



**Wednesday 18 June 2014 – Afternoon**

**A2 GCE MATHEMATICS (MEI)**

**4754/01A** Applications of Advanced Mathematics (C4) Paper A

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

**MODIFIED LANGUAGE**



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (36 marks)

- 1 Express  $\frac{3x}{(2-x)(4+x^2)}$  in partial fractions. [5]
- 2 Find the first three terms in the binomial expansion of  $(4+x)^{\frac{3}{2}}$ . State the set of values of  $x$  for which the expansion is valid. [5]
- 3 Fig. 3 shows the curve  $y = x^3 + \sqrt{(\sin x)}$  for  $0 \leq x \leq \frac{\pi}{4}$ .

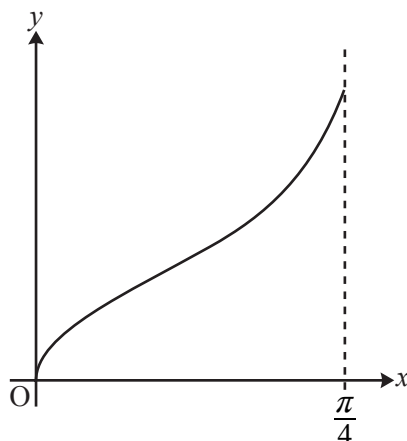


Fig. 3

- (i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the  $x$ -axis and the line  $x = \frac{\pi}{4}$ . Give your answer to 3 decimal places. [4]
- (ii) The number of strips in the trapezium rule is increased. Without doing further calculations, state whether the area estimate increases, decreases, or it is not possible to say. Give a reason for your answer. [1]
- 4 (i) Show that  $\cos(\alpha + \beta) = \frac{1 - \tan \alpha \tan \beta}{\sec \alpha \sec \beta}$ . [3]
- (ii) Hence show that  $\cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$ . [2]
- (iii) Hence or otherwise solve the equation  $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1}{2}$  for  $0^\circ \leq \theta \leq 180^\circ$ . [3]
- 5 A curve has parametric equations  $x = e^{3t}, y = te^{2t}$ .
- (i) Find  $\frac{dy}{dx}$  in terms of  $t$ . Hence find the exact gradient of the curve at the point with parameter  $t = 1$ . [4]
- (ii) Find the cartesian equation of the curve in the form  $y = ax^b \ln x$ , where  $a$  and  $b$  are constants to be determined. [3]

- 6 Fig. 6 shows the region enclosed by the curve  $y = (1 + 2x^2)^{\frac{1}{3}}$  and the line  $y = 2$ .

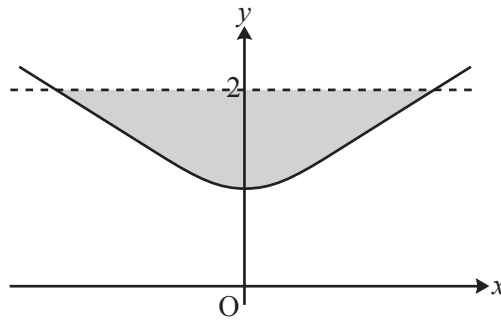


Fig. 6

This region is rotated about the  $y$ -axis. Find the volume of revolution formed. Give your answer as a multiple of  $\pi$ . [6]

Question 7 begins on page 4.

## Section B (36 marks)

- 7 Fig. 7 shows a tetrahedron ABCD. The coordinates of the vertices, with respect to axes Oxyz, are  $A(-3, 0, 0)$ ,  $B(2, 0, -2)$ ,  $C(0, 4, 0)$  and  $D(0, 4, 5)$ .

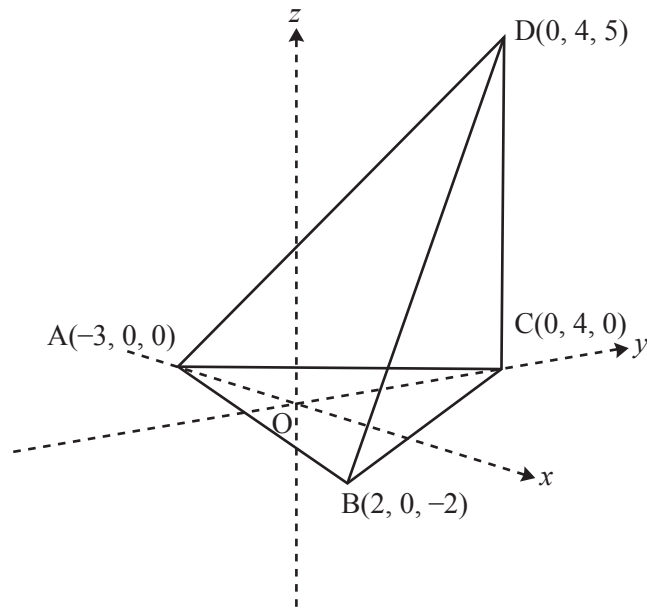
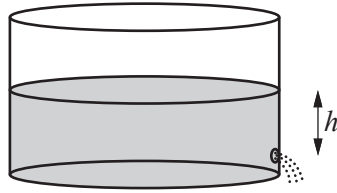


Fig. 7

- (i) Find the lengths of the edges AB and AC, and the size of the angle CAB. Hence calculate the area of triangle ABC. [7]
- (ii) (A) Verify that  $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$  is normal to the plane ABC. [2]
- (B) Hence find the equation of this plane. [2]
- (iii) Write down a vector equation for the line through D perpendicular to the plane ABC. Hence find the point of intersection of this line with the plane ABC. [5]
- The volume of a tetrahedron is  $\frac{1}{3} \times \text{area of base} \times \text{height}$ .
- (iv) Find the volume of the tetrahedron ABCD. [2]

- 8 Fig. 8.1 shows an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.



**Fig. 8.1**

The height of the water surface above the hole  $t$  seconds after opening the hole is  $h$  metres, where

$$\frac{dh}{dt} = -A\sqrt{h}$$

and where  $A$  is a positive constant. Initially the water surface is 1 metre above the hole.

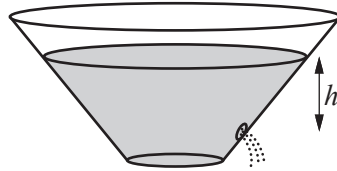
- (i) Verify that the solution to this differential equation is

$$h = \left(1 - \frac{1}{2}At\right)^2. \quad [3]$$

The water stops leaking when  $h = 0$ . This occurs after 20 seconds.

- (ii) Find the value of  $A$ , and the time when the height of the water surface above the hole is 0.5 m. [4]

Fig. 8.2 shows a similar situation with a different barrel;  $h$  is in metres.



**Fig. 8.2**

For this barrel,

$$\frac{dh}{dt} = -B \frac{\sqrt{h}}{(1+h)^2},$$

where  $B$  is a positive constant. When  $t = 0$ ,  $h = 1$ .

- (iii) Solve this differential equation, and hence show that

$$h^{\frac{1}{2}}(30 + 20h + 6h^2) = 56 - 15Bt. \quad [7]$$

- (iv) Given that  $h = 0$  when  $t = 20$ , find  $B$ .

Find also the time when the height of the water surface above the hole is 0.5 m. [4]

**END OF QUESTION PAPER**

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