Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4755**: Further Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

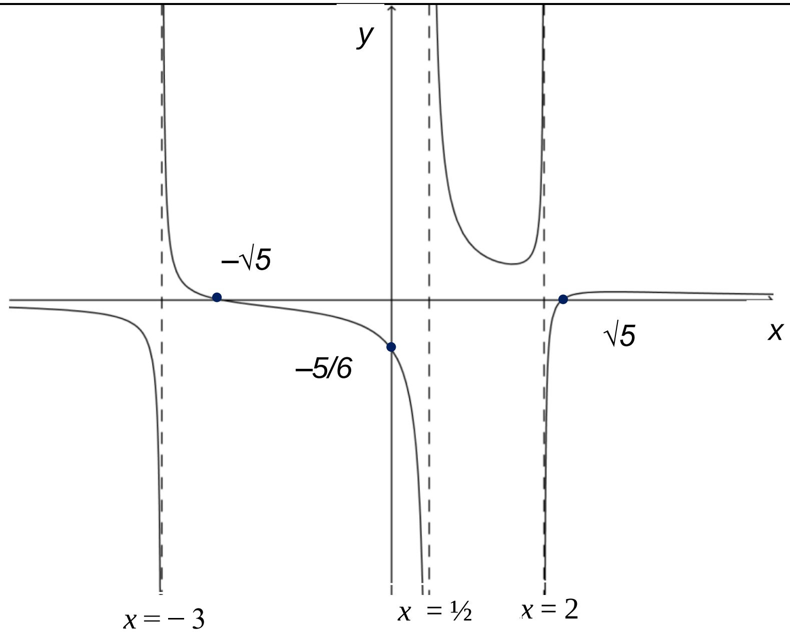
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

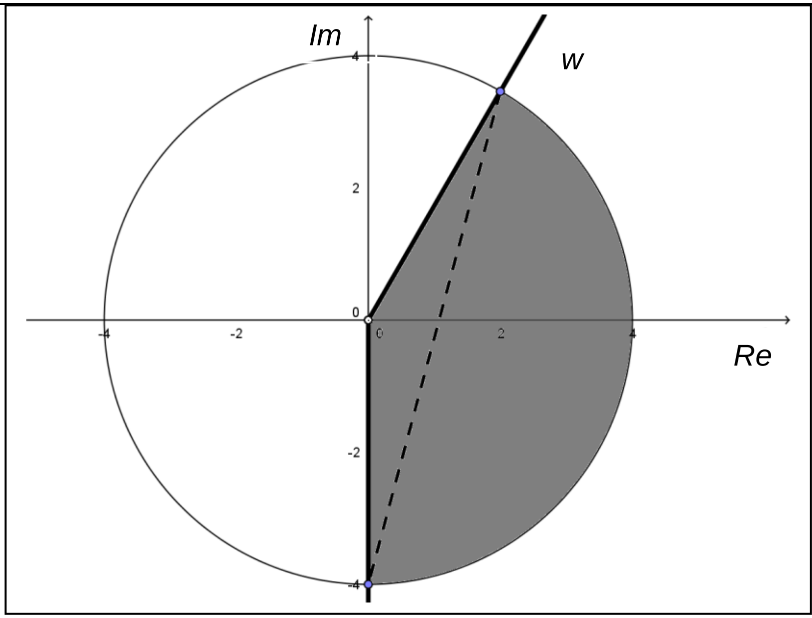
Question			Answer	Marks	Guidance
1			$\sum_{\gamma=1}^{n} r(r-2) = \sum_{\gamma=1}^{n} r^2 - 2\sum_{\gamma}^{n} r$	M1	Separate sum (may be implied)
			$= \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1,A1	1 mark for each part oe
			$= \frac{1}{6}n(n+1)[(2n+1)-6]$	M1	$n(n+1)$(linear factor) seen
			$= \frac{1}{6}n(n+1)(2n-5)$	A1	Or $n(n+1)(2n-5)/6$ only, ie 1/6 must be a factor
				[5]	
2	(i)		$\begin{pmatrix} -3 & -2 \\ -2 & 1 \end{pmatrix}$	B1,B1 **[2]**	1 mark for each column. Must be a 2x2 matrix Condone lack of brackets throughout
2	(ii)		$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1 **[1]**	
2	(iii)		$\begin{pmatrix} -3 & -2 \\ 2 & -1 \end{pmatrix}$	B1,B1 **[2]**	1 mark for each column (no ft). Must be a 2x2 matrix
3			$z = 2-3\text{j}$ is also a root	B1	
			Either		
			$(z-(2+3\text{j}))(z-(2-3\text{j})) = ((z-2)+3\text{j})((z-2)-3\text{j})$	M1	Condone (z+2+3j)(z+2-3j)
			$= z^2 - 4z + 13$	A1	Correct quadratic
			$z^4 - 5z^3 + 15z^2 - 5z - 26 = (z^2 - 4z + 13)(z^2 - z - 2)$	M1 A1	Valid method to find the other quadratic factor. Correct quadratic
			$(z^2 - z - 2) = (z-2)(z+1)$		
			So the other roots are 2 and -1	A1,A1 **[7]**	1 mark for each root, cao

