GCSE (9–1)
Delivery Guide

J560

Accredited

MATHEMATICS

Theme: Number Operations and Integers

April 2015

OCR
Oxford Cambridge and RSA

CONTENTS

Introduction

Delivery guides are designed to represent a body of knowledge about teaching a particular topic and contain:

- Content: a clear outline of the content covered by the delivery guide;
- Thinking Conceptually: expert guidance on the key concepts involved, common difficulties learners may have, approaches to teaching that can help learners understand these concepts and how this topic links conceptually to other areas of the subject;
- Thinking Contextually: a range of suggested teaching activities using a variety of themes so that different activities can be selected that best suit particular classes, learning styles or teaching approaches.

If you have any feedback on this Delivery Guide or suggestions for other resources you would like OCR to develop, please email resources.feedback@ocr.org.uk.

KEY

Click to view associated resources within this document.

Click here   Click to view external resources

Curriculum Content

GCSE content Ref.	Subject content	Initial learning for this qualification will enable learners to…	Foundation tier learners should also be able to…	Higher tier learners should additionally be able to…	DfE Ref.
OCR 1	**Number Operations and Integers**				
1.01	**Calculations with integers**				
1.01a	Four rules	Use non-calculator methods to calculate the sum, difference, product and quotient of positive and negative whole numbers.			N2
1.02	**Whole number theory**				
1.02a	Definitions and terms	Understand and use the terms odd, even, prime, factor (divisor), multiple, common factor (divisor), common multiple, square, cube, root. Understand and use place value.			N2, N4, N6
1.02b	Prime numbers	Identify prime numbers less than 20. Express a whole number as a product of its prime factors. e.g. $24 = 2 \times 2 \times 2 \times 3$ Understand that each number can be expressed as a product of prime factors in only one way.	Identify prime numbers. Use power notation in expressing a whole number as a product of its prime factors. e.g. $600 = 2^3 \times 3 \times 5^2$		N4, N6
1.02c	Highest Common Factor (HCF) and Lowest Common Multiple (LCM)	Find the HCF and LCM of two whole numbers by listing.	Find the HCF and LCM of two whole numbers from their prime factorisations.		N4

Curriculum Content

GCSE content Ref.	Subject content	Initial learning for this qualification will enable learners to…	Foundation tier learners should also be able to…	Higher tier learners should additionally be able to…	DfE Ref.
1.03	**Combining arithmetic operations**				
1.03a	Priority of operations	Know the conventional order for performing calculations involving brackets, four rules and powers, roots and reciprocals.			N3
1.04	**Inverse operations**				
1.04a	Inverse operations	Know that addition and subtraction, multiplication and division, and powers and roots, are inverse operations and use this to simplify and check calculations, for example in reversing arithmetic in "I'm thinking of a number" or "missing digit" problems. e.g. $223 - 98 = 223 + 2 - 100 = 125$ $25 \times 12 = 50 \times 6 = 100 \times 3 = 300$ *[see also Calculation and estimation of powers and roots, 3.01b]*			N3, N6

Curriculum Content

This delivery guide provides support for the delivery of the following topic areas from the curriculum content:

- integer arithmetic
- the composition of integers.

The following activities are designed to give learners practice in each of the basic skills.

Activities	Resources
Place Value The understanding of place value is fundamental to learners' understanding of arithmetic methods. It is important that, prior to ensuring that learners are able to apply the four operations of arithmetic successfully, learners have a good basic understanding of the value of a digit in a number. This process should start with integers.	
[1.02a] Building Number, Place Value This simple PowerPoint assumes that learners are able to read, write and compare numbers up to 999. Using place value, it guides learners in how to read, write and compare larger values, emphasising the connection between columns in the decimal number system. See 'Building Number Place Value' PowerPoint presentation.	▶ Click here
[1.02a] Place Value (National STEM Centre: Nelson Thornes (OUP)) http://www.nationalstemcentre.org.uk/elibrary/resource/10518/place-value Six maths ideas covering the titles given to each column, the importance of place value in expressing values and the problems that can occur when numbers are written incorrectly. Ideas include finding how many numbers can be made using certain digits, what the value of a particular digit represents in a variety of different numbers, questioning the value of zero and exploring the truth of the myths surrounding Popeye. The resources on this website are free to use, but a login is required.	▶ Click here

