GCE

Mathematics

Advanced GCE A2 7890 – 2

Advanced Subsidiary GCE AS 3890 – 2

OCR Report to Centres June 2015

CONTENTS

Advanced GCE Mathematics (7890)

Advanced GCE Pure Mathematics (7891)

Advanced GCE Further Mathematics (7892)

Advanced Subsidiary GCE Mathematics (3890)

Advanced Subsidiary GCE Pure Mathematics (3891)

Advanced Subsidiary GCE Further Mathematics (3892)

OCR REPORT TO CENTRES

4721 Core Mathematics 1

General Comments

The vast majority of candidates were, as usual, very well prepared for this paper. Many candidates achieved very high marks and the proportion of candidates who were unsuitable for this standard was once more very small. As with the last session, the use of additional sheets was uncommon and was usually limited to repeat attempts at parts of questions that were the most demanding. Many of those who did need to repeat a solution indicated so clearly, which was very helpful to markers. A few, however, still leave a choice of answers, which should be discouraged. Any rough work should also be clearly indicated as such.

Many candidates presented very clear and accurate solutions throughout the paper, showing a good understanding of the mathematics needed for this module. The coordinate geometry of straight lines and circle, basic differentiation and the solution of quadratics, including those disguised in another form remain strong on the whole, although the quadratic in question 6 proved difficult for many to factorise. . Accurate description/interpretation of the transformation of graphs, although still a challenge for many, is beginning to improve. The unnecessary use of graph paper for sketches has now almost entirely disappeared, but the quality of sketch graphs remains an area of the syllabus that centres could target for improvement. Many candidates failed to secure full marks in 2(i) and 8(i) despite having a clear understanding of the basic shape of the graphs; they were let down by lack of attention to detail of the finer points. The use of clear sketches to help explain solutions was often very creditable. The other major area of concern was fraction arithmetic, particularly in regards to multiplication, which caused issues both in 5(ii) and 7(ii).

Comments on Individual Questions

1) Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all three marks. Errors were sometimes seen both in evaluating the numerator and the denominator, and occasionally in performing the final division

2) (i) The vast majority of candidates chose the correct quadrants and basic shape for their sketch to earn one mark and many earned the second mark with a good sketch. Often the second mark was withheld due to inaccuracies such as not clearly indicating the graph tended towards the axes as asymptotes; many graphs ran parallel to the axes for a considerable portion of their length. Others touched or even crossed the axes.

(ii) Although there was some confusion with signs amongst the lowest attaining candidates, the majority earned both marks for correctly stating the new equation of the curve. It was relatively rare to see the translation mistakenly performed vertically, which represents a considerable improvement on previous sessions.

(iii) Most candidates knew that this was a stretch and used the correct word, although incorrect descriptions such as ‘squash’, ‘squish’ and ‘enlargement’ were seen again. The scale factor proved more difficult with many erroneously giving 3. There has been an improvement in describing the direction, with ‘parallel to the *x*-axis’(or *y*-axis in this case as either is appropriate for this graph) often seen, although some candidates still use incorrect language such as ‘along the axis.’

3) (i) Almost all candidates secured this easy mark, but the error of $(5^2)^4 = 5^6$ was quite common.

(ii) Again, most candidates were able to gain both marks dealing with both the fractional and negative elements of the index.

(iii) This part of the question proved rather more demanding with a minority of candidates securing both marks. Those who recognised that $\sqrt{5} = 5^{\frac{1}{2}}$ were usually able to go on and complete the question successfully; those who tried to multiply out were less successful.

4) This disguised quadratic was well approached by the vast majority of candidates. The most common approach was to perform a substitution and then to factorise, although some candidates did make their choice of substitution clear, which made it difficult to award partial credit in the cases where errors then occurred. As the resulting quadratic was simple, very few candidates used the quadratic formula and factorisation was usually successful with only a few sign errors seen. Some candidates stopped after solving the quadratic and the number who tried to cube root, rather than to cube, their solutions was comparatively large.

5) (i) Candidates were generally successful in applying Pythagoras' theorem. Use of a diagram was relatively rare. Some candidates gave ± 5 or $\sqrt{25}$, neither of which secured the accuracy mark.

(ii) Candidates generally displayed good understanding of coordinate geometry with many securing all seven marks. Errors in the early stages of solutions were rare, with only occasional sign or calculation errors in finding the mid-point and/or the gradient and perpendicular gradient. It was noticeable, however, that even when candidates were successful in obtaining the equation $y+1=\frac{3}{4}(x-\frac{7}{2})$, the presence of two fractions then caused problems whatever approach was taken to try to simplify. Also, some candidates failed to give the answer in required form apparently misunderstanding the word 'integers'.

6) The vast majority of candidates opted to substitute for *y* and so form a quadratic in *x* as the first step in solving this pair of simultaneous equations. Sign errors meant that not all candidates obtained the correct quadratic and even those who did found it difficult to factorise. Attempts to use the formula were also hampered by the relatively large number 28 and so many candidates got no further. Those who did succeed usually remembered to substitute to find *y*, but sign errors were again quite common in this part. Nonetheless, a significant proportion of candidates produced full, clear and accurate solutions.

7) (a) Although a very small number of candidates attempted the product rule, the vast majority multiplied out the brackets and differentiated accurately. The main cause of error for several candidates was to change all the signs in their expanded expression before differentiating, which resulted in the loss of accuracy marks.

(b) Although the differentiation required here was more demanding than that of part (a), many candidates were able to secure the first two marks. Evaluation of $(-8)^{-\frac{4}{3}}$ proved very challenging, with many ignoring one or both minus signs, not understanding the index or making calculation errors. Even those who were successful often then made errors in finding the product of two unit fractions.

8) (i) Most candidates recognised this as a quadratic and provided an appropriate sketch, although there was a tendency for some to become steep/vertical extremely quickly rather indicate increasing gradient. The points of intersection on the *x*-axis were usually accurate with the occasional sign swaps. Although the *y*-intercept was usually correctly identified as –3, it was very common to see this as vertex of the graph which lost an accuracy mark; candidates were expected to indicate the vertex would be in the correct quadrant for their roots

(ii) Most candidates used their answer to part (i) and chose the correct outside region, although choosing the inside region was a frequently seen error. The notation used to describe the region was usually correct; incorrect language such as joining the two sections with the word ‘and’ lost the accuracy mark.

(iii) This proved demanding for many candidates. Although some secured all three marks, many earned no credit as they either put the discriminant equal to zero or, as was frequently seen, to *k*, making no attempt to rearrange the given equation. Accuracy marks were often lost as candidates failed to deal with the minus signs both in the discriminant and in the expression for *c*. A few candidates found the turning point of their graph either by differentiation or by completing the square but these approaches were far less common.

9) (i) Differentiating and setting to zero and substituting $x = 4$ was the obvious strategy and, although the arithmetic proved troublesome for some, many candidates were able to secure full marks for this part.

(ii) Considering the sign of the second derivative was by far the most common approach for this part and was generally successful. Some candidates equated their second derivative to zero, a confusion that has been common for many sessions.

