (Q.2) were generally well answered. The situations in Q.3 (based on elasticity) presented challenges to very many candidates, because care was required in setting up the equations of motion and the work-energy principle. The centres of mass question (Q.4) presented challenges in the integration and the application to toppling. Most candidates obtained the bulk of their marks from Q.1 and Q.2, and the average mark for the paper was somewhat lower than it has been in the recent past.

Comments on Individual Questions:

Question No. 1 (Dimensional Analysis and Simple Harmonic Motion)

Most candidates answered this question well. In part (a)(i) the dimensions of density were almost always stated correctly and used to find the dimensions of Young's modulus. The only common error was to use $E = \rho v$ instead of of $E = \rho v^2$.
In part (a)(ii) most candidates calculated the value correctly. The relationship between the dimensions found in part (a)(i) and the SI units of Young's modulus was well understood.
In part (a)(iii) most candidates knew how to use dimensional analysis to find the indices in the equation, and very many completed this accurately.
In part (b)(i) most candidates attempted to apply the equation $v^2 = \omega^2(A^2 - x^2)$ at the points X and Y, and a good number followed this through to find the amplitude and the period. A common error was to obtain $\omega = 0.04$ instead of $\omega^2 = 0.04$; another was, after finding ω, to omit finding the period.
In part (b)(ii) most candidates used a suitable displacement-time (or, rarely, velocity-time) equation, with their A and ω, to find the time taken.

Question No. 2 (Circular Motion)

In this question most candidates were able to demonstrate their competence at solving problems involving circular motion and conservation of energy.
In part (a)(i) it was necessary to realise that the tensions in both parts of the string were equal, and both parts had vertical components; candidates then usually completed the calculation successfully.
In part (a)(ii) most candidates wrote down a correct equation of motion. Many found the speed of the ring correctly but did not convert this into the required angular speed.
In part (b)(i) candidates needed to form an equation of motion in the direction of the string, and most candidates did this correctly.
In part (b)(ii) a successful solution required a radial equation of motion and an equation involving kinetic and potential energy. Most candidates formed the radial equation correctly, with others making sign errors or mistakes in resolving the weight. Many candidates made errors in the energy equation, notably in the potential energy. Those who considered potential energy at the start and finish (rather than finding the change in potential energy directly) quite often used two different reference points.
In part (b) (iii) it was again necessary to form a radial equation of motion (with zero tension in the string) and an energy equation. Candidates performed in a similar way to part (b) (ii).

Question No. 3 (Elasticity and Energy)

Most candidates demonstrated their understanding of elastic strings and elastic energy. This question assessed their skills in applying these principles, and produced a wide range of performance.
In part (i) the simplest approach was to resolve parallel to the plane, and candidates who did this were often successful. To earn full marks it was necessary to justify the given value to an accuracy of at least 4 significant figures, so finding the angles to one decimal place, as many candidates did, resulted in the loss of one mark here. A fair number of candidates preferred to resolve horizontally and vertically; this approach was sometimes successful, provided that the normal reaction was included in the equations.
In part (ii) (A) the resultant force is simply the extra applied force of 18 N up the plane. Many candidates started again and repeated much of the work already done in part (i).
In part (ii) (B) most candidates obtained the new tension in the string correctly, and attempted to form an equation of motion parallel to the plane. Common errors in this equation included resolving incorrectly and omitting one of the three forces (usually the 18 N). The question asked for magnitude (which must be positive) and direction, so an answer '−0.584 up the plane' did not earn the final mark. Some candidates persisted in forming vertical and horizontal equations of motion, and it was very difficult (but not impossible) to earn any marks using this approach.
In part (iii) most candidates formed an equation based on energy. The most common error was omission of the work done by the 18 N force. There were also many sign errors, and the change in elastic energy was sometimes calculated incorrectly.

Question No. 4 (Centres of Mass)

Most candidates showed a good understanding of how to find the centre of mass of a solid of revolution in part (i) and a lamina in part (iii). This question also tested integration (notably integration by parts) and an application of the centre of mass in part (ii).
In part (i) there were often errors, such as incorrect signs, in the integration of xe^{-2x}. When a correct expression for the x-coordinate of the centre of mass had been obtained it was necessary to rearrange it into the given form, and this was not always done convincingly.
In part (ii) most candidates attempted to show that the centre of mass was vertically above a point in the base and applied some relevant trigonometry. Many had the radius of the larger circular face as e^{-k} (which is the radius of the smaller circular face) or k (which is the 'height' of the solid) instead of 1. To earn full marks it was necessary to say that the x-coordinate of the centre of mass was always *less than* ½ (for example, *tends to* ½ was not sufficient). Many candidates did not attempt this part at all.
Part (iii) was completed correctly by many candidates. The main difficulty was the integration by parts when finding the x-coordinate.

4764 Mechanics 4

General Comments:

The work on this unit was generally of a good standard. Many of the candidates were very competent and these demonstrated a sound understanding of the principles of mechanics covered in this module. However, a small number of candidates struggled with the majority of the paper and were not able to apply principles appropriate to the situations. Candidates seemed to be particularly confident when solving differential equations and manipulating complicated expressions and most demonstrated a solid knowledge of the techniques and concepts required. The majority of candidates appeared to have sufficient time to complete the paper. The standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions:

Question No. 1 – Variable force

In part (i) the vast majority of candidates correctly applied Newton's second law of motion and used $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for the acceleration. The technique of separation of variables and integrating was well understood and executed well by the majority of candidates. Most went on to use the correct initial conditions to determine the constant of integration and the majority then went on to derive the given result correctly. The most common errors included incorrect signs used in the integration and incorrect application of the laws of logarithms when attempting to get v^2 in the required form. Part (ii) was often done well with the majority of candidates integrating the correct expression between the required limits; others adopted a work-energy approach. The most common errors with the integrating were to use the net force rather than the resistive force or not consider the lower limit at $x = 0$. Those that applied the work-energy principle usually scored full marks.