Question			Answer	Marks	Guidance
			Or		
			$2+3j+2-3j+\gamma+\delta=5$ oe	B1	Sum of roots with substitution of roots $2\pm3j$ for α and β
			$(2+3j)(2-3j)\gamma\delta=-26$	M1	Attempt to obtain equation in $\gamma\delta$ using a root relation and $2\pm3j$
			$\gamma\delta=-2$		
			$\Rightarrow 4+\gamma+\delta=5\Rightarrow\gamma=1-\delta$		
			and $13\gamma\delta=-26\Rightarrow\gamma\delta=-2$		
			$\Rightarrow\delta(1-\delta)=-2\Rightarrow\delta^2-\delta-2=0$	M1 A1	Eliminating γ or δ leading to a quadratic equation Correct equation obtained
			$\Rightarrow(\delta+1)(\delta-2)=0$		
			So the other roots are -1 and 2.	A1,A1	1 mark for each, cao If 2, -1 guessed from $\gamma+\delta=1$ and $\gamma\delta=-2$ give A1 A1 for these equations and A1A1 for the roots. **SC** factor theorem used. M1 for substitution of $z=-1$ (or 2) or division by (z + 1) (or by z - 2), A1 if zero obtained, B1 for the root stated to be -1(or 2). For the other root, similarly but M1A1A1 Max [**7/7**] Answers only get M0M0, max [**1/7**]
				[7]	
4			$\sum_{r=1}^{n}\frac{1}{(2r+3)(2r+5)}=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r+3}-\frac{1}{2r+5}\right]$	M1	Split to partial fractions. Allow missing ½
			$=\frac{1}{2}\left[\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{7}\ldots\right)+\ldots\ldots+\left(\ldots-\frac{1}{2n+5}\right)\right]$	M1 A1	Expand to show pattern of cancelling, at least 4 fractions All correct, allow missing ½, condone r
			$=\frac{1}{2}\left[\frac{1}{5}-\frac{1}{2n+5}\right]=\frac{n}{5(2n+5)}$	M1 A1	Cancel to first minus last term must be in terms of n. oe single fraction
				[5]	