(iii) A small minority omitted this question, but most candidates were comfortable in returning to their expression in (i), equating to zero and finding the other root. An alternative method not seen before by several markers was to equate the second derivative to the negative of the value found in (ii); this is perfectly valid for cubics and was usually successful.

10) (i) Apart from the usual sign error, most candidates were able to identify the centre and calculate the radius of the circle with little apparently difficulty.

(ii) Unsuccessful attempts at implicit differentiation notwithstanding, most candidates were able to present a clear accurate solution to this part of the question. The expected approach of finding the gradient of the radius, its negative reciprocal and then the equation of the line through (8, 2) was performed very well. Some candidates merely rearranged the given equation to find its gradient and re-substituted; this gained no credit.

(ii) Most candidates were able to find both points on the *y*-axis and the best solutions to this included a sketch diagram to aid candidates on their way. Some chose to find the lengths of all the sides of the triangle and multiply together sides that were not perpendicular before halving. Although full marks to this part were comparatively rare, it was noticeable that some lower attaining candidates who did use a good sketch were able to outscore many of the higher attaining students on this particular part.

4722 Core Mathematics 2

General Comments:

The vast majority of candidates were well-prepared for this examination, and able to make an attempt at every question. The work was mostly well-presented, showing clear detail of the method used but this was not always the case. Candidates should ensure that they delete any working that is not intended to form part of their final solution. This was particularly apparent in Q.5 where candidates often included both an integration attempt and use of $y = mx + c$, and it was not always clear as to which was their final attempt.

In questions where the answer is given, candidates should ensure that they show sufficient detail so as to be fully convincing in their method. Candidates should also be aware that method marks can only be awarded if a method is actually shown; examiners cannot try to deduce what may have been attempted when simply presented with an incorrect numerical answer. This was particularly noticeable when evaluating expressions in Q.1 and using limits in Q.6(ii). It is also relevant to candidates who become overly reliant on their calculator; for example, just writing down the two roots obtained is not evidence that they have used a correct method to solve their incorrect quadratic.

There were two questions on this paper that required candidates to sketch a graph, and too many candidates showed a lack of care and precision when doing so, which resulted in their intent being unclear. Where candidates have made two, or more, attempts at a sketch they must ensure that it is clear which is their final attempt. They would be well-advised to make their initial attempts in pencil, which can then be erased if incorrect, and then use a heavier pencil line on their final attempt to ensure that it scans well.

Comments on Individual Questions:

Question No.

1(i) This question was a straightforward start to the paper, and nearly all of the candidates were able to state the correct value for the common ratio. The most common incorrect answer was $r = -9$, indicating a confusion between the definitions of arithmetic and geometric progressions.

(ii) Most candidates knew how to find the eleventh term of the GP, but many were unable to evaluate correctly the expression as it included a negative number. The most successful candidates included brackets in their expression, and then used these in their evaluation. Some candidates included brackets but ignored them in the evaluation, and too many candidates wrote the expression as 3×-2^{10} and duly evaluated this as -3072. At this level, candidates should both be able to use their calculator proficiently and should also consider whether their answer is sensible; they should be aware that a negative number to an even power should give a positive answer.

(iii) Once again, most candidates were able to quote a correct expression for the sum of the first twenty terms, but were unable to correctly evaluate this. A few candidates gave their answer to three significant figures, not appreciating that the instruction on the front of the question paper refers to non-exact numerical answers.

2(i) This question was generally very well done, with the majority of the candidates gaining full marks. Candidates generally showed their method clearly, though the brackets were omitted in some solutions, resulting in an incorrect evaluation of their intended expression. Candidates also need to be careful when copying their work from one line to the next; it was not uncommon to see $\sqrt{19}$ become $\sqrt{9}$. Another common slip was for $\sqrt{16}$ to be evaluated as 4 but then used as $\sqrt{4}$. In an improvement from previous sessions, there were very few candidates who first attempted to integrate the function before applying the trapezium rule.

(ii) The vast majority of candidates gained a mark for either identifying that more strips could be used, or identifying that the width of the strips could be reduced. Benefit of doubt was given to those candidates who gave both reasons, but seemed to think that they were mutually exclusive. The most common error was for candidates to justify why it was inaccurate, referring to it being an underestimate, rather than focusing on the actual question posed.

3(i) This question was very well-answered, and the vast majority of candidates gained all of the marks available. The most efficient method was to work in radians throughout, though there were inevitably candidates who decided to use an equivalent method involving degrees instead. As long as there was no subsequent loss of accuracy, then this method was accepted. A few candidates simply assumed that triangle *OCD* was isosceles, and the other common error was to use the formula for the area of a segment rather than considering carefully the question posed.

(ii) Candidates struggled a little more with this part of the question, but it was still very well-answered. Once again, many efficient and effective solutions were seen. Most candidates gained the first mark for finding correctly the required arc length, and could then attempt the cosine rule. Given that this rule is given in the formula book, it was disappointing to see errors when it was initially quoted. Some candidates made errors when evaluating the expression, and these included treating $(b^2 + c^2 - 2bc)$ as the coefficient of $\cos A$ and omitting to square root the evaluated expression. There was also a loss of accuracy in some solutions from premature approximation, and candidates would be well-advised to make efficient use of their calculator and only round when giving their final answer.

4(i) Most candidates were able to attempt the correct binomial expansion, and there were very few attempts involving a full expansion. The most common error was for the final term to appear as $240ax^2$ rather than the required $240a^2x^2$. In some solutions this was a result of failing to use brackets in the expansion, and in other solutions the brackets were initially stated but subsequently ignored. Some candidates attempted to take out a common factor and then use the expansion for $(1 + x)^n$. This was not always successful, and candidates should appreciate the need to use the most appropriate method for a given problem. Some, otherwise correct, solutions were subsequently spoiled by an attempt to simplify their final answer by dividing through by a common factor.

(ii) Nearly all candidates appreciated the need to use their expansion from the previous part of the question and were then able to attempt the terms required for this part. Some candidates simply picked out the two relevant terms whereas others started with a fuller expansion. No credit was gained in this question until the required two terms, and no others, were identified. A surprisingly common error was to erroneously combine their two terms, with $576ax - 320x$ becoming $256ax$. Some candidates were unsure as to whether to equate the entire terms or just the coefficients; both approaches were condoned as long as it was consistent throughout the entire equation, which was not always the case.

5 Whilst a few of the weaker candidates attempted to use the equation of a straight line, the majority appreciated the need to integrate and were able to make a good attempt at doing so, although the coefficient was not always correct. To gain any further credit, candidates now had to evaluate the constant of integration using the relevant pieces of information. A pleasing number were able to do this, and could then continue to produce a fully correct solution. However a significant minority struggled to progress beyond the first two marks. Some did attempt to find the constant, but used the *y*-coordinate rather than the gradient. Using the correct notation would have been of benefit to a number of candidates. Another common error was to attempt the second integration, but to assume that the second constant of integration would have the same value as the first, giving $y = 4x^{1.5} + cx + c$. It was also surprising that a number of candidates omitted a constant from the result of the first integration, but then included one in the result of the second. Whist a pleasing number of fully correct solutions was seen, it was apparent that a number of candidates struggled with the lack of familiarity of this style of question.