Question No. 2 – Variable mass

This question was found to be the most demanding on the paper and very few candidates made significant progress in either part. In part (i) the majority of candidates stated the mass of counterweight correctly as $m_1 - \lambda t$ but very few could derive the relevant equations of motion for the counterweight and bag correctly. The approach of considering the motion of both parts of the system separately was rarely seen and the majority of candidates attempted to either derive the equation of motion for the whole system from first principles or simply write down the equation of motion for the entire system. The most common errors seen were the inclusion of either incorrect or additional terms involving the quantities m_1, m_2 or λt or the inclusion of an incorrect term involving the velocity of the counterweight (usually present because, when considering the whole system from first principles, the momentum of the sand lost by the counterweight was neglected). Those candidates who did derive the correct differential equation for the velocity of the counterweight usually went on to derive the given result.

Part (ii) was also not well answered by the majority of candidates with many not making any real progress beyond stating the correct relationship between λ and m_1. Many candidates did not realise that the maximum velocity of the counterweight was when $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ and simply substituted the $t = 10$ into their expression for v. Candidates who did find the correct time when the counterweight was at its maximum velocity usually went on to find the correct velocity either exactly or to an acceptable degree of accuracy.

Question No. 3 – Equilibrium

Part (i) was done extremely well by the vast majority of candidates and nearly all handled the necessary calculus and trigonometry accurately.

In part (ii) many candidates did not explicitly show that there was a position of equilibrium when $\theta = \frac{\pi}{2}$ but instead assumed that $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ and attempted to therefore show that $\theta = \frac{\pi}{2}$. Most candidates correctly went on to show that the system was in unstable equilibrium at this value of θ but not all candidates showed sufficient detail in establishing the given result. A number of candidates made incorrect attempts to use the first derivative test to determine the nature of stability.

The responses to part (iii) were mixed; some excellent succinct solutions were seen, either by taking moments, or applying Hooke's law. Those that adopted the former approach were usually more successful as many of those that took a Hooke's law approach left their answers in terms θ. A number of candidates did not realise that the magnitude of the tension in the string and the vertical force at the hinge were equal and therefore they attempted to derive the vertical force from scratch; these attempts were rarely successful. In part (iv), the majority of candidates correctly stated that at the two further positions of equilibrium $mg - \frac{4\lambda}{5}(1 - \sin\theta) = 0$ but failed to justify accurately the given inequality. Part (v) was rarely done well and many candidates made little progress in this part. Stronger candidates realised that at the other positions of equilibrium the second derivative simplified to $\frac{4}{5}\lambda a \cos^2\theta$ but many of these simply stated that this expression was positive in the given interval with the majority failing to consider the case when this expression might be zero.

Question No. 4 - Rotation

Part (a) discriminated quite well and while many candidates scored full marks a number made very little progress beyond stating the moment of inertia of the central wooden cylinder. A number of candidates attempted to derive the moment of inertia of the outer ring of steel from first principles but few did so successfully. Those that correctly found the moment of inertia of the outer ring of steel in terms of the radius of the central cylinder usually went on to find this radius successfully.

Part (b) (i) was done well and many candidates correctly found the kinetic and potential energies of the two blocks in terms of t and went on correctly to derive the given result. Most candidates who attempted a method involving Newton's second law failed to give enough detail to establish convincingly a given answer. Most candidates correctly applied the principle of conservation of energy to derive the stated moment of inertia of the pulley.

Most candidates started part (b) (ii) well with a significant number of them correctly applying the rotational form of Newton's second law to set up the correct differential equation for the angular velocity of the pulley. However, a number of candidates introduced sign errors (mostly that of the resistive couple). While the vast majority of candidates separated the variables of the resulting differential equation and went on to integrate correctly many candidates then tried to find the constant of integration by incorrectly assuming that at $t = 3$ the angular velocity was 0 (rather than the correct value of 24). Those that did apply the correct initial conditions usually went on to derive the correct given result.

The responses to part (b) (iii) were mixed; many candidates either made no response or only attempted part (b) (iii) (A). However, most of those that did attempt part (b) (iii) (A) found the correct value for k at which the pulley continues to rotate with constant angular velocity. It was rare to see the correct set of values for k in either parts (b) (iii) (B) or (C) and even when the correct critical value of 36.75 was seen it was rare to see it used with the correct interval in these two parts.

4766 Statistics 1

General Comments:

The majority of candidates coped well with this paper. A good number of candidates scored at least 60 marks out of 72 and there were quite a number who achieved full marks. There was no evidence of candidates being unable to complete the paper in the allocated time. As in previous years, only a small minority of candidates attempted parts of questions in answer sections intended for a different question/part and most candidates had adequate space in the answer booklet without having to use additional sheets.

Surprisingly many candidates seemed to cope better on the topics which are not part of GCSE than they did on Question 1, which is was a very standard GCSE topic. Candidates performed rather better on the conditional probability question, than in the past, although this topic still causes difficulties for many. The majority of candidates found Q4(ii) very difficult, with the many scoring at most 1 mark out of 3. In Question 5, many candidates did not provide a convincing explanation of why $k = 0.09$, with quite a number substituting $k = 0.09$ into the given formula and trying to show that the sum of the probabilities was 1. This was only given credit if there was very convincing working. The earlier parts of Question 7 on the binomial distribution and hypothesis testing was fairly well answered, with many candidates defining the hypotheses correctly, and also carrying out the hypothesis test correctly. In the last part of this question, candidates often found $P(X \le 0)$ for $n = 3$ but omitted $P(X \le 0)$ for $n = 2$, and so only scored one mark out of three. Most candidates supported their numerical answers with appropriate working, but when written explanations were required, as in Q6(v), the poor handwriting and in some cases the poor use of English of some candidates made it difficult to determine what they were trying to say.

Fortunately, rather fewer candidates lost marks due to over specification of some of their answers, than in past sessions. A number of candidates, did however over specify some of their answers, particularly in Q6(ii), where candidates often gave an answer of 63.416, some adding ‘to 3dp’, which they thought was appropriate accuracy. Of course it is the number of significant figures rather than the number of decimal places that is important, and giving an estimated mean to 5 significant figures is not sensible and so attracted a penalty.