Curriculum Content

Activities	Resources
Four Rules with Positive Integers Learners need to have good mental arithmetic skills before tackling written methods, since written methods naturally use some mental skills. Various approaches will be helpful – counting on and back for values that are close together – as well as using numbers that pair to make 10. Learners who find recalling multiplication tables a challenge would benefit from having a greater appreciation of the fundamentals of multiplication – such as the order of multiplication not affecting the outcome. Also learners who are able to double are able to use this skill effectively to multiply by 4 and 8 and so on, often with much greater speed than if they had attempted to count up, as is often the case amongst this group. It is often helpful to encourage learners to create their own multiplication grid to use each time they are working on a topic that requires a lot of multiplication. This will help reinforce the connection between multiplication and repeated addition, the commutative nature of multiplication, as well as the inverse relationship between multiplication and division. Before moving on to consider negative integers, develop good, basic skills in applying the four rules to positive integers.	
[1.01a] Pairs of Numbers that Make 10 (Resource: Playing cards) This exercise is for practising the fundamentals of sound mental addition. It can be used as a starter exercise for a competent group of learners or as a main teaching tool for a group that needs more in-depth consideration of the method. Provide learners with a set of playing cards to play either 'snap' or 'matching pairs', where pairs have to form number bonds to 10.	
[1.01a] Multiplication of Integers using the Lattice Method A simple PowerPoint that demonstrates how to multiply integers using the lattice method. Using this method greatly reduces the need for carrying and is only reliant on the ability to multiply single digit numbers and position the results correctly in the box. Learners have the opportunity to see the skill demonstrated with animated worked solutions and then practise on given questions, which they may check using a calculator or other valid method. Weaker learners may benefit from being given a small number of pre-drawn copies of the grids, allowing their first attempts to be focused on learning the method rather than struggling to draw the diagonal lines accurately. See 'Multiplication-Lattice method Integers' PowerPoint presentation.	▶ Click here

Curriculum Content

Activities	Resources
[1.01a] [1.04a] Division by Inverse Multiplication and Repeated Subtraction Learners who use the grid method with powers of 10 will benefit from considering this method of division, should they not already have a successful method. The PowerPoint links division with multiplication in order that learners may make use of their knowledge of multiplication tables; it then considers some examples using repeated subtraction. Learners have the opportunity to see the skill demonstrated with animated worked solutions and then practise on given questions, which they may check using a calculator or other valid method. See 'Division by Inverse Multiplication Repeated Subtraction' PowerPoint presentation.	▶ Click here
[1.01a] [1.04a] Division by Inverse Multiplication and Traditional Short Division Learners must have a good understanding of place value in order to follow the logical steps in this traditional method. The PowerPoint links division with multiplication in order that learners may make use of their knowledge of multiplication tables; it then considers some examples using traditional short division. Learners have the opportunity to see the skill demonstrated with animated worked solutions and then practise on given questions, which they may check using a calculator or other valid method. See 'Division by Inverse Multiplication Traditional Short Division' PowerPoint presentation.	▶ Click here
Four Rules with Negative Integers Once arithmetic with positive values has been mastered, learners are able to access the arithmetic of negative values. Visual presentations should be helpful. It may be useful for some learners to consider that, for example, negative 1 is the same distance from 0 as positive 1, but it is on the opposite side of 0. This can then be extended to explain, for example, that the opposite of 1 is −1 and, as a consequence of using the − sign as meaning opposite, − (−1) is 1 as the opposite of −1 is 1. Number lines should both be vertical and horizontal and the location and direction of positives and negatives can then be explored. After an initial introduction, learners will benefit from exposure to a context such as money or temperature. The distributive law of number can be used to demonstrate examples to explain why the product of two negative integers is a positive integer.	
[1.01a] Add and Subtract with Negatives (Manga High.com) https://www.mangahigh.com/en-gb/maths_games/number/negative_numbers/add_and_subtract_with_negatives Perform addition and subtraction with negative and positive values. The resources on this website are free to use without a login, but they are not supported by all browsers.	▶ Click here
[1.01a] Multiply and Divide with Negatives (Manga High.com) https://www.mangahigh.com/en-gb/maths_games/number/negative_numbers/multiply_and_divide_with_negatives Perform multiplication and division with negative and positive values. The resources on this website are free to use without a login, but they are not supported by all browsers.	▶ Click here