6(i) This proved to be a straightforward question for many candidates, and the majority gained full credit. A variety of methods continue to be seen when factorising a cubic, and inspection is becoming increasingly common. Coefficient matching continues to be seen as a routine method that candidates of all abilities can employ successfully. More candidates are attempting to use algebraic long division, but errors tend to be more common as some candidates can be confused as to whether to add or subtract within the division. The lack of an x^2 term also caused problems for some. It was clear within some solutions that an alternative method had been attempted when the initial one failed. When this is the case candidates should ensure that they delete any working that does not form part of their final solution.

(ii) The integration attempt was invariably correct, and most candidates were able to attempt the correct use of limits. However, evaluating an expression involving negative numbers once again caused problems for a significant minority of candidates and it was relatively common for F(-5) to be incorrect. As long as there was evidence of the limits having been correctly attempted then candidates were awarded the method mark for this. However if candidates simply write down an incorrect numerical evaluation, with no evidence to support this, then examiners cannot speculate as to what may or may not have been attempted, and no credit can be awarded.

(iii) For the first mark candidates were expected to provide a sketch of the cubic and many were able to do so, though there was a disappointing lack of care in the sketches. For the second mark candidates had to demonstrate an appreciation that an area under the *x*-axis would give a negative result when found by integration. Expressing this idea clearly proved to be beyond the capabilities of many of the candidates. Many of the answers could identify that it was the region below the *x*-axis that was at issue, but they failed to explain what the issue was. A number of solutions stated that this region would be ignored in the integration, and others used shading to show that 'the area under the curve' would be infinite for this region. Some solutions referred to 'areas cancelling out', but lacked detail or precision, and others described how the integration should be done without explaining why this method had failed. Whilst a number of concise and detailed explanations were seen, it was disappointing that so many candidates were unable to express their reasoning with the clarity required.

7(i) This question was invariably correct, with most candidates using the formula for the *n*th term of an AP. Other methods included firstly generating an *n*th term expression for the sequence, and some just resorted to manually listing the terms.

(ii) The purpose of this part of the question was to assist candidates in finding an appropriate strategy with which to attempt the final part. Because the answer was given, candidates were expected to show full detail of their method and too many solutions did not address this. Most candidates gained at least the first mark for attempting the sum of the first twenty terms, but a number then struggled to make any further progress. Subtracting the sum of the first ten terms was the most common error; some candidates gave up at this point, whereas others made a valiant, but not always convincing, attempt to obtain the given 517. Some candidates listed, and summed, the relevant eleven terms. This approach gained full credit in this part of the question, but was not a method that could then be replicated in part (iiii).

(iii) This final part of the question proved to be challenging for even the most able candidates, and fully correct solutions were in the minority. It was disappointing that so few candidates made the link between what they had been asked to do in the previous part of the question and what was now required of them. The first mark was available for finding the sum of the first $2N$ terms, and this was gained by just over half of the candidates. To make any further progress candidates now had to consider the sum of the first N - 1 terms, and then equate the difference to 2750. Only a minority actually attempted this, with the most common error being to subtract the sum of the first N terms. A lack of care with brackets meant that some candidates could not obtain the correct, simplified, quadratic despite the initial part of the solution being correct. An elegant alternative method that was sometimes seen considered the sum of N + 1 terms, starting on the Nth term and finishing on the $2N$th term, and an equally efficient method used the nth term definition of the sequence.

8(a) Candidates have become increasingly proficient when solving straightforward equations involving logarithms and this was true on this question, with most candidates gaining all of the available marks with ease. The most common approach was to use logarithms to base 2, although solutions involving base 10, or even some unspecified base, were also seen.

(b) This part of the question proved to be more challenging, and a variety of different approaches were seen. The most effective method tended to be to remove the logarithms as a first step and then solve the resulting simultaneous equations. The most common method however was to use equations that still involved logarithms. Candidates usually gained the first two method marks, for using a log law and eliminating a variable, with ease. However the resulting equation of $\log_2 x^2 + \log_2 x - 15 = 0$ was seen as a quadratic, and a solution attempt made based on this misunderstanding. Candidates who dropped the index on the first term, either at this stage or earlier in their solution, tended to then produce a fully correct solution. Candidates would be well advised to show their method clearly and not attempt to do more than one step at a time. It was quite common to see $\log_2 x + \log_2 y = 8$ become $xy = 3$ in the next line. Whilst this may suggest that a correct log law was used before the logs were incorrectly removed, with no clear evidence of this the method mark cannot be awarded.

9(i) Candidates found this question very challenging, and only a small proportion gained any credit. Solutions indicated that many candidates appreciated that the period was no longer 2π, but were still unable to identify what it now was. It was surprising that a number of candidates correctly marked the period as 6π on the graph in part (ii), but were unable to gain any marks in part (i). Partial credit was allowed for unsimplified answers and/or those in degrees, which helped a few candidates.

(ii) As with the other graph-sketching question on this paper, too many attempts lacked care or clarity which meant that examiners were unable to discern the true intent of the candidate. The first mark was for appreciating that the period of the curve that they were sketching would be the same as that of the given curve. Some leeway was given when making this judgment, but a sizeable minority offered curves that seemed to have no intention of finishing at the correct point. The second mark was for their curve to have an amplitude that was half of the given curve. Whilst some excellent solutions were seen, a number of curves did not make this intention clear and some had an amplitude that was not consistent throughout the curve.

(iii) Most candidates could equate the two curves, but many then struggled to make any further progress. Whilst correct generic identities were quoted, they were not always applied accurately. The most common error was for the coefficients of x to be cancelled, resulting in $\tan x = 2$. The attempted use of $\sin^2 x + \cos^2 x = 1$ was rarely successful; in some attempts it was used as $\sin x + \cos x = 1$, whereas attempts at squaring often resulted in the coefficient of x also being squared. Nevertheless, just under half of the candidates did manage to obtain a correct equation, usually $\tan(\frac{1}{3}x) = 2$ but sometimes another acceptable alternative. Most of these candidates could then find at least one correct root, though only the best candidates were able to also find the second root. Some candidates spoiled an otherwise correct solution by giving their answers in degrees not radians.

4723 Core Mathematics 3

General Comments

This paper proved accessible to the vast majority of candidates and the more demanding questions did include some straightforward requests. Questions 4, 8 and 9 were those with aspects that challenged even the more competent candidates but most candidates were able to earn a few marks from each. Examiners were pleased to note a significant number of candidates recording full marks on the paper; 8% of the candidates recorded a mark of 70 or more. There were a few candidates who produced little of merit; only 1% of the candidates recorded a mark of 10 or fewer. Time was not a problem and all candidates appeared to have sufficient time to tackle all the questions.

Many candidates did not take sufficient care with the presentation of their work. Sometimes this was not helped by the methods adopted. Candidates generally fared best when they simplified their work as they went along. An example occurred in question 8 where some candidates persisted in writing $\ln(\mathrm{e}^N - 3 + 3)$ over many lines of working. Some never realised that this could be simplified; others realised but at a relatively late stage and by then errors might have crept in. The comments below on question 9 also stress the need for some attention to the way in which work is presented.

The topics that presented the most problems to candidates were the modulus function, logarithms and trigonometry. Many candidates seemed to have no understanding of the basic definition of the modulus function and were far too ready to fall back on some inappropriate routine, usually involving squaring. Question 8 revealed many uncertainties about logarithmic and exponential functions. The properties of logarithms were generally known but examiners would often find candidates inventing their own properties such as $p + \ln q = \ln r$ leading to $\mathrm{e}^p + q = r$. The basic trigonometric identities were known but many candidates struggled when needing to adapt one to particular circumstances. Candidates generally knew an identity for $\cos 2\theta$ but did not realise how it could be adapted to become an identity for $\cos 4\theta$. Incorrect versions included $\cos 4\theta = \cos^4\theta - \sin^4\theta$ and $\cos 4\theta = \cos 2\theta \cos 2\theta$.

Comments on Individual Questions

Question 1

This opening question was answered very well in general with 74% of the candidates recording full marks. The majority applied the quotient rule accurately although lack of care with brackets in the numerator did lead to some sign errors. Some candidates opted for use of the product rule and this was not handled quite so convincingly. There were some cases where candidates stopped as soon as they had found the gradient but, in general, candidates proceeded without difficulty to produce the equation of the tangent and to present it in an acceptable form.

Question 2

The instruction 'Without using a calculator' in this question meant that candidates were required to supply sufficient detail and this was the case with the vast majority of candidates; there were just a few cases of 4.12 appearing as the answer in part (ii). Part (i) was answered very well; there were a few candidates who apparently did not know that $\tan 45°$ is 1 and occasionally the solution $\tan(\theta + 45°) = \tan\theta + 1 = \frac{5}{4}$ was noted.

Part (ii) presented a few more problems and some candidates wrote down various identities, but not the crucial one, in the hope of finding a way to the value of $\operatorname{cosec}\theta$. Many candidates made efficient and concise use of the identity $\operatorname{cosec}^2\theta \equiv 1+\cot^2\theta$; another popular approach was to use a right-angled triangle to find the length of the hypotenuse. Many candidates gave their final answer as $\pm\sqrt{17}$ and this did not earn the second mark; they were expected to note that θ was an acute angle.

Question 3

Questions such as this one placed in a context have to be presented in an accessible form. Despite a necessary simplification of a real situation, candidates should expect that the model will give sensible and realistic answers. Candidates who concluded that the depth of the water will be increasing at 372900 metres per hour gave no indication that they recognised that this answer was absurd; such unlikely answers were by no means uncommon.

The vast majority of candidates realised that differentiation was needed but there were various errors that occurred in finding $\frac{\mathrm{d}V}{\mathrm{d}h}$. Sometimes the -192 was retained; in a few cases the power 5 was lost. More often the factor $\frac{1}{2}h^{-\frac{1}{2}}$ did not appear. Some candidates then offered an answer without making any use of 150. Most did realise that the numerical value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ had to be combined with 150 to find the appropriate rate of change but the correct approach of 150 divided by that numerical value eluded many.

55% of the candidates did earn full marks and it was often the case that successful candidates were careful and accurate with their notation; knowing that their differentiation gave $\frac{\mathrm{d}V}{\mathrm{d}h}$, that the question gave $\frac{\mathrm{d}V}{\mathrm{d}t}=150$ and that the request was to find $\frac{\mathrm{d}h}{\mathrm{d}t}$ made it far more likely that the correct conclusion was reached.

Question 4

This question proved to be one of the more demanding requests in the paper and only 38% of the candidates recorded full marks. The slightly unfamiliar nature of the request and the presence of *a* were presumably factors causing the difficulties. It was also plain that many candidates did not understand the meaning of the modulus function. Successful candidates were able to complete the answer in just a few lines, obtaining the two possible values of *x* from two simple linear equations and then substituting each value into $|x+7a|-|x-7a|$ without fuss. For example, $|-8a+7a|-|-8a-7a|=|-a|-|-15a|=a-15a=-14a$ as part of the solution left no doubt that the candidate really understood what was needed.

Not all candidates realised that a sensible strategy was to start by finding possible values of *x*. They started by trying to manipulate $|x+7a|-|x-7a|$, often squaring the two terms and simplifying to obtain $28ax$. Sketch graphs were employed by some candidates but were seldom of any help. A majority of the candidates did solve $|x+3a|=5a$ correctly to obtain $-8a$ and $2a$, although those adopting a method involving squaring were prone to errors, forgetting to square the right-hand side or being unable to deal with a quadratic equation including both *x* and *a*. A few candidates, apparently thinking everything had to be positive in a question involving modulus, rejected $-8a$ as a possibility or changed it to $8a$.

For many of the candidates who had found the two possible values of *x*, the latter part of the question revealed their uncertainties with this topic. As well as the presence of modulus signs tending to prompt a spurious process of squaring, their presence also sometimes prompted the haphazard deployment of $\pm$ signs. Many candidates proceeded to provide a long list of possible values by evaluating $|x+7a|-|x-7a|$, using $x=2a, x=-2a, x=8a, x=-8a$ in turn and even, in some cases using different values of *x* in the same evaluation, evaluating for example $|2a+7a|-|-8a-7a|$. Sometimes the modulus signs were treated just as if they were brackets, and final answers presented as $|x|=4a$ and $x=|4a|$ were not convincing either.

Question 5

For a question involving routine techniques, it was disappointing that only 37% of the candidates recorded all eight marks. Almost all candidates differentiated correctly in part (i) but then many struggled to find the *x*-coordinate of the minimum point. The equation $3e^{3x}-12e^{2x}=0$ prompted some to a next incorrect step of $\ln(3e^{3x})-\ln(12e^{2x})=0$; others followed $\ln e^{3x}=\ln 4e^{2x}$ with $3x=2x\ln 4$. Those with an approach involving factorisation such as $3e^{2x}(e^{x}-4)=0$ often included extra incorrect roots such as 0 or $\frac{1}{2}$. Confirmation that the minimum point lies on the *x*-axis required a little more detail than the mere statement $e^{3\ln 4}-6e^{2\ln 4}+32=0$ and, as a result, some candidates did not earn the final mark of part (i). Some candidates also found the second derivative but no confirmation that the stationary point is indeed a minimum was needed.

There was more success with part (ii). Integration was handled efficiently and the area was produced in a suitably simplified form. There were occasional sign errors and some answers were not exact. Surprisingly, there were a few cases where $\int \pi y^2 \, dx$ was attempted.

Question 6

There was evidence that some candidates were not particularly familiar with the inverse sine function in this question; manipulation and evaluation of expressions were not always carried out accurately. Answers to part (i) demonstrated this unease as some responses involved π or ± 8. Many candidates were able to write down the two correct values of *a* and *b* immediately. Others identified the stretch and translation of the curve $y=\sin^{-1}x$ required to produce the given curve before determining the two values. Another approach involved solving two equations.

Part (ii)(a) is a routine verification of the location of the root but only 57% of the candidates earned all the marks. Some candidates used their calculators in degree mode and no marks were available in this part. A more significant problem concerned those candidates who evaluated $8\sin^{-1}(x-\frac{3}{2})$ at the two values. Obtaining 1.61 and 2.44, many were clearly surprised not to find a sign change; some then realised that subtraction of 1.7 and 1.8 respectively was needed or that an observation that $1.61<1.7$ whereas $2.44>1.8$ was required. Evidence of calculations was needed; candidates who merely stated that there would be a sign change earned no marks in this part.

Use of iteration to find a root is usually a good source of marks for candidates in this unit. But on this occasion, this was not always the case; only 55% of the candidates earned all the marks in part (ii)(b) and 21% recorded no marks. Some did not know how to set up the iterative formula or there were errors in establishing the values of *p* and *q*; it was not uncommon for $q=8$ to be stated. There was limited credit available for those candidates using degrees. There was also confusion between significant figures and decimal places and those candidates offering 1.7124 as their final answer did not earn the final mark.

Question 7

This question was a good source of marks for candidates with all three parts being answered confidently by many candidates. 61% of the candidates earned all nine marks. Part (i) presented no problems and the only errors to occur with any frequency were an incorrect coefficient of $(7x+1)^{\frac{4}{3}}$ and a failure to provide an exact answer.

There were a few more problems with part (ii); some candidates evidently found it strange that Simpson's rule should include an exact value and could not resist using an approximation to $\sqrt[3]{36}$ in their answers. A few did not use the correct *x*-values of 1, 5 and 9 and substitution into $\frac{4}{3}(y_0+4y_1+2y_2)$ occurred on occasions. Lack of care with use of brackets led to a conclusion of $8+4\sqrt[3]{16}$ in some instances. A curious error occurred so often that it was remarked on by examiners; that was $2+4=8$.

Candidates generally knew the manipulation required in part (iii) although earlier errors meant that the approximate value found was not very close in some cases.

Question 8

Some of the marks in this question were easily earned but there were other more demanding aspects. Only 12% of the candidates managed to earn all eleven marks. Part (i) was answered well, particularly by those candidates who simplified $\mathrm{f}(\mathrm{e}^4-3)$ as their first step. Those who formed $\mathrm{gf}(x)$ and attempted to expand $a[2+\ln(x+3)]^2$ before any substitution for *x* had more scope for error. Almost all candidates recognised the composition of functions and applied f and g in the correct order. A moment's carelessness led a surprising number of candidates to follow $36a=9$ with $a=4$.

The vast majority of candidates had no difficulty in finding $\mathrm{f}^{-1}(x)$ but few were successful in stating the correct domain. Common responses were all real numbers, $x \geq 0$ and $x \geq -3$. Those candidates who recognised the link between the range of f and the domain of its inverse usually provided the correct answer.

Part (iii) was more challenging and 43% of the candidates earned all five marks. Different approaches were seen. Some started by forming $\mathrm{ff}(x)$ before substituting e^N-3 whereas another common method involved finding and simplifying $\mathrm{f}(\mathrm{e}^N-3)$ before applying f for a second time. Candidates who simplified at each step fared better than those who constructed complicated expressions before attempting to deal with them. Attention to detail was needed too and some candidates made things difficult for themselves by not adding or subtracting 3 at appropriate stages. After applying ff successfully, candidates were faced with the equation $2+\ln(N+5)=\ln(53\mathrm{e}^2)$ and many were unable to deal with this correctly. All too common was a next step of $\mathrm{e}^2+(N+5)=53\mathrm{e}^2$. A few candidates demonstrated a sound understanding of functions by taking the result in part (ii) to use $\mathrm{e}^N-3=\mathrm{f}^{-1}\mathrm{f}^{-1}[\ln(53\mathrm{e}^2)]$ as the method for finding the value of *N*.

Question 9

This question contained challenges for even the best candidates and only 13% of the candidates recorded all thirteen marks. The first two marks of part (i) were obtained by most but convincing and concise responses to the subsequent proof were not so common. Many candidates did not take the trouble to present solutions in such a way that they were easy to follow, or indeed to read. On some scripts, it was often difficult for examiners to decide whether candidates had written $\cos 2\theta$ or $\cos^2\theta$. In other cases, parts of the proof were scattered around the page and efforts to reassemble the parts did not always succeed. The main difficulty was dealing with $\cos 4\theta$. Some decided that, since $\cos 2\theta = \cos^2\theta - \sin^2\theta$, $\cos 4\theta$ must be $\cos^4\theta - \sin^4\theta$. Many did state $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ but use of this did lead to involved expressions involving $\cos\theta$ and $\sin\theta$; considerable care was then needed to reach a successful conclusion. The best solutions usually involved use of $\cos 4\theta = 2\cos^2 2\theta - 1$ and $\cos 2\theta = 2\cos^2\theta - 1$.

Part (ii)(a) proved demanding for many; about as many earned no marks as earned all three. A few carelessly considered $\dfrac{1}{8\cos^4\theta - 3}$. For those dealing with the correct $\dfrac{1}{8\cos^4\theta + 4}$, the value $\frac{1}{12}$ usually appeared but many candidates mistakenly decided that the other requested value would result from $\cos^4\theta$ being -1.

Many candidates saw no connection between the equation in part (ii)(b) and the results in part (i). Their attempts involved starting afresh and it was very seldom that any significant progress was made. Some made a connection with the first result from part (i) and formed the equation $\cos 12\alpha + 4\cos 6\alpha = 1$. Not all knew how to deal with this; for those who did, replacement of 6α by another letter sometimes meant that the solution of the equation was not completed correctly. The other successful approach involved recognising the link with the main result from part (i). However, the attempt to solve the corresponding equation $\cos^4(3\alpha) = \frac{1}{2}$ frequently led to only one value of α as candidates omitted the value corresponding to $\cos(3\alpha) = -\sqrt[4]{\frac{1}{2}}$.

4724 Core Mathematics 4

General Comments:

The paper proved accessible to almost all of the candidates. There were many examples of well-presented response and some excellent work scoring full marks was seen by most examiners. There were some instances of poorly presented work, however. In particular candidates are reminded not to present alternative responses to the same question, and are encouraged to keep all the work relating to a particular question in one place – in some cases candidates made several continuations to answers making their work difficult to follow.

Some candidates demonstrated good understanding of Core 4 syllabus material, but failed to do themselves justice in the examination either because they didn't answer the question fully or because of poor (GCSE level) algebra and careless arithmetical slips.

A surprising number of candidates were unable to relate work done in the early part of a question to the demand of a subsequent part. It is advisable to read through the whole question first rather than attempt a solution in a piecemeal fashion.

When there is a 'show that' request in the question, the onus is on the candidate to show sufficient working to convince the examiner that the result has been demonstrated satisfactorily, and that it hasn't simply been back-engineered.

Comments on Individual Questions:

Question No. 1

Part (i)

This proved accessible to nearly all candidates, with most scoring full marks. A few slipped up with arithmetic and lost the accuracy mark, but zero marks was very rare.

Part (ii)

A minority of candidates ignored the request for lowest terms and simply multiplied everything out. This approach didn't score. Most correctly factorised the numerator, however, and usually successfully cancelled out at least one pair of terms. Surprisingly, only a minority successfully reached the final answer.

Question No. 2

Part (i)

Most candidates successfully found $\overrightarrow{AB}$ and realised the need to find $\overrightarrow{CP}$ and use a scalar product. Many candidates made sign errors or slipped up with the arithmetic, and gave up. A few ignored the demand to 'show that' and simply stated that the scalar product was zero, which was insufficient.