Comments on Individual Questions:

Question No. 1(i)
Many candidates gained full credit. A common error which resulted in the loss of 2 marks was to plot the correct height but at mid-points. Only a few used the lower class boundaries. Some candidates drew cumulative frequency bars and a small number just plotted frequency against midpoints. Some candidates forgot to label their axes or more often omitted the word “cumulative” on their vertical axis.

Question No. 1 (ii)
This part was very well answered with many candidates picking up the follow through marks for correctly identifying the median and quartiles from their mid-point plotted graph.

Question No. 2 (i)
The vast majority of candidates were able to correctly construct the tree diagram although it did appear that quite a few needed two attempts (it looked as though there may have been some rubbing out under the final version). Only a very small number of candidates omitted any of the required labels or mixed up some of the probabilities, but these candidates were able to gain

follow through marks in subsequent parts of the question. A few candidates omitted the middle set of branches, or added extra sets following ‘Accept’ or ‘Reject’.

Question No. 2 (ii)
This was generally very well answered.

Question No. 2 (iii)
Candidates found this part much more difficult and many gave an answer of 0.096, which is simply the probability that a candidate for the job is retested at least once and accepted, so not a conditional probability at all. This scored zero unless it was as the numerator of a fraction. Other candidates did have a fraction with the correct denominator but their numerator was incorrect.

Question No. 3 (i)
The majority of candidates who scored this mark showed that $P(L \cap R) = 0.099 \neq P(L) \times P(R) = 0.033$. Very few candidates gave the simplest explanation which is that $P(L|R) \neq P(L)$. For the former, candidates had to quote the correct probabilities, but for the latter the symbolic representation was adequate, as the probabilities were given in the question.

Question No. 3 (ii)
There were three common answers here. The majority correctly obtained 0.099, but some candidates multiplied the wrong probabilities together to obtain 0.033 or 0.0675. Brief working was generally given both for the correct and the incorrect answers

Question No. 3 (iii)
Most candidates gained full credit here, often from a follow through of a wrong answer to part (ii). Some candidates failed to subtract $P(L \cap R)$ away from $P(L)$ and $P(R)$ and but were still able to score one mark for the two labelled circles.

Question No. 4 (i)
This was generally well answered but those candidates who did struggle with this question often still managed to score the first mark for $^{16}/_{30}$ multiplied by another probability. There were very few over specified answers seen. A very small minority of candidates mixed up boys and girls but still gained SC2. Rather fewer candidates used the combinations method than the probability method, but those who did were usually successful.

Question No. 4 (ii)
This part was found to be rather difficult. The most successful method was to add together the probabilities of ‘no boys’ and ‘no girls’ then take the sum from 1. However, a significant number of candidates took each probability from 1 and then multiplied the resulting answers, which only scored one mark. Those considering the three possibilities 1g3b, 2g2b, 3g1b, often either omitted the coefficients of 4, 6 and 4 altogether or got at least one of them wrong, usually the middle coefficient, replacing 6 with either 4 or 5.

Question No. 5 (i)
A surprising number of candidates could not cope with the algebra required for this part, and whilst credit was given for the substitution method (if all working was shown) it is not a suitable method at this level. A significant number of candidates omitted the summation equal to 1 and so could only gain one mark out of three if their table was correct. A small number of candidates forgot to include the table.

Question No. 5 (ii)
This part was very well answered by the vast majority of candidates with many scoring all 5 marks. Solutions were well laid out, formulae quoted, and correct values for $E(X)$ and $Var(X)$ obtained. It is very pleasing to note that very few candidates made the mistake of dividing by 5,

as was more frequently seen in the past. Fortunately most candidates used the $E(X)^2$ – $E(X^2)$ method rather than the alternative – these latter often making calculation errors. A number of candidates had wrong probabilities. If their probabilities added to 1 they could still score three marks, but if not only two marks. Candidates should be advised always to check that their probabilities do actually add up to 1 in probability distribution questions.

Question No. 6 (i)
Most candidates found the frequency densities correctly. They usually then went on to draw the axes correctly although a few failed to start the frequency density scale at zero or to label the axes. A few candidates used inequalities on the horizontal axis, which attracted a penalty of one mark. The choice of scales on the vertical axis was not always ideal, and this left some candidates vulnerable to drawing the heights at incorrect positions. In particular the height of the first bar was frequently incorrectly plotted at 0.5 rather than 0.55.

Question No. 6 (ii)
The calculation of the mean of the grouped data was in most cases accurately performed using correct mid-points. The calculation of the standard deviation was less well executed. Whilst there were many correct solutions seen, some forgot to factor in the frequencies and worked with Σx^2 rather than Σfx^2. Over specification of either or both of the answers caused some candidates to lose one mark.

Question No. 6 (iii)
Most candidates scored at least the first two marks. However many omitted the fact that there were definitely no outliers at the top end of the data and/or stated that there were definitely some outliers present at the bottom end, thus missing the final mark.

Question No. 6 (iv)
This was generally very well answered.

Question No. 6 (v)
For this type of question candidates should be taught to discuss ‘average’ and ‘variation’. Simply stating for example that the mean of A is lower than the mean of B does not attract any credit.

Question No. 7(I) a
This was generally very well answered.

Question No. 7(i) b
Although most candidates answered this correctly, some gave $P(X \leq 12)$ rather than $P(X \leq 11)$, and some found the required probability but then subtracted it from 1.

Question No. 7 (ii)
Most candidates wrote down the correct hypotheses using the correct notation. It is encouraging to report that rather more candidates gave a correct definition of p than was the case in previous years.

Question No. 7 (iii)
Those candidates who calculated $P(X \leq 13)$ were generally more successful than those using a critical region method. Those who used the latter method often got the critical region wrong, thereby losing credit. In general conclusions were given more clearly than in previous sessions, although not always in context. There was also rather less use of point probabilities than in the past.

Question No. 7 (iv)
Many candidates, despite having answered the previous part correctly, reverted to point probabilities in this part, using their calculator to find $P(X = 33)$. This of course gained no credit.

Others made a correct comparison ($33 < 35$) but were not always sure what this meant in the context of the test.