Curriculum Content

Activities	Resources
Inverse Operations Encourage learners to use inverse operations to check answers to calculations as this is not only good practice, but also gives learners strong pre-algebra skills.	
[1.04a] Inverse Operation: Addition and Subtraction (TES Connect) http://www.tes.co.uk/ResourceDetail.aspx?storyCode=6094952& A series of tasks with random inverse/missing link addition and subtraction questions. All are short presentations, ideal for use as a starter activity or mental maths test. These are PowerPoint presentations with audio sound effects added, so if used, headphones or the mute button may be required. The resources on this website are free to use, but a login is required.	▶ Click here
Order of Operations Many learners use BODMAS or BIDMAS as a memory aid. It is vital, however, that learners understand that division and multiplication are equally weighted, as are addition and subtraction, and that if a calculation contains operations of equal weighting they should simply be tackled in order from left to right. It is also important for learners to realise that, for example, the fraction bar and square root can be extended for grouping purposes.	
[1.03a] Order of Operations (Centre for Innovation in Maths Teaching, Plymouth) http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i4/bk8_4i3.htm Interactive 'quiz-type' webpage comprising some practice questions with worked answers for learners to reveal and then a quiz to try with a 'How you did' grid at the bottom.	▶ Click here
Definitions: odd, even, prime, factor (divisor), multiple, common factor (divisor), common multiple, square, cube, root. Most learners will have a basic understanding of odd and even numbers. This can be built upon so that other terms and definitions have meaning and to facilitate recall. Learners may find divisibility tests useful for factorising and generally enjoy working with them.	
[1.02a] [1.01a] Divisibility Rules (Maths is Fun) http://www.mathsisfun.com/divisibility-rules.html This gives the rules for checking whether a number is divisible by numbers up to 12 and it then has a short, interactive multiple-choice quiz at the bottom of the page.	▶ Click here

Curriculum Content

Activities	Resources
[1.02a] Factors, Multiples and Primes: Follow Me Cards (TES Connect) http://www.tes.co.uk/teaching-resource/Factors-Multiples-and-Primes-Follow-me-Cards-6030114/ A nice follow-me 'loop card' starter activity for exercising learners' knowledge of factors, multiples and primes. This activity is suitable for KS3 and KS4 learners. There are 32 cards to be printed that can be adapted and changed to suit different levels. The resources on this website are free, but a login is required.	▶ Click here
[1.02a] Factors and Multiples True/False Quiz (TES Connect) http://www.tes.co.uk/ResourceDetail.aspx?storyCode=3013481& Instructions, questions and answers for a short true/false quiz on factors and multiples. An outline of the activity and extension work are provided. It is suitable for KS3 and KS4 and can be used as a starter or plenary. The resources on this website are free, but a login is required.	▶ Click here
Prime Factorisation The fundamental fact that all integers can be written as a product of their prime factors is better explained using a visual presentation such as a factor tree or factor ladder. This topic should reinforce the definition of a prime number and a product. Writing in index form should be covered once learners are familiar with indices – or indices could be introduced in conjunction with this topic, if appropriate for the level of learners.	
[1.02b] Prime Factors (Centre for Innovation in Maths Teaching, Plymouth) http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i2/bk8_2i2.htm Simple interactive page which can be used to introduce factor trees. The method demonstrated is based on knowing any pair of factors as a starting point. This may work well for some learners, however other learners may prefer a slightly more structured approach to choosing the factors, as illustrated in the following PowerPoint presentation.	▶ Click here