Question			Answer	Marks	Guidance
5			**Either**		
			$y=3x-1 \Rightarrow x=\dfrac{y+1}{3}$	M1*	Change of variable, condone $\dfrac{y-1}{3}, \dfrac{y}{3} \pm 1$.
			$\Rightarrow 3\left(\dfrac{y+1}{3}\right)^3 - 9\left(\dfrac{y+1}{3}\right)^2 + \left(\dfrac{y+1}{3}\right) - 1 = 0$	M1dep*	Substitute into cubic expression
				A1	Correct
			Correct coefficients in cubic expression (may be fractions)	A3ft	ft their substitution (-1 each error)
			$\Rightarrow y^3 - 6y^2 - 12y - 14 = 0$	A1	cao. Must be an equation with integer coefficients
				[7]	
			Or		
			$\alpha+\beta+\gamma = \dfrac{9}{3} = 3$	M1	All three root relations, condone incorrect signs
			$\alpha\beta+\alpha\gamma+\beta\gamma = \dfrac{1}{3}$		
			$\alpha\beta\gamma = \dfrac{1}{3}$	A1	All correct
			Let new roots be k, l, m then $k+l+m = 3(\alpha+\beta+\gamma)-3 = 6$ $kl+km+lm = 9(\alpha\beta+\alpha\gamma+\beta\gamma)-6(\alpha+\beta+\gamma)+3 = -12$	M1	Using (3α-1) etc in $\sum k, \sum kl, klm$, at least two attempted, and using $\sum\alpha, \sum\alpha\beta, \alpha\beta\gamma$
			$klm = 27\alpha\beta\gamma - 9(\alpha\beta+\beta\gamma+\beta\gamma)+3(\alpha+\beta+\gamma)-1 = 14$	A3ft	One each for 6, -12, 14, ft their $3, \dfrac{1}{3}, \dfrac{1}{3}$.
			$\Rightarrow y^3 - 6y^2 - 12y - 14 = 0$	A1	cao. Must be an equation with integer coefficients
				[7]	

Question			Answer	Marks	Guidance
6			When $n = 1$, $\frac{1}{1\times 3}=\frac{1}{3}$ and $\frac{n}{2n+1}=\frac{1}{3}$, so true for $n = 1$	B1	Condone eg "$\frac{1}{3}=\frac{1}{3}$ "
			Assume true for $n = k$	E1	Assuming true for k, (some work to follow) If in doubt look for unambiguous "if…then" at next E1 Statement of assumed result not essential but further work should be seen
			Sum of $k + 1$ terms		NB "last term =sum of terms" seen anywhere earns final E0
			$=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$	M1	Adding correct $(k+1)$th term to sum for k terms
			$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$	M1	Combining their fractions
			$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$		
			$=\frac{(k+1)(2k+1)}{(2k+1)(2k+3)}=\frac{k+1}{2k+3}$	A1	Complete accurate work
			which is $\frac{n}{2n+1}$ with $n=k+1$		May be shown earlier
			Therefore if true for $n = k$ it is also true for $n = k+1$.	E1	Dependent on A1 and previous E1.
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1 **[7]**	Dependent on B1 and previous E1 E0 if "last term"= "sum of terms " seen above

Question			Answer	Marks	Guidance
7	(i)		$\left(0,\ -\frac{5}{6}\right)$	B1	Allow for both $x=0$ and $y=-\dfrac{5}{6}$ seen
			$\left(\sqrt{5},\ 0\right),\ \left(-\sqrt{5},\ 0\right)$	B1	(both) Allow $(\pm\sqrt{5},0)$ or for both $y=0$ and $x=\pm\sqrt{5}$ seen
				[2]	
	(ii)		$a = 2$	B1	
			$y = 0$	B1	
			$x = -3,\ x = 2$	B1 [3]	Must be two equations
	(iii)			B1 B1 B1 B1 [4]	Two outer branches correctly placed Inner branches correctly placed Correct asymptotes and intercepts labelled For good drawing. Dep all 3 marks above Look for a clear maximum point on the right-hand branch, (not really shown here). Condone turning points in $-\sqrt{5}<x<\dfrac{1}{2}, y<0$
	(iv)		$-3<x<-\sqrt{5},\ \dfrac{1}{2}<x<2,\ x>\sqrt{5}$	B3 [3]	One mark for each. Strict inequalities. Allow 2.24 for $\sqrt{5}$ (if B3 then -1 if more than 3 inequalities)