Question No. 7 (v)
Most candidates who knew how to tackle this question wrote down 'for $n = 3$, $P(X = 0) = 0.0034 < 0.01$'. However many did not then justify their answer by writing down $P(X = 0)$ for $n = 2$ and thus only gained one mark. There were very few successful attempts using logarithms.

4767 Statistics 2

General Comments

The overall performance of candidates taking this paper was very good. It was particularly pleasing to see candidates taking care over the wording used in hypothesis tests, with the majority providing non-assertive conclusions referring to the alternative hypothesis. Most candidates demonstrated awareness of the difference between parameters relating to an underlying population and statistics relating to sample data, though there are still many who seem unsure in this area. In general, statistical calculations were accurately handled and, where statistical tables were required, these were used appropriately. More candidates took care not to provide over-specified answers than in previous years. As in previous years, questions requiring some form of interpretation provided the most diverse range of responses; if candidates could be advised to limit explanations to statistical considerations then their answers would be more likely to gain credit.

Comments on Individual Questions:

Question No. 1

(i) Well answered. Responses to the first part this question saw most candidates correctly rank the data and then accurately use the appropriate formula to find the correct figure to an appropriate degree of accuracy. Of the few errors, mostly it was forgetting the "1 -..." even if the formula had been quoted correctly initially.

(ii) Also well answered, with correctly worded hypotheses and conclusions commonly seen. There were a few cases of the final statement being too assertive [eg "there **is** positive association between..."]. Most candidates obtained the correct critical value and used it appropriately. A small proportion of candidates seemed unsure whether to use "association" or "correlation" in this part, and expressions given in terms of ρ appeared alongside otherwise correct hypotheses in many cases.

(iii) The need for the elliptical nature of the scatter diagram and the relevance of this for the underlying population distribution seemed to be known by many candidates although the language explaining the relevance was sometimes not as precise as it might have been. Many incorrect spellings of "ellipse" and "bivariate" were seen. Candidates referring to the distribution of the "data" rather than "underlying population" were penalised.

(iv) Use of the correct symbol ρ, in hypotheses was common, as was its definition. Though most candidates gave fully correct final statements, there were many who were too assertive in their summing up or who focussed on the null hypothesis [eg "the evidence suggests there is **no association** between…].

Question No. 2

(i) The vast majority of candidates got the mark for identifying the Poisson distribution though most struggled to provide a statistical interpretation of "independent". Many attempts went along the lines of "independent means not dependent". Those attempting to mention probability in their explanations usually picked up this mark.

(ii) Most candidates earned both marks. The few errors seen usually involved accuracy – either over-specification or incorrect rounding.

(iii) Most candidates earned all three marks. Fairly common errors included using $\lambda = 7$, or finding P(7 defects).

(iv) Most candidates provided convincing methods leading to the given answer. Those who stated that $\lambda = 5$ tended to obtain all three marks. A small proportion stated incorrectly that $P(X \geq 8) = 1 - P(X < 7)$. Since the answer was provided, candidates who just wrote "1 – 0.8666" without first writing "$1 - P(X \leq 7)$" were not given full credit.

(v) Mostly correct answers were seen, though Poisson or Normal distributions were suggested by a small proportion of candidates.

(vi) Many were awarded full marks here; it was clear that candidates were familiar with the method. Common errors included not applying the continuity correction or mistakes with the standard deviation (such as using $\sigma = 11.56$ or $\sigma = \sqrt{13.34}$).

Question No. 3

(i) Apart from the occasional use of a spurious continuity correction, or using the variance as the standard deviation, this part of the question caused few problems for candidates. It was pleasing to note that there were fewer cases of the use of the form $\frac{\text{mean - value}}{\sigma}$ when standardising. Some candidates evaluated $\Phi(1.308)$ rather than $1 - \Phi(1.308)$ for their final answer.

(ii) Most obtained the value 2.326 and then used it correctly in the relevant equation, avoiding continuity corrections, to find the value of k. Common errors included using -2.326 or 1.282 in place of +2.326.

(iii) Many candidates correctly identified the appropriate range of values as 130.5 to 131.5 and went on to find the appropriate probability, though other ranges were seen frequently.

(iv) Having used the correct symbol, μ, in the hypotheses (although a few believed μ related to the population of English male blackbirds), most candidates used the correct distribution of the sample mean, $N\left(130.5, \frac{11.84}{20}\right)$ when answering this question. However, the inclusion of units with the population variance led to confusion for some. Most candidates successfully completed the test by providing suitable conclusions, with over-assertiveness leading to loss of the final mark in only a few cases.

(v) This part of the question proved to be difficult for many candidates. Those who made the connection that the move from 5% to 10% meant it was more likely to reject H_0 tended to provide suitable comments. Most candidates did not clearly distinguish between their advantage and disadvantage.

Question No. 4

(i) This Chi-squared test was well handled by the majority of candidates. Most obtained the mark for stating hypotheses though some use "relationship" here in place of "association"; many candidates provided hypotheses as an afterthought rather than at the beginning of their solution as desired. Most followed the instruction to "include a table showing the contributions of each cell to the test statistic" and, in general, a suitable level of accuracy in working was observed. Most obtained the marks for stating the number of degrees of freedom and critical value. Most provided appropriate conclusions here.

(ii) Despite following the instruction to "include a table showing the contributions of each cell to the test statistic" in part (i), it was evident that many candidates were unaware that this instruction was to assist in answering part (ii). Many of those who did make the connection tended to earn all three marks.

(iii) Well answered, though conclusions relating to the null hypothesis were seen frequently.

(iv) Many candidates found it difficult to provide succinct answers to this question, though many plausible comments were seen.

4768 Statistics 3

General Comments:

As might be expected on a paper at this level, the scripts indicated that most candidates knew what they were doing most of the time. In addition, there were few scripts which showed evidence of candidates running out of time. Candidates seemed to be far more comfortable carrying out calculations than with the other requirements of the paper such as producing hypotheses and conclusions, interpreting results and providing definitions. In addition, as in previous years, many scripts suffered from a lack of precision. This manifested itself in many ways, inadequate hypotheses, over-assertive conclusions, over-specified final answers yet too little accuracy carried forward in calculations, inaccurate reading of tables, and finally a large number of scripts which were very difficult to read.