Curriculum Content

Activities	Resources
[1.02b] Prime Factorisation This resource presumes that learners understand the definitions of a factor and a prime number. It has a slightly more methodical approach in that it always starts by considering whether a number is even before applying any knowledge of factors. Factors are identified by recognising which multiplication table the number is located in. See 'Prime Factorisation' PowerPoint presentation. There are lots of opportunities for display work around prime factor trees. The diagrams can easily be turned into 'pot plants' with the starting value in the 'pot', the prime numbers circled and turned into 'flowers' and the lines making stems. Alternatively as a Christmas activity, the starting number can be written in the star on top of a tree, the lines used to form the structure of the tree and tinsel, and the prime numbers circled to make baubles. Learners can also be encouraged to create their own themed prime factor trees. An extension to this could be to investigate the types of numbers that will form symmetrical trees.	▶ Click here
HCF/LCM The process of listing factors and multiples and ringing the HCF or LCM for two or more integers is a useful and meaningful process, albeit slow. It also provides a further opportunity to reinforce the difference between a factor and a multiple. However, prime factorisation is a much quicker process and learners should be encouraged to adopt this approach if possible.	
[1.02c] Highest Common Factor and Lowest Common Multiple (Centre for Innovation in Maths Teaching, Plymouth) http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i2/bk8_2i4.htm Simple interactive page that introduces finding the HCF and LCM by listing factors and multiples respectively. It then progresses to considering the prime factorisation of numbers and using that to find HCFs and LCMs.	▶ Click here

Thinking Conceptually

Approaches to teaching the content

Number is of such importance that it is vital that all learners develop a clear and full understanding of the decimal system and its structure, as well as fluent arithmetic skills. Learners who understand **why** a method works, rather than simply being able to execute it, will have much greater success when problem solving.

Learners whose number skills are insecure are likely to be unable to access large areas of the mathematics syllabus and likewise encounter difficulties in other subjects, such as science and social sciences.

Common misconceptions or difficulties learners may have

With place value, learners commonly:

- work competently with H T U but are confused by larger numbers – particularly in the tens and hundreds of thousands. Learners may be helped by appreciating that the H T U pattern is repeated in the thousands and in the millions.

With arithmetic, learners commonly:

- attempt to calculate all solutions to problems **mentally** when **written** methods are appropriate. Learners should be encouraged to be secure in the application of both mental and written methods and be able to apply them appropriately.

With negative numbers, learners commonly:

- confuse the difference between the operation of subtraction and a negative sign used to indicate a negative number, since the same sign is used for both.

- misinterpret the notation, for example, 'minus minus 3' as opposed to 'subtract negative 3'.
- confuse when to use the 'rules' they have learned for multiplication and division with simple sums and differences.

Also, learners often

- confuse factors and multiples – to address this issue help learners distinguish by using: factor × factor = multiple.
- think that 1 is a prime number, or that all prime numbers are odd, or that all odd numbers are prime.
- think that the inverse of squaring is division by 2 and the inverse of cubing is division by 3.

Conceptual links to other areas of the specification – useful ways to approach this topic to set learners up for topics later in the course

Number – learners need a good understanding of factors and multiples in order to be successful when manipulating fractions. It may be helpful for learners to observe that factors are also called divisors.

Algebra – for example, learners need to be confident in calculating $2 - 3$ before they can attempt $2x - 3x$. Likewise, using inverse operations – essential for algebraic processes such as changing the subject of a formula and solving equations.

Statistics – numerical situations are often used as a basis from which to establish a sample space of possible outcomes, from which probabilities are then evaluated.

The following activities are designed to give learners a more in depth understanding of a process, rather than simply practising the basic skill.