Question			Answer	Marks	Guidance
8	(i)		$\lvert w\rvert = \sqrt{\left(2^2 + \left(2\sqrt{3}\right)^2\right)} = 4$	B1	
			$\arg w = \arctan\frac{2\sqrt{3}}{2} = \frac{\pi}{3}$	M1	
			$w = 4\left(\cos\frac{\pi}{3} + \text{j}\sin\frac{\pi}{3}\right)$	A1	Accept $\left(4,\ \frac{\pi}{3}\right)$, 1.05 rad, 60° in place of $\frac{\pi}{3}$, or $4e^{j\frac{\pi}{3}}$
				[3]	
	(ii)			B1 B1 B1 B1 B1 B1	Circle, or arc of circle, centre the origin Radius 4 Half line from origin $\frac{\pi}{4} < \text{angle} < \frac{\pi}{2}$ with positive real axis or acute angle labelled as $\pi/3$ Use of negative Im axis clearly indicated Correct region indicated. Dependent on first 4 B marks. Ignore placing of *w*. *w* at intersection of $\frac{\pi}{3}$ line and circle (dep 1st 3 B marks)

Question			Answer	Marks	Guidance
			Maximum $\lvert z-w\rvert = \sqrt{\left(2^2+\left(4+2\sqrt{3}\right)^2\right)}$ = 7.73 (3 s.f.) Or 2x 4cos15° = $2\sqrt{6}+2\sqrt{2}$	B1 M1 A1 **[9]**	Maximum $\lvert z-w\rvert$ indicated by chord on diagram oe or sight of $-4j-(2+2\sqrt{3}j)$ oe Valid attempt to calculate maximum $\lvert z-w\rvert$ allow $\sqrt{32+16\sqrt{3}}$ oe (accept 2 s.f. or better)
9	**(i)**		$\beta=(-1)(3\alpha-1)+5\alpha+(-1)(2\alpha+1)$ $=-3\alpha+1+5\alpha-2\alpha-1=0$	M1 A1 **[2]**	multiply second row of **A** with first column of **B** Correct
	(ii)		$\gamma=(1)(3\alpha-1)+15+(-1)(2\alpha+1)$ $=\alpha+13$	M1 A1 **[2]**	Attempt to multiply relevant row of **A** with relevant column of **B**. Condone use of **BA** instead Correct
	(iii)		When $\alpha=2,\ \gamma=15$ $\mathbf{A}^{-1}=\frac{1}{15}\begin{pmatrix}5 & -8 & -1\\5 & 1 & 2\\5 & -5 & 5\end{pmatrix}$ $\mathbf{A}^{-1}$ does not exist when $\alpha=-13$	M1 A1 B1ft **[3]**	Multiplication of **B** by $\frac{1}{\text{their } \gamma}$,($\gamma\neq 1$) using $\alpha=2$ in both Correct elements in matrix and correct γ. ft their $\gamma=0$. Condone "$\alpha\neq-13$"

Question			Answer	Marks	Guidance
	(iv)		$\frac{1}{15}\begin{pmatrix}5 & -8 & -1\\ 5 & 1 & 2\\ 5 & -5 & 5\end{pmatrix}\begin{pmatrix}25\\ 11\\ -23\end{pmatrix}=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$	M1	Set-up of pre-multiplication by their 3x3 $\mathbf{A}^{-1}$, or by **B** (using $\alpha = 2$)
			$=\frac{1}{15}\begin{pmatrix}60\\ 90\\ -45\end{pmatrix}=\begin{pmatrix}4\\ 6\\ -3\end{pmatrix}$	B1	$(60\ \ 90\ \ -45)'$ soi need not be fully evaluated
			$\Rightarrow x = 4,\ y = 6,\ z = -3$	A3	cao A1 for each explicit identification of x, y, z in a vector or a list. (-1 unidentified) Answers only or solution by other method, M0A0
				[5]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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Head office
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Facsimile: 01223 552553

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