OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4754/01A

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

WEDNESDAY 18 JUNE 2014: Afternoon

DURATION: 1 hour 30 minutes plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4754/01A
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (SJF/CGW) 4050073916 99964/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

This paper will be followed by PAPER B: COMPREHENSION.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 Express $\dfrac{3x}{(2-x)(4+x^2)}$ in partial fractions. [5]

2 Find the first three terms in the binomial expansion of $(4+x)^{\frac{3}{2}}$. State the set of values of x for which the expansion is valid. [5]

3 Fig. 3 below shows the curve $y = x^3 + \sqrt{(\sin x)}$ for $0 \leqslant x \leqslant \frac{\pi}{4}$.

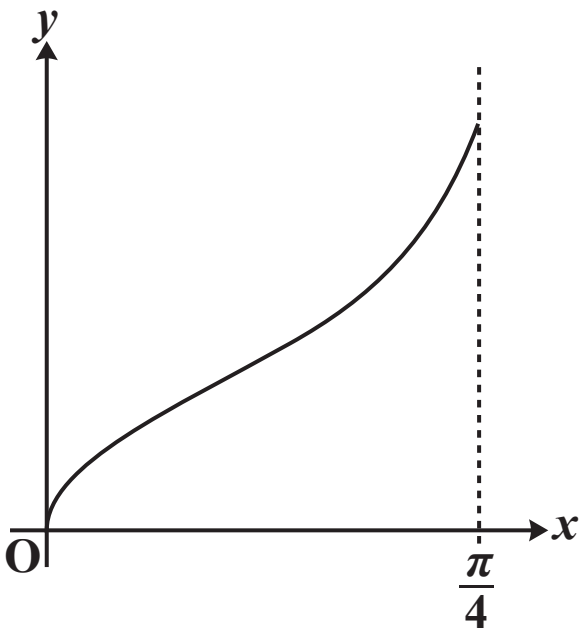

FIG. 3

(i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, giving your answer to 3 decimal places. [4]

(ii) Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say. [1]

4 (i) Show that $\cos(\alpha+\beta)=\dfrac{1-\tan\alpha\tan\beta}{\sec\alpha\sec\beta}$. **[3]**

(ii) Hence show that $\cos 2\alpha=\dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$. **[2]**

(iii) Hence or otherwise solve the equation $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\dfrac{1}{2}$ **for** $0^\circ \leqslant \theta \leqslant 180^\circ$. **[3]**

5 A curve has parametric equations $x=\mathrm{e}^{3t}, y=t\mathrm{e}^{2t}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ **in terms of** t. **Hence find the exact gradient of the curve at the point with parameter** t **= 1. [4]**

(ii) Find the cartesian equation of the curve in the form $y=ax^b\ln x$, **where** a **and** b **are constants to be determined. [3]**

6 Fig. 6 below shows the region enclosed by the curve $y = (1 + 2x^2)^{\frac{1}{3}}$ and the line $y = 2$.

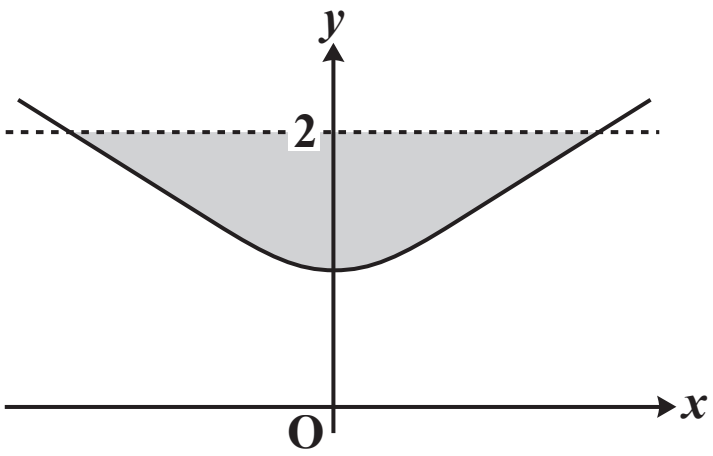

FIG. 6

This region is rotated about the y-axis. Find the volume of revolution formed, giving your answer as a multiple of π. [6]

SECTION B (36 marks)

7 Fig. 7 below shows a tetrahedron ABCD. The diagram is a 2 dimensional representation of the 3 dimensional tetrahedron. x, y and z axes are also shown. The coordinates of the vertices, with respect to axes Oxyz, are A(–3, 0, 0), B(2, 0, –2), C(0, 4, 0) and D(0, 4, 5).

