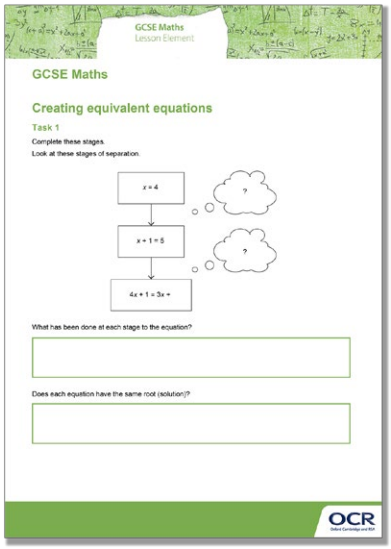
GCSE Maths

Creating equivalent equations

Instructions and answers for teachers

These instructions should accompany the OCR resource 'Creating equivalent equations' *which supports OCR GCSE Maths.*



The Activity:

This activity offers an opportunity for maths skills development.

This Lesson Element may assist teachers in covering manipulation and solving of linear equations, which is includes on the Foundation and Higher tiers of both the GCSE Mathematics A (J562) specification and the GCSE Mathematics B (J567) specification.

Associated materials:

'Creating equivalent equations' Lesson Element learner activity sheet.

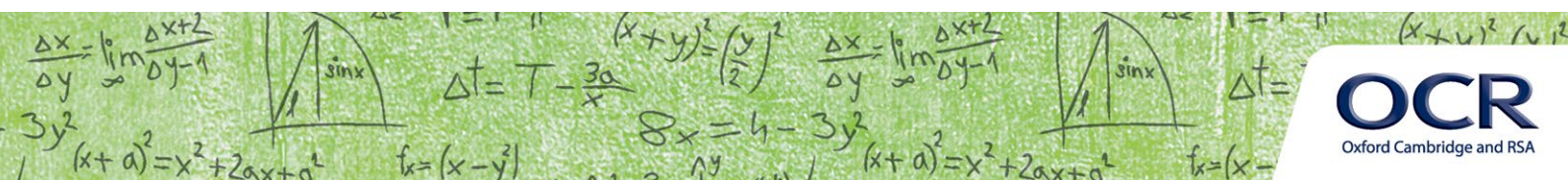
Teacher Guide

The aim of this lesson element is to get students to use simple transformations to create equivalent equations, which all share a common root or answer. The key steps are that you must do the same transformation to both sides of the equation, that you can add or subtract numbers or algebraic terms and that you can multiply or divide by numbers. The resultant equations have the same root and can be termed equivalent equations. In examinations students tend to forget these rules. Brackets can be treated in the same way, or expanded; it is a useful exercise to attempt both approaches. For example, an equation such as $5(x - 3) = 40$ can be solved by dividing by 5 to $x - 3 = 8$, or by expanding the brackets to give $5x - 15 = 40$. Which method is better, or which method is quicker? This can also encourage students to check their solutions.

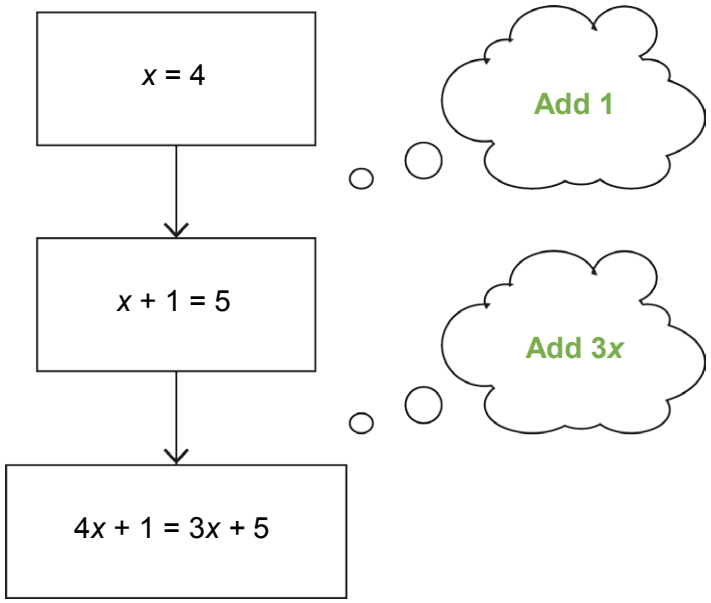
Students should be able to work backwards from the root to produce equations, 'here is the answer, now find the question' style. It is more open-ended and you may find that you will have to deal with brackets, order of operations or even powers of x; do not avoid any of these and do try to use negative numbers whenever possible.