Part (ii)

Very few candidates made the connection between the work in part (i) and the base and perpendicular height of the required triangle. Most started again, finding an angle using the Cosine Rule or scalar product, and then using $\frac{1}{2}ab\sin C$ – and most lost accuracy somewhere along the way.

Question No. 3

The differentiation was very well done by nearly all candidates, and an overwhelming majority set the derivative equal to zero and successfully identified tan$x$ = 2. Thereafter many lost accuracy or omitted either the y- values or one of the x-value. Only a few candidates found incorrect finite values from $e^{2x} = 0$, rather more failed to recognise that tan$x$ was available, and worked with $\sin^2 x$ or $\cos^2 x$, thus nearly always introducing incorrect extra values in the specified range.

A very small number of candidates integrated instead of differentiating.

Question No. 4

Part (i)

This was very well-done, with most candidates scoring at least three out of four marks. A few had difficulty dealing with $8^{2/3}$ and some made sign errors.

Part (ii)

Many careless slips were seen, such as $x < {}^8/_9$ and $|x| < -{}^8/_9$ but the correct answer was seen in the full range of scripts.

Question No. 5

Most candidates earned the first two marks. Thereafter a surprising number either didn't recognise the need to use integration by parts, or attributed the variables the wrong way round and made no further progress. The many candidates who did use integration by parts usually went on to score five marks in total – most either missed off the constant of integration, or neglected to substitute back in for x.

Question No. 6

Part (i)

This was very well-done, with most candidates achieving full marks. A few showed insufficient working and lost a mark, and a small minority either misquoted the Quotient Rule or the relevant trigonometric identity.

Part (ii)

A surprisingly high proportion of candidates did not recognise that double angle formulae were needed here, and went round in circles trying to use integration by parts or achieve a logarithmic form. Some of those who did successfully use the correct identities to produce a multiple of the function in part (ii) didn't make the connection between the two parts and either ran out of steam or produced reams of incorrect work.

That said, there were many examples of excellent work: well-presented, succinct solutions with sufficient detail to meet the show that' demand.

Question No. 7

Very many candidates showed mastery of implicit differentiation, and an overwhelming majority earned the first 4 marks on this question. Many went on successfully to score full marks. However, some weaker candidates set $\frac{dy}{dx}$ equal to zero and made no further progress, or lost the accuracy mark either because their value of *y* was incorrect or because their attempt to make $\frac{dy}{dx}$ the subject of the formula went astray.

A small number of candidates attempted to make *y* the subject of the equation before differentiating. This was nearly always unsuccessful as the crucial branch of the curve was usually ignored.

Question 8

Part (i)

A surprising number of candidates did not seem to understand inverse proportion, and setting up the initial equation elicited a wide range of incorrect responses. Those who did set up the equation correctly usually went on to earn at least four marks out of six. Finding *k* caused more difficulty than expected: many candidates mistakenly assuming that $t = 1$, $P = 101$ was a valid pair of values, instead of working with the information given in the question. In some cases rearranging to make *P* the subject of the formula proved troublesome.

Part (ii)

Full marks were rarely achieved in this case. Those who did find the correct values often speculated on future trends rather than commenting on the two values they had found. It was necessary to have earned at least four marks in part (i) to score in this part.

Question 9

Part (i)

This was very well-done. The question was accessible to nearly all candidates and many achieved full marks. A few made sign errors or slipped up with arithmetic.

Part (ii)

Most candidates adopted the correct strategy and attempted to use the correct scalar product. Many went astray in the arithmetic; it was surprising to see $\sqrt{(1^2 + (-1)^2 + \alpha^2)}$ become $\sqrt{2}\alpha$ so many times. Many candidates attempted to square both sides of the equation, but bracket errors were frequent, so full marks was usually only achieved by the best candidates.

Question 10

Part (i)

Most recognised the correct form of partial fractions and successfully cleared the fractions to produce a fully correct solution.

Part (ii)

Most candidates used long division and successfully found the quotient and the remainder. Many then used their answer to part (i) to produce a correct solution. A variety of other approaches were also successful, but a significant minority of those who equated coefficients went astray in the algebra.

A small number of candidates tried to divide by x and x + 2 separately, and were rarely successful.

Part (iii)

There were many well laid out, perfectly correct responses to this question. However, it proved to be surprisingly difficult for many. Sometimes a formula for t in terms of x and t was substituted in, which didn't lead anywhere. In other cases the expression for t contained a sign error or an algebraic slip. Often candidates persisted with a clearly incorrect formula, instead of checking the early part of their work. A few candidates verified the result by substitution, which was a convoluted approach and did not earn full marks.

Part (iv)

There were many excellent responses to this part of the question. Most candidates spotted the link with part (ii) and went on to earn three or four marks. Those who started from scratch were almost never successful.

4725 Further Pure Mathematics 1

General Comments:

Although completely correct solutions to all questions were seen, this year the standard of presentation and poor handwriting often led to marks being lost, simple arithmetic or algebraic errors thus occurring.

There was no evidence of candidates being under time pressure, with most making some attempt at the majority of the questions and some candidates made more than one attempt. However, in a number of questions, there was the opportunity to check a solution, but few candidates did this and so could not identify that an error had occurred and have the chance to rectify it.

Comments on Individual Questions:

Question No.

1 The conjugate was known by nearly all candidates, but there were many errors in the modulus. Some thought that $|z| = \sqrt{2}$ or $x^2 + y^2$, while many expanded *zz** and obtained an expression with terms involving "i" or obtained $x^2 - y^2$.

2 Few candidates quoted an incorrect expression for $\sum r^2$ and most subtracted *5n* rather than 5. Most made a sensible attempt to factorise, but a significant number stopped at $\frac{1}{2}n(2n^2+3n-9)$. Those who expanded to obtain a cubic expression usually then made an algebraic error in their attempt to factorise.

3(i) This question was answered correctly by most candidates. Omission of the determinant was the most common error.

3(ii) This question was generally answered correctly. The most common error was finding $\mathbf{A}^{-1}\mathbf{B}$.

4 This was not done particularly well. Most could establish the truth of the result for $n = 1$, but then did not add on correct $(k + 1)$th term, or omitted brackets and so obtained an expression that could not be simplified. Those who had a correct expression often showed insufficient working to justify the truth for $k + 1$. Many lost the final mark by not giving a clear statement of how induction works.

5(i) Many did not see that the centre of the circle was on the *x*-axis and those who did often were not careful enough to make their circle touch the *y*-axis at the origin. The half line was usually correct, but a significant minority thought that $\frac{5\pi}{6}$ was an acute angle.

5(ii) Some tried to find the Cartesian equations of the loci and then solve them, usually with little success. Those who used a simple trigonometrical approach often made errors in the real part, $-\sqrt{3}$ instead of $(-2-\sqrt{3})$. Many gave the answers as a pair of coordinates, thus not answering the question, which required a complex number.

5(iii) Most candidates recognised that the area inside the circle was required, but the correct sector was often not shown correctly.