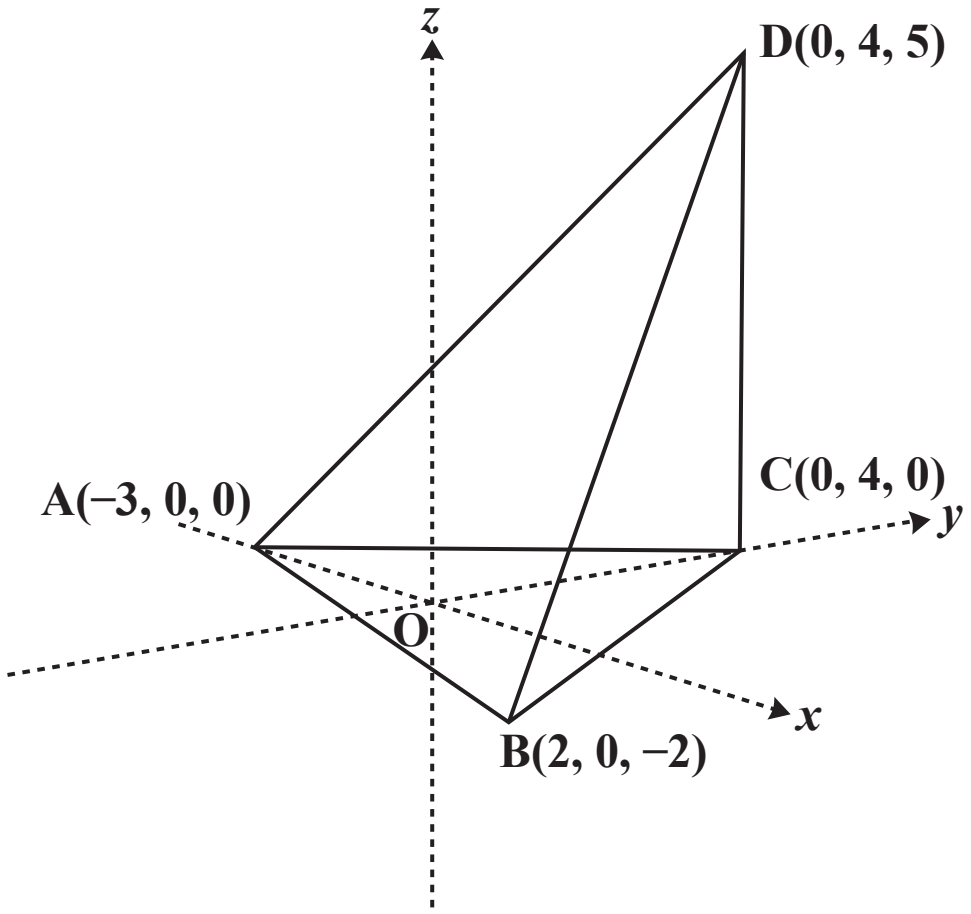

FIG. 7

(i) Find the lengths of the edges AB and AC, and the size of the angle CAB. Hence calculate the area of triangle ABC. [7]

(ii) (*A*) Verify that $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$ is normal to the plane ABC. [2]

(*B*) Hence find the equation of this plane. [2]

(iii) Write down a vector equation for the line through D perpendicular to the plane ABC. Hence find the point of intersection of this line with the plane ABC. [5]

The volume of a tetrahedron is
$\frac{1}{3}$ × area of base × height.

(iv) Find the volume of the tetrahedron ABCD. [2]

8 Fig. 8.1 below shows the side view of an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.

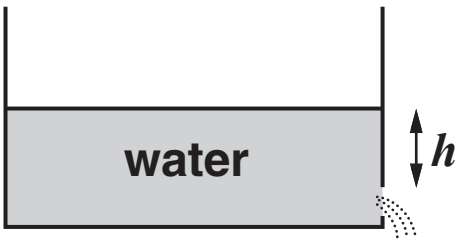

FIG. 8.1

The height of the water surface above the hole t seconds after opening the hole is h metres, where

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -A\sqrt{h}$$

and where A is a positive constant. Initially the water surface is 1 metre above the hole.

(i) Verify that the solution to this differential equation is

$$h = \left(1 - \frac{1}{2}At\right)^2. \quad [3]$$

The water stops leaking when $h = 0$. This occurs after 20 seconds.

(ii) Find the value of A, and the time when the height of the water surface above the hole is 0.5 m. [4]

Fig. 8.2 below shows a similar situation with a different barrel; h is in metres.

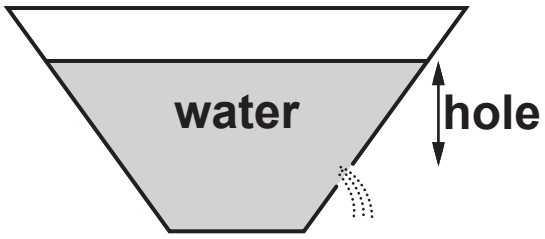

FIG. 8.2

For this barrel,

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -B\frac{\sqrt{h}}{(1+h)^2}$$

where B is a positive constant. When $t = 0$, $h = 1$.

(iii) Solve this differential equation, and hence show that

$$h^{\frac{1}{2}}(30 + 20h + 6h^2) = 56 - 15Bt. \qquad [7]$$

(iv) Given that h = 0 when t = 20, find B.

Find also the time when the height of the water surface above the hole is 0.5 m. [4]

END OF QUESTION PAPER