Through creating equivalent equations, students should begin to see how equations can be solved and manipulated using the reverse, or inverse, of processes. The ultimate aim is to get them to solve equations by doing the same transformational process to both sides of the equation.

Further work or homework could involve attempting one of the activities, though it is recommended that Activity 4 is done in the classroom because it addresses the aim of this lesson. Try to avoid setting too much repetitive work on solving equations.

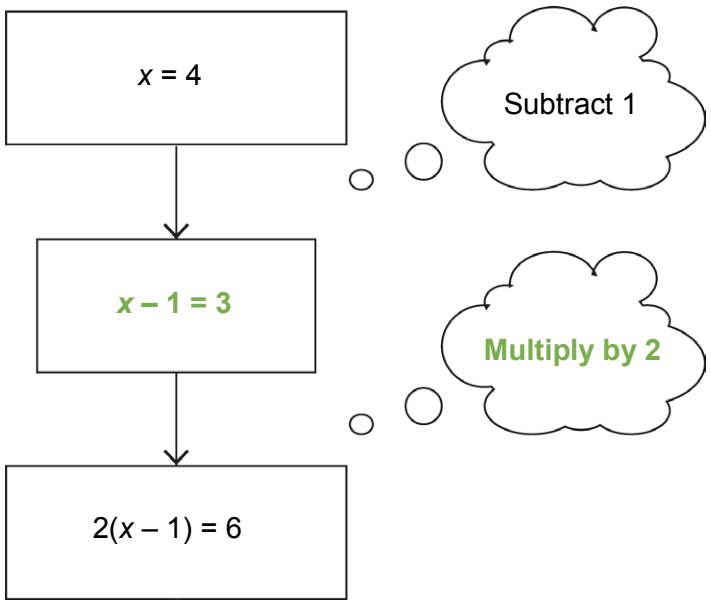


Task 1



Write the first example from the 'Creating equivalent equations' sheet on the display board and ask students what operation has been carried out on one equation to get the next one. At many stages keep asking "Do they all have the same root?"; this is important to encourage students to check that their solutions actually work. The answers are "add 1" and "add 3*x*". You can draw the template on the board and try some more examples if you wish. At this stage do not prevent any suggestions and do not try to distinguish between them, so adding numbers and adding *x* terms are treated as the same standard. The check should be "Yes", all the equations have the same root, ie 4.