6(i) Many candidates did not show sufficient detail. Coordinates were not clearly displayed or scales shown clearly on the axes to indicate them. Some just plotted the image points, thus not showing that the image of the unit square is a rectangle.

6(ii) Most candidates identified that the pair of transformations was a rotation and a stretch, but made errors in the direction of rotation or stretch. Many who got a correct pair of transformations could then not give the correct matrix that represented them.

7(i) Most candidates knew the method for finding the square roots of a complex number, the most common error was finding e.g. $x^2 = 9$, but then not square rooting to get the correct real part.

7(ii) Those candidates who completed the square to solve the quadratic usually solved correctly seeing the answers from (i) could be used. Those who tried the quadratic formula often made sign errors, so could see that “hence” was not suitable. This should have prompted them to check their working. Some gave 4 answers from using all combinations of ± signs.

8(i) This part was well-done by most candidates, with only minor algebraic slips occurring.

8(ii) Most candidates used the method of differences correctly, but some started at $r = 1$, missing that this gave an infinite value for the first term and some finished at $r = n + 1$.

8(iii) This part was less well-done. Many simply let $n \to \infty$ in the answer to (ii) or subtracted too many terms from the sum to infinity. Those who started the method of differences at the correct term found the correct answer quite easily.

9(i) Most candidates realised that the determinant had to be found and apart from minor sign errors were able to find, correctly, the values for singularity. However, those who solved a quadratic equation and found non-integer answers generally did not go back to see if an error had been made somewhere.

9(ii) This part prove quite testing. The value of the determinant needs to be considered first as this determines uniqueness or not. If non-unique then the solution of the equations must be investigated to determine if they are consistent or not.

10(i) Most candidates substituted correctly, but then did not know how to rearrange their equation before squaring it to obtain the new cubic equation. Many simply squared each term of their new equation.

10(ii) Those candidates who used the symmetric functions of the new cubic equation usually made good progress as the required identity is fairly easy to establish. Those who tried to use the symmetric functions of the original equation were usually unable to derive a correct identity and just produced many lines of incorrect working.

4726 Further Pure Mathematics 2

General comments

There were similar numbers of candidates this series as last year and the mean mark was marginally higher.

A number of comments on the standard of presentation and mathematical notation were made last year and all of these could be repeated.

There are a number of places in the paper where what seems like a demand to do a question in a particular way is actually a hint as to how to go about the question. In other words, there are places where it might have been appropriate to write ‘or otherwise’ in the question. Candidates doing a question by a different method are not penalised providing the method is valid and will lead to the right conclusion. However, it is often the case that an ‘otherwise’ method is rather longer and prone to more errors than the expected method. In going down these routes, candidate expose themselves to a greater chance of error and will penalise themselves in terms of time. Usually the number of marks to be awarded will give a clue as to how much work is required. Comments on what we have seen in the scripts are made in the individual questions below.

Comments on individual questions

Q1
This standard question was usually well done, but often variables became muddled and candidates got lost.

Q2
In this question it was indicated that candidates should use the standard expansions for $\ln(1+x)$ and $\sin x$. Those that did so found the question straightforward. The only recurrent problem was the failure to take the ln expansion to the cubic term.

In this question there was an alternative method, which was to find the 1st, 2nd and 3rd derivatives and substitute into the general Maclaurin series. This was perfectly acceptable and many candidates obtained full marks. However, most found this method rather longer and the process of differentiating increasingly complicated expressions resulted in most candidates making an error and therefore not achieving the correct result. It was not helped by the fact that many did not simplify the 2nd derivative before differentiation again.

Q3
Unfortunately the answers to this part were marred by the inability to complete the square, a procedure that is first tested in GCSE. The other common error that caused a loss of marks was to fail to deal effectively with a double negative which was often just ignored on the assumption that the answer was positive.

A very neat alternative method to this question was to make the substitution $x = 1 + \sin\theta$. This was seen rarely but always produced the correct answer and full marks.

Q4 (i)
This was a standard reduction formula, which was usually done well – 95% of candidates achieved full marks.

Q4(ii)
This was done less well but still a good source of marks for most candidates.

Q4(iii)

This was completed by only about 30% of candidates. Those that started with I_{11}, used the reduction formula three times found that 990 I_8 was involved from which the answer came easily.

A significant number thought that the demand to find I_3 in (ii) was a hint to keep going up to I_{11}. This involved a great deal of work with huge numbers and it was remarkable how many achieved the right result in the end. This method, of course, was a poor use of time given that there were only three marks available.

Q5(i)

It was surprising how many candidates could not take the formula given in the list of formulae book to get the correct derivative.

Q5(ii)

Notation and the layout of the question were often very poor. It was acceptable to find the left hand side and the right hand side separately and to show that they were equal. However, in a significant number of cases it was not at all clear that what was being written was one of the sides. For instance, to multiply the second derivative by $\left(1-4x^2\right)$ and show that it is equal to something else without even saying that this is $\left(1-4x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ leaves the examiner not knowing what is being done.

Q5(iii)

The more able candidates completed this part the easy way by taking the equation in (ii) and differentiating. Others took the second derivative and differentiated to find the third derivative and then attempted to substitute all three expressions into the left hand side to show that it came to 0. There were many failed attempts and a lot of fudging.

Q6 (i)

Because the expression for x_{n+1} was given, candidates needed to take care not to 'skip over' lines of working, in particular the combining of two fractions into one. This is a common problem for candidates when there is a 'show that' question. Examiners need to see that the process to produce the result really has been shown rather than the given end result being written down. Suffices were sometimes missing and this was penalised.

Q6(ii)

Many candidates identified values for x_1 that converged to wrong root. Nearly all those who drew a tangent in to illustrate the situation had correct x_1.

Q6 (iii)

Most candidates showed the first few iterations and several described the sequence as alternating between the two values –1 and 0. There was evidence that some candidates were confusing a Newton-Raphson method with a rearrangement method and tried to construct a web or staircase diagram.

Q6(iv)

The simplest way to establish the given relationship was to eliminate k by dividing $d_4 = kd_3{}^2$ by $d_3 = kd_2{}^2$ and then multiplying both sides by d_3. Several candidates reduced both sides to expressions involving k and d_2 or k and d_1, sometimes accumulating errors in powers as they did so.

The values of d_4 and $\frac{d_3^{\ 3}}{d_2^{\ 2}}$ were usually correct, although sometimes the minus sign was dropped and some candidates rounded to 2 sf or even 2 dp.

Q6(iv)
Most candidates found the root to 5dp correctly, though many showed no working at all. Candidates should be reminded that if there is no working and the answer is wrong then no intermediate marks can be given so the result is full marks or 0.

Q7(i)
Some candidates did not recognise that the fraction was improper and started by using the form $\frac{A}{(x-1)}+\frac{B}{(x+2)}$ instead of $\frac{A}{(x-1)}+\frac{B}{(x+2)}+C$. Many divided out to get $1-\frac{x+23}{(x-1)(x+2)}$ but then lost the minus sign.
Candidates who tried using the form $\frac{A}{(x-1)}+\frac{Bx+C}{(x+2)}$ were only credited with the method mark if they subsequently split the second fraction; most did not.

