OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS GCE

4755/01

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

FRIDAY 16 MAY 2014: Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Booklet, or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4755/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CW/SW) 3962374425 96927/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 Use standard series formulae to find $\sum_{r=1}^{n} r(r-2)$, factorising your answer as far as possible. [5]

2 Fig. 2 below shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

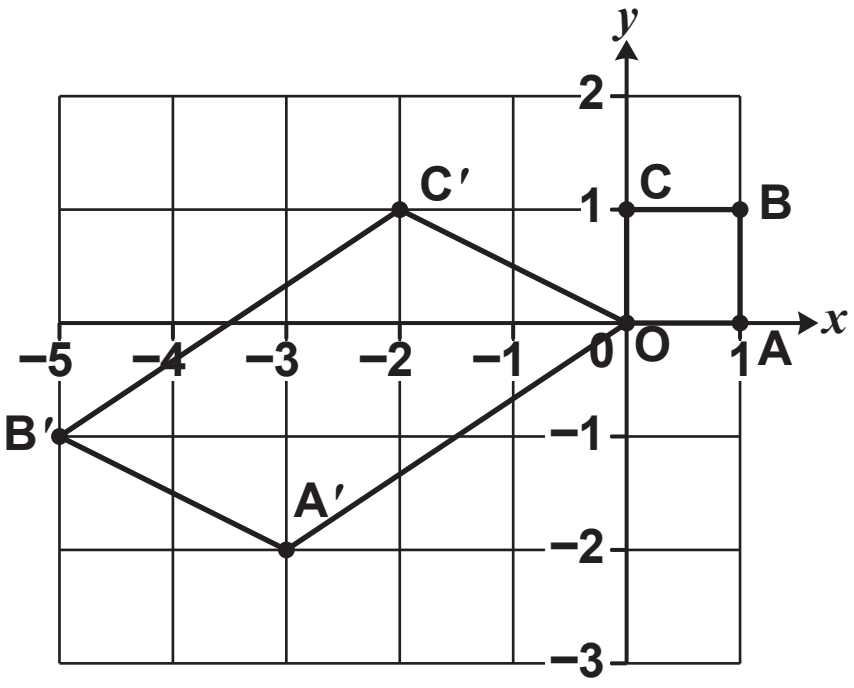

Fig. 2

(i) Write down the matrix T representing this transformation. [2]

The quadrilateral OA′B′C′ is reflected in the x-axis to give a new quadrilateral, OA″B″C″.

(ii) Write down the matrix representing reflection in the x-axis. [1]

(iii) Find the single matrix that will transform OABC onto OA″B″C″. [2]

3 You are given that $z = 2 + 3\mathrm{j}$ is a root of the quartic equation $z^4 - 5z^3 + 15z^2 - 5z - 26 = 0$. Find the other roots. [7]

4 Use the identity $\frac{1}{2r+3} - \frac{1}{2r+5} \equiv \frac{2}{(2r+3)(2r+5)}$ and the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2r+3)(2r+5)}$, expressing your answer as a single fraction. [5]

5 The roots of the cubic equation $3x^3 - 9x^2 + x - 1 = 0$ are α, β and γ. Find the cubic equation whose roots are $3\alpha - 1, 3\beta - 1$ and $3\gamma - 1$, expressing your answer in a form with integer coefficients. [7]

6 Prove by induction that $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$. [7]

SECTION B (36 marks)

7 A curve has equation $y = \dfrac{x^2 - 5}{(x+3)(x-2)(ax-1)}$, where a is a constant.

(i) Find the coordinates of the points where the curve crosses the x-axis and the y-axis. [2]

(ii) You are given that the curve has a vertical asymptote at $x = \frac{1}{2}$. Write down the value of a and the equations of the other asymptotes. [3]

(iii) Sketch the curve. [4]

(iv) Find the set of values of x for which $y > 0$. [3]

8 You are given the complex number $w = 2 + 2\sqrt{3}\,\mathrm{j}$.

(i) Express w in modulus-argument form. [3]

(ii) Indicate on an Argand diagram the set of points, z, which satisfy both of the following inequalities.

$-\frac{\pi}{2} \leqslant \arg z \leqslant \frac{\pi}{3}$ and $|z| \leqslant 4$

Mark w on your Argand diagram and find the greatest value of $|z - w|$. [9]

9 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 3 & -1 \\ -1 & \alpha & -1 \\ -2 & -1 & 3 \end{pmatrix}$,

$\mathbf{B} = \begin{pmatrix} 3\alpha - 1 & -8 & \alpha - 3 \\ 5 & 1 & 2 \\ 2\alpha + 1 & -5 & \alpha + 3 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} \gamma & 0 & 0 \\ \beta & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix}$.

(i) Show that $\beta = 0$. **[2]**

(ii) Find γ in terms of α. **[2]**

(iii) Write down $\mathbf{A}^{-1}$ for the case when $\alpha = 2$. State the value of α for which $\mathbf{A}^{-1}$ does not exist. **[3]**

(iv) Use your answer to part (iii) to solve the following simultaneous equations.

$$\begin{aligned} x + 3y - z &= 25 \\ -x + 2y - z &= 11 \\ -2x - y + 3z &= -23 \end{aligned}$$ **[5]**

END OF QUESTION PAPER

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.