



Thursday 12 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

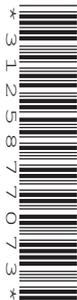
OCR supplied materials:

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 30 \cos 2t.$$

- (i) Find the general solution. [9]

The particle is initially at the origin, travelling with velocity 10 m s^{-1} .

- (ii) Find the particular solution. [4]

- (iii) Find the amplitude of the oscillations of the particle for large values of t . [2]

Consider now the differential equation

$$\frac{d^3x}{dt^3} + 2\frac{d^2x}{dt^2} + 2\frac{dx}{dt} = 0.$$

- (iv) Show that 0 is a root of the auxiliary equation and write down the other roots. [2]

- (v) Find the particular solution of this differential equation subject to the initial conditions

$$x = 0, \frac{dx}{dt} = 10 \text{ and } \frac{d^2x}{dt^2} = 4 \text{ when } t = 0. \quad [5]$$

- (vi) Sketch the graph of this solution. [2]

- 2 The population, P , of a species at time t hours is to be modelled by a differential equation. The initial population is 100.

At first, the model $\frac{dP}{dt} - 0.25P = 0$ is used.

- (i) Find P in terms of t and comment on the suitability of this model. [4]

To allow for certain environmental effects, the model is refined to

$$\frac{dP}{dt} - 0.25P = -18e^{-0.5t}.$$

- (ii) Write down the complementary function for this differential equation. Find a particular integral and hence state the general solution. [6]

- (iii) Find the solution subject to the given initial condition and comment on the suitability of this refined model. [3]

The following mathematical model for the population is now used.

$$\frac{dP}{dt} = 6 \times 10^{-4}P(400 - P)$$

- (iv) Solve this differential equation subject to the given initial condition, expressing P in terms of t . [8]

- (v) Show that the time T hours at which $P = 200$ is given by

$$T = \frac{25}{6} \ln 3. \quad [1]$$

- (vi) What does this model predict for the population of the species in the long term? [2]

- 3 (a) The equation of a curve in the x - y plane satisfies the differential equation

$$(x+1)\frac{dy}{dx} - xy = e^{2x}$$

for $x > -1$.

- (i) Show that an integrating factor for this differential equation is $e^{-x}(1+x)$ and hence find the general solution for y in terms of x . [11]

The curve passes through the point $(0, -2)$.

- (ii) Find the equation of this curve. [2]

- (b) The differential equation

$$\frac{dy}{dx} = \frac{1}{x^2 + y^2}$$

is to be solved approximately, first by using a tangent field and then by Euler's method.

- (i) Show that the isocline for which $\frac{dy}{dx} = 4$ is a circle and state its centre and radius. [2]

- (ii) Sketch the isoclines for the cases $\frac{dy}{dx} = \frac{1}{4}$, $\frac{dy}{dx} = 1$ and $\frac{dy}{dx} = 4$. Use these isoclines to draw a tangent field. [3]

- (iii) Sketch the solution curve through $(0, 1)$. [2]

Euler's method is now used, starting at $x = 0$, $y = 1$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$.

- (iv) Use a step length of 0.05 to estimate y when $x = 0.15$. [4]

Question 4 begins on page 4

4 The simultaneous differential equations

$$\frac{dx}{dt} + 2x = 4y + e^{-2t}$$

$$\frac{dy}{dt} + 3x = 5y + 2e^{-2t}$$

are to be solved.

(i) Obtain a second order differential equation for x in terms of t . Hence find the general solution for x . [12]

(ii) Find the corresponding general solution for y . [4]

When $t = 0$, $y = -\frac{2}{3}$ and $\frac{dy}{dt} = 0$.

(iii) Find the particular solutions for x and y . [5]

(iv) Find the set of values of t for which $y > x$. [3]

END OF QUESTION PAPER



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