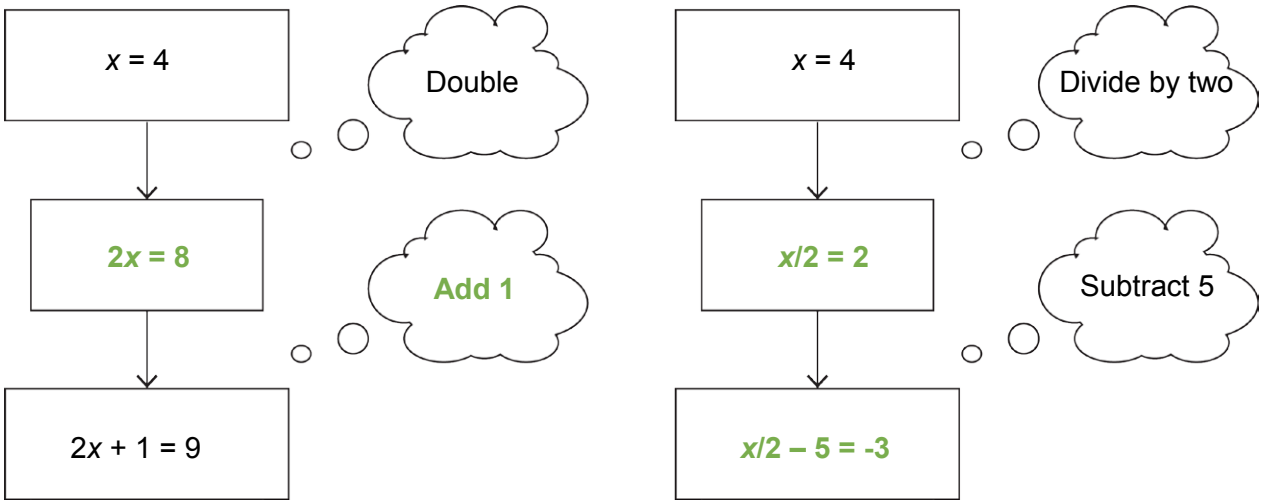
Task 2



The answers are "x – 1 = 3" and "double" or "multiply by 2". The check on the solution should reveal a "Yes". At this stage you could challenge students to write their own equations using x = 4 as the root; they could then be written on the board and the class asked what the connections are. It may be that your class is able to start to write their own equations from this starting point (if not, there is further practice in Task 3 below). You could also stipulate equations with one, two, three, or more, stages, with the definition of a stage given below by the example.

	$x = 4$	
<add 3>	$x + 3 = 7$	(one stage)
<multiply by 4>	$4(x + 3) = 28$	(two stage)
<subtract 5>	$4(x + 3) – 5 = 23$	(three stage)

Task 3



The answers are "2*x* = 8" and "add 1" on the left hand side and "*x*/2 = 2" and "*x*/2 – 5 = -3" on the right hand side. You may find you can replace this with the students own examples, which they write on the board. At this stage, it is the students' turn to write some equations and you can specify how many stages if you like. The complexity of their equations will show you the level they are working at. A one stage equation is a low grade Foundation level, possibly grade E or F. A two stage equation is around grade D or E, depending on whether there are *x* terms on both sides. A three stage, especially with brackets, is more like grade C or D. The other activities are there to stretch those who have picked up the basics. Activity 4 is the key activity, because it addresses the aim of the lesson. It is explained fully below.

Activity 1

You can set students to work on creating their own family of equivalent equations with a different root, say *x* = 5 or *x* = -3. This is recommended to ensure that they do not get used to one root equation or for groups that might need the practice. Always keep stressing to check that their equations have the same root equation.

Activity 2

Multiplying both sides by *x* will lead to two root equations, the first is still the same one, the second being *x* = 0. They could investigate whether *x* = 0 will always be a root of these equations.

Activity 3

You can remove the suggestions for squaring to see if students come up with the idea by themselves. Squaring both sides, say for $x = 4$, which transforms to $x^2 = 16$, still has the same root equation, but also has $x = -4$ as a root; it has two root equations. Why is this, perhaps a look into what happens when a negative number is squared? Will this always remain so? A look into possible transforms from that one could give $x^2 + 1 = 17$, which does have the same two root equations. Will the root equations always be of the form $x = \pm a$? This activity will especially benefit those on the Higher Tier course.

Activity 4

Writing the steps backwards is the method for solving them and that stage can be delayed until later. However it works very well if you can provide the students with card cut into rectangles and they can write their connected equations onto the card. Then all they do is to reverse the positions of the cards. The challenge is to explain the processes and, more importantly, why they occur in that order. This will lead to using BODMAS (or similar). It will assist the students if the order of operations was covered in a previous lesson, or if it was displayed on the classroom wall as a reminder.

If you use cards then they can put a one stage equation on two cards, reverse the cards and then have to state the connection between the two equations, which will be the inverse of the operation that they used originally. When you put a two stage equation on three cards and they reverse them, then they will have to use BODMAS to determine the order of operations, which will be the opposite of the order they used to generate the equations originally. Always ask them to check that their root equation is the root equation of all their equations on the cards in front of them. When they are ready you can give them a fresh equation to find the root equation, for example $6x - 5 = 13$ should lead to the root $x = 3$.

Activity 5

This is a chance for them to write down what they have learned, in particular that an equation maintains the same root equation if you add or subtract numbers or x terms, or multiply or divide by numbers, providing that you do the same operation to both sides. Squaring or multiplying by x terms usually keeps that root equation and adds a second one.