Comments on Individual Questions:

Question No. 1

Linear combinations of random variables

This question was done well by most candidates and many scored full marks for parts ii, iii, and iv.

Q1i nearly all candidates gained the first 2 marks correctly, showing that they could recognise the difference between adding two different values of X together, and multiplying one value of X by 2. Most candidates could not put this distinction into words, and few gained the third mark.

Q1ii This part was well done. The only errors of note were calculating z to only one or two decimal places, or the introduction of a spurious continuity correction.

Q1ii Again this part was done well and most candidates were able to sum the variables correctly and find the correct tail probability. The most common error was the lack of accuracy in calculating z, and then in reading the statistical tables.

Q1iv In this part, more candidates were unable to find the variance of $2X + Y$, but this was still rare. Most candidates were able to identify 2.054 as the relevant 2% point and thereby gave a final value in the correct tail. Some candidates used the value -2.054 and so ended up in the wrong tail. In this part many candidates gave their answers to 5 or more significant figures.

Q1v Many candidates in this part incorrectly started with the statement $W – 0.6X > 0$, but then a large proportion recovered at the end, often using a statement like “and so the proportion who did not achieve this =”. Candidates who took this approach often calculated the mean and variance correctly. Some candidates tried to deal with W and $0.6X$ separately.

Answers (ii) 0.0561 (iii) 0.0168 (iv) 108.3 (v) 0.0204

Question 2 Single sample *t* test and Wilcoxon test

Most candidates know how to carry out these tests, and most carried out the appropriate test in each case. Many candidates lost marks through taking insufficient care with hypotheses and conclusions. Symbols used in hypotheses need to be defined and in context. Conclusions also need to be in context and must not be over assertive.

Q2i Many candidates gave accurate definitions stating that every sample of a given size has the same chance of being selected. A significant number stated that every member of the population has the same chance of being selected, and a few simply described how a random sample might be carried out.

Q2ii Most candidates were able to calculate $\bar{x}$ and *s* correctly, although a significant number used truncated values in what followed. The test statistic was usually correctly carried out, although on occasion $\sqrt{10}$ was missing. The majority of candidates then chose the correct critical value of *t*, although occasionally two tailed values were seen, as were values of *z*.
The great majority of candidates made the correct decision in terms of rejecting the null hypothesis.

Q2iii This part was very well done, the most common error was not defining the population median in the hypotheses. The vast majority correctly found the differences from 1.05, although a few found the differences from 1.057, the median of the data. Most candidates identified 8 as the critical value, although a significant number gave 10 as the value. A few candidates carried out another t test here.

Question 3 Paired t test and goodness of fit test

Most candidates again knew how to carry out these tests. The comments in question 2 about hypotheses and conclusions are equally appropriate here.

Q3a Most candidates were able to calculate the mean and standard deviation correctly, although a few calculated “water – beetroot” rather than the other way around as specified in the question. Candidates usually calculated the mean and standard deviation correctly, but again many used truncated values. The test statistic was usually correctly carried out, although on occasion $\sqrt{12}$ was missing. The majority of candidates then chose the correct critical value of t, although occasionally two tailed values were seen, as were values of *z*.
The great majority of candidates made the correct decision in terms of rejecting the null hypothesis. A few candidates carried out a Wilcoxon test here

Q3b In goodness of fit tests it is particularly important to work to a high level of accuracy, and a significant minority did not do this. Most candidates were able to work out the Poisson probabilities correctly, but some calculated Prob($X = 7$) instead of Prob($X \geq 7$). Most candidates the multiplied by 60 to obtain the expected values, but a few multiplied by 100. Most candidates merged the first two and last two cells, although a few failed to merge any cells and a few merged only the last two cells. Those who had worked out Prob($X = 7$) often merged the last three cells. Most candidates were able to calculate the statistic correctly. The majority of candidates used the correct number of degrees of freedom, but a few failed to subtract 1 because the mean had been calculated from the data. A few candidates looked at the wrong tail in the chi-squared tables.

Question 4 Two part pdf and confidence interval

This question proved to be the most difficult for the candidates.

Q4i The great majority of candidates were able to sketch the correct shape. The horizontal axis was virtually always correct, but many either gave no value on the vertical axis or an incorrect value. Most candidates realised the key feature of the distribution was its symmetry, although a few stated that it was the fact that the distribution achieved its maximum value at a.

Q4ii The better candidates used the fact that the area of a triangle could be used at this point, or that the first integral should have a value of 0.5. These candidates invariably gained full marks. Those who stated that the sum of two integrals should equal 1 often made errors. These errors involved dealing with k and with signs. As the value of k was given, these candidates invariably ended with the correct value.

Q4iii Only the best candidates were able to obtain the correct value of $a^2/6$. Many candidates did not know how to calculate $E(X^2)$ and many were unable to deal with k, inserting the limits, and the signs correctly. In addition some omitted to subtract a^2 at the end.

Q4iv Most candidates knew how to calculate a confidence interval. Here the main errors seen were the use of 1.645 or 2.009 instead of 1.96 and giving the final interval to 5 or more significant figures.

Q4v Most candidates were able to give a correct definition of a 95% confidence interval, but a significant number still felt that the non-statistician was correct.

4771 Decision Mathematics 1

General Comments:

There are a few general points regarding candidate performances in the 2014 examination

- Candidates should ensure they write their answers in the allocated section of the answer book, where this is not possible a continuation sheet should be used.

- The quality of written communication is often very poor, and when that is combined with losing sight of the maths, it usually fails to gain credit. There are times when interpretation is required, so that some description of the circumstances is needed. Such writing needs to be concise and precise. There are other times when modelling is needed, when mathematics is to be extracted from the given scenario; see comments on Qu1 (ii) below, and on Qu3 (i).

Comments on Individual Questions:

Question No 1.

(i) Quite a lot of candidates omitted arcs.

(ii) Very few students thought about translating the problem into mathematics. Almost all will have heard about odd nodes, but they either failed to enter into the modelling, or harboured confusions, as indicated by quotes such as ... "*There's an even number of vertices*".
Some of the writing was very poor indeed, e.g. "*ts is looped turns to get out of that loop you must use that entrance again (ts to pa), As you go in and come back*"

(iii) Many candidates were unable to put into practice what is in the specification. Without essential structuring, using odd nodes, it will have been very difficult for them. With it, it was easy.