Thinking Conceptually

Activities	Resources
Four Rules with Positive Integers Before moving on to consider negative integers, develop good, basic skills in applying the four rules to positive integers.	
[1.01a] Dicey Operations (NRICH) http://NRICH.maths.org/6606 This activity from NRICH is an excellent way to allow learners to develop confidence and apply problem solving techniques in the guise of game playing. Various options are given for 2-player games involving throwing a die and recording the scores obtained on a grid with the winner being the one whose sum, difference, product or quotient is nearest to 1000. The games encourage discussion of place value, mental calculations and estimation, alongside valuable strategic mathematical thinking.	▶ Click here
The following activity links the Four Rules with Negative Integers to Probability (Sample Spaces): **[1.01a] Connect Three (NRICH)** http://NRICH.maths.org/5864 This is a noughts and crosses type game that uses two dice (one with the numbers 1, 2, 3, −4, −5, −6 and the other with the numbers −1, −2, −3, 4, 5, 6). It offers learners a chance to analyse a game which involves adding and subtracting positive and negative numbers, and requires them to work out the probability of the different possible outcomes.	▶ Click here
[1.01a] Evaluating Directed Number Statements (National STEM Centre: Department for Education) http://www.nationalstemcentre.org.uk/elibrary/maths/resource/1966/evaluating-directed-number-statements-n9 In this resource learners learn to make valid generalisations about the effect of operations on directed numbers. It is helpful if learners have already attempted to use directed quantities in contexts (e.g. money or temperature) before attempting this session. In this session, learners are provided with a collection of statements. They have to decide whether these statements are always, sometimes or never true and justify their choices with examples and counter-examples. The resources on this website are free to use, but a login is required.	▶ Click here

Thinking Conceptually

Activities	Resources
Recognising Numbers by name linking to Probability (Venn Diagrams) **[1.02a] Various Venns (NRICH)** http://NRICH.maths.org/5721 The first two parts of this problem provide an opportunity for learners to become familiar with Venn diagrams, whilst reinforcing their knowledge of number properties. The final part introduces slightly higher-level thinking as learners then work 'backwards'. This activity could be extended into much more challenging areas and is a useful basis for many topics.	▶ Click here
Prime Numbers Logic Puzzles 1 **[1.02b] Strange Numbers (NRICH)** http://NRICH.maths.org/722 A challenging logic puzzle involving prime numbers. This would be a good activity for a confident group of learners.	▶ Click here
Prime Numbers Logic Puzzles 2 **[1.02b] Prime Magic (NRICH)** http://NRICH.maths.org/846 A magic square type puzzle – place the numbers 1, 2, 3, ...9, one on each square of a 3 by 3 grid so that all the rows and columns add up to a prime number.	▶ Click here
[1.02c] Venn Diagrams – LCM, HCF This PowerPoint resource presumes that learners have worked on prime factorisations of numbers and understand the use of factor trees to find them. It demonstrates a straightforward method to find HCF and LCM. See 'Venn Diagrams-LCM HCF' PowerPoint presentation. The activity is a nice link to the use of Venn diagrams, which is new to the content; there is an expectation that the learners will have seen Venn diagrams prior to this lesson, although a brief description of the basic idea is given.	▶ Click here

Thinking Contextually

It is increasingly important that learners see the relevance of their studies to real life everyday events. The result of this for the learner should be greater motivation, better retention of skills and, therefore, improved confidence.

Choosing scenarios that are meaningful to the group of learners is essential to maintain their interest, attention and help them focus on the mathematics required to complete the task – giving them an outline or a basis for study and allowing them to bring their own context to the task would provide for greater, more memorable impact and therefore better retention of skills.

Using Positive Numbers in Context

Activities	Resources
Money and Measures Some of the most natural contexts for arithmetic are provided by problems about money or measures. In this curriculum section, the problems given should be based on integer values only, with decimals considered in the 'Fractions, Decimals and Percentages' section of the content.	
[1.01a] Treasure Hunt – Addition in Context (TES Connect) http://www.tes.co.uk/teaching-resource/Treasure-Hunt-Addition-in-Context-6184000/ A treasure hunt based around finding the answers to various contextual addition questions. Simply print the sheets and stick them up around your classroom or school building in a random order. Print enough answer sheets for all your learners (working in pairs works well). They can start anywhere, answer a question, put the answer in the box on their sheet, then find that answer in the room or building and tackle the new question below it. Learners who complete the treasure hunt will end up back where they started. The resources on this website are free to use, but a login is required.	▶ Click here

