OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4776/01

MATHEMATICS (MEI)

Numerical Methods

QUESTION PAPER

MONDAY 16 JUNE 2014: Morning

DURATION: 1 hour 30 minutes

plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book, or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4776/01

MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (SJF) 3980263919 99921/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 You are given that the equation

$$x^2 + \frac{1}{x} - 5 = 0$$

has one root, α, in the interval (0.2, 0.3) and another root, β, in the interval (2, 2.2).

(i) Use the method of false position with starting values 0.2 and 0.3 to find a first estimate of α. Apply false position again to find a second estimate of α.

A third application of the false position method gives 0.201 640 as an estimate of α. Give the value of α to the accuracy that appears to be justified. [5]

(ii) Suppose that β is to be found using the bisection method, starting with the interval (2, 2.2).

Write down the initial estimate of β and the associated maximum possible error.

Determine how many further steps of the bisection method would be required to find β with a maximum possible error of less than 0.005. (You are not required to carry out this bisection process.) [3]

2 **The table below shows several values of the mid-point rule (M) and trapezium rule (T) estimates of an integral for different values of h.**

h	M	T
0.5	0.536 650	0.594 027
0.25		
0.125	0.555 282	0.558 599

(i) Calculate the two missing values in the table. Calculate also three Simpson's rule estimates of the integral. [7]

(ii) Give the value of the integral to the accuracy that appears justified. [1]

3 **In computer science, powers of 2 occur frequently. It is quite common to approximate 2^{10k} as 10^{3k}.**

(i) Find the relative error in this approximation for each of $k = 1$, $k = 2$ and $k = 3$. [4]

(ii) You are reminded that if $X = x(1 + r)$, where r is small, then $X^n \approx x^n(1 + nr)$.

How does this result relate to your answers in part (i)? [2]

4 The function $\mathrm{f}(x)$ has the values shown in the table below.

x	1.9	1.95	2	2.05	2.1
$\mathrm{f}(x)$	0.385 570	0.677 625	1	1.356 076	1.749 638

(i) Calculate two estimates of $\mathrm{f}'(2)$ using the forward difference method. [2]

(ii) Calculate two estimates of $\mathrm{f}'(2)$ using the central difference method. [2]

(iii) Give the value of $\mathrm{f}'(2)$ to the accuracy that appears justified, explaining your reasoning. [2]

5 The table below gives some values of the function $\mathrm{g}(x)$ correct to 6 decimal places.

x	1	1.1	1.2
$\mathrm{g}(x)$	1.188 395	1.234 281	1.287 500

(i) Estimate g(1.14) using linear interpolation. [2]

(ii) Use Newton's forward difference interpolation formula to write down a quadratic approximation to $\mathrm{g}(x)$. You need not simplify your answer. [4]

(iii) Hence obtain a second estimate of g(1.14). [2]

SECTION B (36 marks)

6 **You are given that the equation**

$$6x^5 - 3x^2 - 2x + 1 = 0$$

has exactly 3 real roots. Let these roots be α, β, γ where $\alpha < \beta < \gamma$.

(i) Show that there is one root in the interval (–1,0) and two roots in the interval (0,1). **[3]**

(ii) Use the iteration

$$x_{r+1} = \left(\frac{1}{6}\left(3x_r^2 + 2x_r - 1\right)\right)^{\frac{1}{5}}$$

to find α correct to 2 decimal places. [4]

(iii) Use the iteration

$$x_{r+1} = \frac{1}{2}\left(6x_r^5 - 3x_r^2 + 1\right) \quad (*)$$

to verify that $\beta \approx 0.34$.

By considering a suitable derivative, show that the convergence of (*) to β will be slow. [6]

(iv) Use the Newton-Raphson method to find γ correct to 4 decimal places. [5]

7 **A function $f(x)$ is known to have the following values correct to 3 decimal places.**

x	0	0.5	1	1.5
$f(x)$	1.693	1.405	1.288	1.253

The value of the integral $I = \int_0^{1.5} f(x)\,dx$ is required.

You should give all numerical answers in this question correct to 5 decimal places.

(i) Find the best estimate possible of I using just the trapezium rule. [2]

(ii) By drawing a suitable graph, determine whether your answer in part (i) is likely to be an underestimate or an overestimate. [4]

(iii) Obtain two further estimates of I, each of them using both the trapezium rule and Simpson's rule. With reference to your graph, explain which of these estimates seems likely to be more accurate. [6]

(iv) The integration rule

$$\int_0^{3h} \mathrm{f}(x)\,\mathrm{d}x \approx \frac{3h}{8}\left(\mathrm{f}(0) + 3\,\mathrm{f}(h) + 3\,\mathrm{f}(2h) + \mathrm{f}(3h)\right)$$

is designed to be used when 4 equally spaced values of a function are known.

Find the value of I given by this rule.

You should now assume that the value of I you have just calculated is correct to 3 decimal places. Find the approximate errors in your three earlier estimates of I, and hence determine whether your conclusions in parts (ii) and (iii) are confirmed. [6]

END OF QUESTION PAPER

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