(iv) As above ... with the extra path, Joanna can either avoid repeating any path by starting/finishing at fs/ol, or she can start and finish at a vertex of her choice by repeating fs to ol. Candidates usually gave amended stories about parts of her walk, failing to make any reference as to how her implied objective would be affected.

Question No 2.

(i) Most candidates could do this.

(ii) Again, most could do this, although one or two ended up with 9 drinks as a consequence of having to reject a random number.

(iii) A small minority failed to understand the question, and failed to give two rules.

(iv) Candidates who did not have two rules in part (iii) could not gain any credit here.

(v) Candidates who had, say 5/10 coffees in part (ii) and 8/10 coffees in part (iv), mostly successfully manipulated this to 13/20 in part (v). Given these inputs, a few well-drilled candidates (drilled, that is, in the addition of fractions) produced a proportion (!) of 13/10. Others who knew something about probability (and how to multiply fractions) gave an answer of 0.4.

Question No 3.

(i) There were two marks for this, and two things to do to get them. Few candidates failed both to give a shorter indirect route, and to make an appropriate comment regarding the triangle inequality. Candidates were not expected to quote the triangle inequality, but the essential maths is that for straight connections AD ≤ AC + CD (or AD ≤ AF + FD). But then roads are not often straight! That observation would have been enough.

(ii) This was answered well by a substantial majority of candidates. The most common failing was the failure to number columns as they were included in the set of columns under consideration.

Question No 4.

(i) The CPA network was generally well constructed. The most common error was to have activities F and G share the same i node and same j node.

(ii) The forward and backward passes were also well done. The common error was to have a late time of 6 at C's j-node / D's i-node.

(iii) Some candidates failed to answer the question in this part. Schedules and cascade diagrams were seen, but without indication of who was doing what. In some cases that might have been due to candidates attempting to use colouring, or other shading, which could not be seen on scanned scripts.

(iv) The question could have been phrased as "*Give the new minimum completion time, critical activities and ...*", and it is probable that a better response would have followed. But there are no apologies for having asked "*How does this delay affect the minimum completion time, critical activities and ... ?*" There is a little bit of modelling implicit in that, and it certainly found some candidates wanting.

Question No 5.

(a)(i/ii) A surprisingly large number of candidates failed with this simple algorithm. Some terminated after two passes around the loop. It was very common to see 24 following 8 in (i), presumably because up to that point the two rules had been alternating. Others seemed not to be able to do the arithmetic.

(a)(iii) Very few succeeded in locating their stop instruction in an appropriate place, i.e. with a step number between 21 and 29.

(a)(iv) Many candidates took an almost moral stance to this question ... they required an algorithm to have a purpose! Others were concerned about a stopping condition, even though they had just provided one in part (iii), and had that clarified in the question. Few correct answers were seen.

(b)(i) Most succeeded in answering the question, which included a requirement to state how many boxes had been used.

(b)(ii/iii) Most candidates continued to score well on these two packing questions, but some forfeited all marks by getting their "increasing" and "decreasing" confused. It's not always the case that questions can have a good storyline, but most do, including this question. Candidates sensitive to that might have spotted such an error.

(b)(iv) Most candidates found this challenging. It is to their credit that they realised what was needed, but many generated large random sets of weights, and hoped. Few thought it through to minimalist sets such as 3, 3, 3, 3, 2, 2, 2, 2.

(b)(v) Fewer correct solutions were seen than might have been hoped for. Of course, there were those who scaled up in proportion, but there were several who modelled quartic complexity instead of quadratic, and many others who confused items and times in trying to compute scale factors.

Question No 6.

(i) Many candidates failed to get to grips with the underlying variables, confusing stew and soup with carrots, beans and tomatoes. To add to their confusion there were both litres and tens of litres to handle.
Through it all shone those who had the clarity of thought to identify and define their variables clearly and unambiguously. As in every report, it is repeated here that the identification should start with "Let ... be the number of ..."

(ii) Many candidates who struggled through part (i) managed to score well in part (ii). Quite a lot failed to label and scale their axes. A few produced broken scales, and scored zero.

(iii) The usual optimisation requirement had a sting in the tail for those who had chosen their variables to represent tens of litres of the products ... they had to do some interpreting ... not 20 litres of soup, but 200.

(iv) A challenging last part question. A few candidates succeeded with it.

4772 Decision Mathematics 2

General Comments:

There are no general points.

Comments on Individual Questions:

Question No 1.

(i) This was done well.

(ii) Fewer than half of the candidates could correctly compute the EMV. Many were distracted by computing losses. Others were totally confused.

(iii) This was done well.

(iv) Only about half showed a utility × probability computation.

(v) This was done well.

(vi) Candidates scored well on this, although some were poor in handling the inequality. The phrasing of the question allowed them to get away with poor algebra.

(vii) Few candidates answered the question. Most made obvious statements which did not answer the question.

Question No 2.

(a)(i) This was difficult. The essence was that either the ball hit the bat or it did not (total probability). It was essential to recognise this explicitly. Successful candidates received reinforcement in (a)(v).

(a)(ii) About half of the candidates had the implication reversed.

(a)(iii) About half of the candidates had the implication reversed.

(a)(iv) About half of the candidates had the implication reversed.

(a)(v) Only a very few candidates could correctly negate both sides and reverse the implication to get the contrapositive.

(b)(i) There are several equivalent alternatives for the answer ... $d \Rightarrow (a \vee b \vee c)$ or $\sim(a \vee b \vee c) \Rightarrow \sim d$ or $(\sim a \wedge \sim b \wedge \sim c) \Rightarrow \sim d$ or $\sim(d \wedge \sim(a \vee b \vee c))$ or $\sim d \vee a \vee b \vee c$ ... but most candidates got none of these.

(b)(ii) To gain the method mark, the truth table needed 16 lines. Some candidates used complex unbracketed expressions, so that the column giving the overall truth value was not known, making “following” difficult.

(b)(iii) Most scored 2 of the 3 marks here.

Question No 3.

(i) The modal mark was 5. This was because hardly any candidates correctly identified the variables. For instance, a common offering was "Let *a* be the amount of *A* ...", or similar. That would allow *a* to be, for example, 1000kg. But in the algebra of number, and in the modelling which is given, letters represent numbers.
This point has been emphasised in examiner reports every year, but still it eludes candidates. Variable definitions MUST be in terms of "... number of ...", in this case "Let *a* be the number of kg of *A* ...". This is not being picky. Precise variable definition is absolutely crucial to correct modelling.
Some candidates tried to answer this in words ... possible, but in practice this did not lead to many marks.

(ii) The simplex was done well. However, the interpretation was often incomplete or had units missing, which were vital in this question.

(iii) Again, interpretations were often incomplete.

(iv) The question asked for incorporation into initial tableau, but candidates often lost marks by failing to do this.

Question No 4.

(a)(i) This was done well, although some candidates were upset that Floyd was not tested. Some were very critical, even though the specification is quite clear on requiring either Floyd or the repeated application of Dijkstra. Lack of appreciation of this led to inefficiency on a grand scale. Having found all shortest routes from A in the first application, A and its arcs can be deleted from subsequent networks. Similarly for B, etc, so that the work involved is halved. No candidate did this.

(a)(ii) This was done well.

(a)(iii) More than half of the candidates incorrectly computed the total of shortest distances from each vertex in turn, and then minimised that.

(a)(iv) This was, arguably, the hardest mark on paper. Only by considering distances to D could it be scored.

(b)(i) Many candidates failed to answer the question, for instance, failing to give the connections.

(b)(ii) This was badly done. Some candidates who were successful with (b)(i), did not seem to be able to do (b)(ii).

4776 Numerical Methods (Written Examination)

General Comments:

Broadly speaking, candidates showed a sound grasp of the theory and methods required in this paper. Numerical accuracy was generally good.

It remains the case, however, that some candidates work in very inefficient ways. Scattering calculations about the page makes working more difficult to follow. This in turn means that candidates are more likely to make mistakes, and there is a danger that poor presentation will make it difficult for the examiner to identify correct work.

Numerical methods lend themselves to a tabular layout, as would be obtained from a spreadsheet. Setting out working like this, and clearly identifying answers, is much the best approach to adopt.

A handful of candidates answered questions in the wrong part of the answer book. Examiners will always do their best to deal with such situations, but candidates should realise that this is a risky behaviour. Sometimes it is simply not possible to tell, from looking at a few numbers, which question is being answered. So answers should be written in the space provided or in an additional answer book.

Comments on Individual Questions:

Question No. 1

Part (i) was frequently done well, though a sizable minority of candidates confused the method of false position with the secant method. These two methods are distinct: false position works with two values that bracket the root, while the secant method works with the two most recent values which may not bracket the root.

Part (ii) was done well by most.

Question No. 2

This was a test of knowledge about the relationships between the various integration rules. Almost everyone could obtain the second T value, but there many who couldn't work backwards from the third M and T values to get the second M value. Obtaining the S values was very straightforward, though a small minority of candidates appeared not to know how S is related to M and T.

Question No. 3

Part (i) was found to be easy, though some candidates confused the exact and approximate values in calculating the relative error, thereby losing a mark.

Part (ii) was found hard. Very few candidates spotted that squaring and cubing gave double and triples the relative errors.

Question No. 4

This question was answered well by most candidates. The two different methods were understood, and in part (iii) the majority of candidates were able to comment that the central difference method is more accurate than the forward difference method. A few candidates, however, compared all four estimates and so gave a very low precision answer.

Question No. 5

The linear interpolation in part (i) proved a little more difficult than it should. It seems that candidates who know how to do higher order interpolation sometimes fail to realise that linear interpolation can be done the same way. They therefore rely on ‘first principles’ methods; that is fine if they get it right.

Parts (ii) and (iii) were straightforward for many. However, some did not know the correct formula and so omitted this part or used the Lagrange formula instead thereby gaining no marks.

Question No. 6

Candidates knew pretty much what to do in this question, but some lost marks or took excessive time through not following instructions carefully.

In part (i), some candidates argued that as the function is positive at 0 and at 1 there must be two roots in the interval. This is not sufficient of course. It is necessary to show that the function is negative at some point between 0 and 1.

Part (ii) was generally well done, though some appeared to have keyed the iterative formula into their calculators incorrectly.

In part (iii), candidates were expected to start with 0.34 and show that the next couple of iterates are close to 0.34. However, many chose to start some distance away from 0.34 and iterate a large number of times to get close to 0.34. The second request in this part required finding the derivative of the function on the right of the iterative formula and evaluating at 0.34. The value obtained is, in modulus, close to 1 so the iteration will be slow.

Part (iv) was straightforward for most.

Question No. 7

Part (i), using the trapezium rule, was done well.

In part (ii) candidates should have drawn a smooth curve through the given points and then drawn in the straight lines that represent the trapezium rule. In very many cases the sketches were so poor that it was not possible to distinguish the straight lines from the curve. Despite that, most candidates deduced correctly that the trapezium rule is likely to be an overestimate.

Part (iii) defeated almost everyone. The intended idea was to use the trapezium rule on (0, 0.5) and Simpson’s rule on (0.5, 1.5), then Simpson’s rule on (0, 1) and the trapezium rule on (1, 1.5). Most candidates floundered and it was common to see invented integration rules or additional values being read off the graph.

In part (iv) the supplied rule was used correctly by most, though a common error was to misread the last term in the brackets as 3f(3*h*). Candidates were able to confirm their conclusions from part (ii), but not from part (iii) for the reasons indicated.

Coursework

Administration

Administration difficulties caused by the failure of centres to adhere to instructions was minimal this year. There were few clerical errors and the vast majority enclosed the Authentication Form, CCS160. It was also helpful to have work submitted in good time, for some well before the deadline set by the board. This all made the process of external moderation very much easier. Centres are once again reminded that it is also a great help to have the cover sheets filled in properly. This means

- Full candidate name and candidate number,
- Marks given by criteria rather than domain,
- Comments to help the external moderator determine which marks have been awarded and which have been withheld,
- An oral communication report,
- The criteria marks summed correctly to give a final total which is correctly transferred onto the MS1 and which is the mark submitted to OCR.