Thinking Contextually

Activities	Resources
Time Counting on and back using timetables gives learners the opportunity to practise simple arithmetic skills using a different number system.	
[1.01a] Timetables and Charts (Manga High.com) https://www.mangahigh.com/en-gb/math_games/number/time_timetables_and_charts/timetables_and_charts A simple, interactive online game involving the following skills: read and write the time in 12 and 24 hour formats; find the difference between two times; read information from charts (deciding what information is and is not required); and interpret 2 way distance/price charts and timetables in 12 and 24 hour formats. The resources on this website are free to use without a login, but they are not supported by all browsers.	▶ Click here

Using Negative Numbers in Context

Activities	Resources
[1.01a] Sea Level (NRICH) http://NRICH.maths.org/5929 This task is a simple way to increase familiarity with negative numbers on a number line. In answering the questions, learners begin to calculate with negative numbers in a context that will give them confidence.	▶ Click here
[1.01a] Using Directed Numbers in Context N8 (National STEM Centre: Department for Education (Standards Unit: Improving learning in Mathematics)) http://www.nationalstemcentre.org.uk/elibrary/resource/1965/using-directed-numbers-in-context-n8 In this resource from the DfE Standards Unit learners use directed numbers in the context of temperatures. Learners are provided with the temperatures in four cities; temperature differences between these and four other cities are also given. Learners are asked to find missing temperatures and temperature differences. The resources on this website are free to use, but a login is required.	▶ Click here

Thinking Contextually

Activities	Resources
[1.01a] Maths Vegas! Negative Numbers (TES Connect) http://www.tes.co.uk/teaching-resource/Maths-Vegas-Negative-Numbers-6174685/ Learners have a stake and a set of rules to follow to play this simple game comprised of ten questions. The game is played in groups of 4. Each group starts with £50 and before each question they must decide on their stake (min £1, max £20); if they answer the question correct the stake is added to their pot, but if they are wrong they lose their stake. The resources on this website are free to use, but a login is required.	▶ Click here
[1.01a] Using Positive and Negative Numbers in Context (Mathematics Assessment Resource Service) http://map.mathshell.org.uk/materials/download.php?fileid=1304 This resource is designed to address statements in the US Common Core State Standards for Maths. However, the resource is entirely appropriate for use with learners in the UK as it is intended to help identify and aid learners who have difficulties in ordering, comparing, adding, and subtracting positive and negative integers. Particular attention is paid to the use of negative numbers on number lines to explore the structures: • starting temperature + change in temperature = final temperature • final temperature – change in temperature = starting temperature • final temperature – starting temperature = change in temperature	▶ Click here

Thinking Contextually

Practical Applications of HCF and LCM

Activities	Resources
[1.02c] Real Highest Common Factor/Lowest Common Multiple (TES Connect) http://www.tes.co.uk/teaching-resource/Real-Highest-Common-Factor-Lowest-Common-Multiple-6305435/ An A–Z card activity based on real life applications of highest common factor and lowest common multiple. The resources on this website are free to use, but a login is required.	▶ Click here
[1.02a] Maths and Numeracy – Exploring Digital Devices (Place Value and the Decimal System)(National STEM Centre: NCETM Maths and Numeracy – Exploring Digital Devices) http://www.nationalstemcentre.org.uk/elibrary/maths/resource/5067/issue-3-maths-and-numeracy-exploring-digital-devices Using the context of digital devices, this article – from the National Centre for Excellence in the Teaching of Mathematics (NCETM) FE online magazine – explores how learners might be helped to understand how decimal numbers work by considering binary numbers and manipulating them. Tables are used to explain the link between base 2 and base 10. The resources on this website are free to use, but a login is required.	▶ Click here

We'd like to know your view on the resources we produce. By clicking on the 'Like' or 'Dislike' button you can help us to ensure that our resources work for you. When the email template pops up please add additional comments if you wish and then just click 'Send'. Thank you.

If you do not currently offer this OCR qualification but would like to do so, please complete the Expression of Interest Form which can be found here: www.ocr.org.uk/expression-of-interest

OCR customer contact centre

General qualifications
Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk