OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4756/01

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

MONDAY 23 JUNE 2014: MORNING

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book, or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4756/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LEG) 4064182887 100148/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (54 marks)

1 (a) Given that $f(x) = \arccos x$,

(i) sketch the graph of $y = f(x)$, [2]

(ii) show that $f'(x) = -\frac{1}{\sqrt{1-x^2}}$, [3]

(iii) obtain the Maclaurin series for $f(x)$ as far as the term in x^3. [7]

(b) A curve has polar equation $r = \theta + \sin\theta, \theta \geqslant 0$.

(i) By considering $\frac{dr}{d\theta}$ show that r increases as θ increases.

Sketch the curve for $0 \leqslant \theta \leqslant 4\pi$. [4]

(ii) You are given that $\sin\theta \approx \theta$ for small θ. Find in terms of α the approximate area bounded by the curve and the lines $\theta = 0$ and $\theta = \alpha$, where α is small. [3]

2 **(a)** **The infinite series C and S are defined as follows.**

$$C = a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \ldots,$$

$$S = a\sin\theta + a^2\sin 2\theta + a^3\sin 3\theta + \ldots,$$

where a is a real number and $|a| < 1$.

By considering $C + \mathrm{j}S$, show that

$$S = \frac{a\sin\theta}{1 - 2a\cos\theta + a^2}.$$

Find a corresponding expression for C. [8]

(b) **P is one vertex of a regular hexagon in an Argand diagram. The centre of the hexagon is at the origin. P corresponds to the complex number $\sqrt{3} + \mathrm{j}$.**

(i) **Find, in the form $x + \mathrm{j}y$, the complex numbers corresponding to the other vertices of the hexagon. [5]**

(ii) **The six complex numbers corresponding to the vertices of the hexagon are squared to form the vertices of a new figure. Find, in the form $x + \mathrm{j}y$, the vertices of the new figure. Find the area of the new figure. [4]**

3 (a) (i) Find the eigenvalues and corresponding eigenvectors for the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 6 & -3 \\ 4 & -1 \end{pmatrix}.$$ [5]

(ii) Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [2]

(b) (i) The 3×3 matrix $\mathbf{B}$ has characteristic equation $\lambda^3 - 4\lambda^2 - 3\lambda - 10 = 0$.
Show that 5 is an eigenvalue of $\mathbf{B}$. Show that $\mathbf{B}$ has no other real eigenvalues. [4]

(ii) An eigenvector corresponding to the eigenvalue 5 is $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$.

Evaluate $\mathbf{B}\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{B}^2\begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix}$.

Solve the equation $\mathbf{B}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \\ 40 \end{pmatrix}$ for x, y, z. [4]

(iii) Show that $\mathbf{B}^4 = 19\,\mathbf{B}^2 + 22\,\mathbf{B} + 40\,\mathbf{I}$. [3]

SECTION B (18 marks)

4 **(i)** Given that $\sinh y = x$, show that

$$y = \ln\left(x + \sqrt{1 + x^2}\right). \qquad (*)$$

Differentiate (*) to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1+x^2}}. \quad [8]$$

(ii) Find $\displaystyle\int \frac{1}{\sqrt{25+4x^2}}\,\mathrm{d}x$, expressing your answer in logarithmic form. [3]

(iii) Use integration by substitution with $2x = 5\sinh u$ to show that

$$\int \sqrt{25+4x^2}\,\mathrm{d}x = \frac{25}{4}\left(\ln\left(\frac{2x}{5} + \sqrt{1+\frac{4x^2}{25}}\right) + \frac{2x}{5}\sqrt{1+\frac{4x^2}{25}}\right) + c,$$

where c is an arbitrary constant. [7]

END OF QUESTION PAPER