It is also helpful to have the work annotated, particularly in places where the work has been checked. Assessors are asked not to tick work that they have not checked.

The marks of candidates in most centres were appropriate and acknowledgement is made of the amount of work that this involves to mark and internally moderate. The unit specific comments are offered for the sake of centres who have had their marks adjusted for some reason.

Teachers should note that all the comments offered have been made before. These reports should provide a valuable aid to the marking process and we would urge all Heads of Departments to ensure that these reports are read by all those involved in the assessment of coursework. It is a concern to us that we have to report the same problems each year. More than one centre repeated the error of last year, causing a significant reduction of marks again this year. It is clear that in these (very rare) cases it is not only this general report that is not being read but the centre specific report also and this is a discouragement to the moderators.

4753 Methods for Advanced Mathematics (C3)

There are a significant number of centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance.

A general comment on graphical work.

- A graph of the function being used does not constitute an illustration of the method. Furthermore, the graphical work often lacks detail with not enough annotation or “zooms” in the decimal search method, tangents in the Newton-Raphson method which do not touch the curve because of the scale used, and staircase or cobweb diagrams in the rearrangement method which do not match the iterates.

Change of Sign

- Trivial equations should not be used to demonstrate failure.
- If a table of values actually finds the root then the method has not failed.
- Graphs which candidates claim crosses the axis or just touch but don’t (these should be checked).

- The answer is given as an interval rather than an actual value with error bounds.

Newton Raphson

- Candidates who use equations with only one root should not be credited the second mark.
- Iterates should be given for success and failure.
- A print out from "Autograph" is not sufficient. Candidates should derive the formula by differentiation and algebra. This should be for the equation being used rather than theory.
- Poor illustrations (for example, an "Autograph" generated tangent with no annotation or just a single tangent).
- The graph used to demonstrate the method should match the iterates.
- Error bounds should be established by a change of sign.
- Failure should be demonstrated "despite a starting value close to the root". If the starting value is too far away from the root or too artificial then the mark should not be awarded.
- The roots in this domain should be given to 5 significant figures. Teachers should note that the default position of "Autograph" is only 4 significant figures so candidates need to make some adjustment before the use of the software is satisfactory.

Rearrangement

- Incorrect rearrangements are often not spotted and marked as correct.
- Graphs do not match iterates.
- The method should be explained by a graph which may need to be annotated.
- Weak discussions of g′(x) are often given. Candidates should not just quote the criterion without linking it to their function.

Comparison

- The methods need to be compared by finding the same root of the same equation with the same starting value to the same degree of accuracy.
- The discussions are often thin and omit references to the software actually being used.

Notation

- Equations, functions, expressions still cause confusion to candidates and teachers! Candidates who assert that they are going solve $y = x^3 + x + 7$ or that they are going to solve $x^3 + x + 7$ should be penalised.

4758 Differential Equations

The majority of changes, particularly large changes this series, were for students investigating Aeroplane Landing. Some centres are still awarding full marks in Domain 2 when the motion of the aeroplane is only considered for the first 9 seconds which is before the brakes are applied. On the basis of this, the model is rejected. This should be penalised. It is not valid to reject a model based on only part of the motion.

Often centres will award marks for criteria in the first four domains wherever they appear in the script. Domains 1,2 and 4 apply only to the initial model and not the revised one.

When investigating situations which involve air resistance, e.g Aeroplane Landing, Paper Cups etc. students will often consider the case where there is constant resistance. Whilst this is acceptable, and may well help the narrative, it cannot be used as an initial model in terms of the marking criteria. The differential equations developed from the model should be one that is part of this unit.

For those students investigating Aeroplane Landing, a consideration of the accuracy of the given data would help towards meeting the criteria which relates to the variation of the parameters, which often seems to cause a problem.

Finally, some students assumed that air resistance is proportional to v^n or, in Cascades, assumed that the rate of flow is proportional to h^n. They then found the optimum value for n. This is curve fitting and not modelling and should be penalised. Curve fitting, although tempting, should be resisted since it does not rely upon the formulation and later modification of the assumptions. The choice of parameters, based on guesswork or shape of the data also often seems to occur in 'Interacting Species'.

The essential function of the coursework element of this module is to test the candidates' ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

Finally, when using the experimental / modelling cycle care must be taken to avoid circular arguments. That is, the data used to produce the parameters should not then be used to compare with the predicted values. It is better, where possible, to calculate the parameters from one set of data and compare the predictions with another set.

4776 Numerical Methods

The most popular task continues to be to find the value of an integral numerically. The following comments are offered – it is to be hoped that those teaching and assessing will take note so that the problems do not continue to occur with such regularity!

Domain 1.
There is a basic requirement to make a formal statement of the problem in mathematical terms. Sometimes candidates will say what they intend to do in words; most often the "dx" is missing from their integral.

Domain 2
Most candidates describe what method they are to use but fail to say why. The criteria require some justification of the process to be adopted.

Domain 3
Finding numerical values for one of the methods up to at least 64 strips is a requirement for a substantial application.

Domain 4
It is not enough to state what software is being used. A clear description of how the algorithm has been implemented is required, usually by presenting an annotated spreadsheet printout.

Domain 5
It is accepted that candidates might use a function that they are unable to integrate (because of where they are in the course) but which is integrable. However, it is not then appropriate to state a value found by direct integration. It is helpful to the moderator if the assessor comments on whether the function used by the candidate is integrable in their experience.
Many candidates will state the value to which the ratio of differences is converging without justification from their values. Indeed, some candidates use the "theoretical" value regardless of the values they are getting (or not if they do not work the ratio of differences) far too early giving inaccurate solutions.

Domain 6
Most of the marks in this domain are dependent on satisfactory work in the error analysis domain and so often a rather generous assessment of that domain led also to a rather generous assessment here as well. Teachers should note that comments justifying the accuracy of the solution are appropriate here, but comments on the limitations of Excel are not usually creditworthy.

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