Q7(ii)
Most candidates knew that the vertical asymptotes were at x = 1 and x = –2. Many also knew that there was a horizontal asymptote at y = 1 although some gave y = 0 and some tried for an oblique asymptote. However, 90% of candidates obtained full marks in this part.

Q7(iii)
Again, a significant majority achieved full marks. Those who did not usually equated the function to 0 rather than 1 in spite of getting (ii) correct.

Q7 (iv)
A few excellent graphs were seen, many graphs showing the four parts fairly well. Only a minority showed the part on the left-hand side crossing the horizontal lines y = ±1 and then approaching y = 1 from above and y = –1 from below.

Q8(i)
A majority of candidates differentiated correctly and then worked through to the answer in exponentials or by finding a value for tanh$x$ and using the standard ln form. A few attempted to obtain a quadratic in sinh$x$ or cosh$x$. Some quadratics were correct but no candidate was able to progress to find x.

Far too often the value of y was given as an approximation, and sometime not a very good one.

Q8(ii)
Once again the method to find a quadratic in sinh$x$ or cosh$x$ proved unfruitful, but the majority found a quadratic in e^x which as straightforward to solve.

Q9(i)
Most attempted to draw an enclosed loop of some description. Very few drew loops in other quadrants. Those who had found tangents at the pole and marked them on graph produced better diagrams and avoided making $\theta=\frac{\pi}{2}$ look like a tangent.

Q9(ii)

This part was largely well-done. Errors included the failure to find an expression for $\sin^2 3\theta$ which could be integrated.

Q9(iii)

This last part was often done quite well, though a surprising number could not find an expression for $\sin 3\theta$. Once again there was a fair amount of fudging where the correct, but given, answer appeared at the end.

4727 Further Pure Mathematics 3

General Comments:

Overall this paper was found to be slightly harder than recent ones for well-prepared candidates, although it still produced a good spread of marks. Most candidates were able to attempt all questions, and the time available appeared to be sufficient. Many of the questions allowed weaker candidates to demonstrate basic techniques, but also contained parts to stretch the most able, particularly in the area of group theory.

Once more, questions where demonstration or proof is required were only consistently well-answered by the very best candidates. As in previous years attention should be drawn to assessment objective 2 (AO2) and the need, particularly at this level, to "construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference".

There were many candidates who had been well prepared for this exam with a sound knowledge of each topic area; there were also fewer candidates who were unable to tackle questions on whole areas of the syllabus.

Comments on Individual Questions:

Question No.

Q 1

Most candidates had been well-prepared for solving differential equations. The most common errors occurred in calculations made whilst solving the simultaneous equations. The ability to use a calculator, which solves simple equations, would obviously have been helpful to those prone to this type of error. Another, not infrequent, error occurred through poor transcription from complementary function to general solution. Other occasional errors seen included using a complementary function of the form $\cos 3x + i\sin 3x$ or $e^{-2}(A\cos x + B\sin x)$ and using trial functions $y = ax\cos x + bx\sin x$ or $y = a\sin x$. It was rarer, this year, for candidates to omit "$y =$" from their general solution. However, expressions beginning G.S. = y = ... were sometimes seen; although not penalised on this occasion, candidates should be encouraged to be precise in their use of the equals symbol.

Q 2

This year there were fewer candidates who appeared completely unfamiliar with groups, although this is still the topic that causes the greatest difficulties.

In parts (i) and (ii), almost all candidates were successful. Those that failed to score marks on part (ii) usually assumed, erroneously, that negative values exist within the group, giving an answer of $-3-2x-x^2$

(iii) This part was often answered correctly but the following incorrect values were regularly seen: 5, 25 and 60 (from 5P3) and 120 (from 5!)

(iv) This part was a good differentiator between candidates. Good answers were those that carefully justified each step required to reach the conclusion that the order was 25. Some candidates were not sufficiently clear in showing that they knew what distinguished proper subgroups from trivial ones. A common alternative approach was to claim, with varying degrees of corroboration, that *H* consisted of all elements of the form $a+bx$. Unfortunately, most candidates taking this approach were unable to adequately justify it; often they assumed, without substantiating it, that the two given elements formed a generating set. Some candidates demonstrated poor conceptual understanding of “order”, claiming that the order of the group was 5 because both 1+x and 2+x were of order 5.

Q 3

Vector notation was, again this year, generally answered to a good standard. This question was well answered by many candidates.

(i) Most candidates tackled the question by finding vectors in the plane and thus finding the cross product of these. Where this approach was followed, virtually the only errors seen were where candidates misread signs in either the question or in their own working. Those who looked to form equations for x, y and z in terms of two parameters sometimes found it difficult to eliminate their parameters.

(ii) Virtually all candidates were aware of the correct methodology for this part. Even those who made an error in (i) usually followed through with the correct method, gaining two marks.

(iii) Most successful candidates used the scalar product to solve this problem with roughly equal numbers of those using $\cos\alpha$ to find ‘90 – required angle’ and those directly using $\sin\theta$. A few used the vector product equally effectively. A number of candidates found the angle between *l* and the normal to the plane but then stopped rather than finding the angle between the plane and the line. A few of them, in error, found the scalar product using the position vector for *l* rather than its direction vector.

Q 4

(i) Good Argand diagrams were drawn with attention to detail so that axes were labelled but not scaled, angle AOB was clearly less than 45°, OA and OB looked to be the same length and the triangle was clearly neither equilateral nor right angled. The majority of candidates started by placing A on the x axis which is, of course, a quite permissible location; but by doing so, they tended to lose the sense of it being a general complex number. As a result, in attempting to show that the triangle was isosceles, it was common to see candidates claim that the two moduli or lengths were equal to z (rather than $|z|$). Some only considered the specific case |z| = 1. A number of others believed that what was required was to demonstrate that the triangle was not equilateral in order to fully justify their conclusion. Centres should stress the hierarchical nature of the subject means that isosceles includes equilateral. There was also a number of candidates who attempted to calculate the area of their triangle – quite why this occurred is unclear.

(ii) Strong candidates produced neat and concise solutions. Often the best solutions had a clear grasp of the relationship between vector methods and the Argand diagram. Others obtained good marks by meandering methods that sought to evaluate anything and everything that might be useful until an appropriate solution was found, even if this was only as a decimal approximation rather than, as required, the exact value. In weaker scripts, frequently the only mark picked up was that for the conversion of $e^{i\pi/6}$ to $a+bi$ form. Many tried to rotate $5+2i$ (rather than $4+i$), or, having rotated $4+i$, failed to translate the resulting expression. A number obtained an equation of the form $a^2+b^2=17$ (where $w=a+bi$) but were unable to gain further information about a and b.

Q 5

As with Question 1, students were, in general, very well prepared on the topic of differential equations, with plenty of them following the standard approach outlined in the markscheme. A few also successfully used CF+PI method, although some who took this approach erroneously formed an auxiliary equation for a homogeneous equation without constant coefficients. The most common errors were failing initially to divide the RHS by *x* and writing $\frac{A}{x^3} = k$ (a new constant). A few candidates falsely gave the LHS, after multiplication by IF, as $\frac{d}{dx}(3x^2 y)$.

Q 6

Solutions for this question were generally very good with most candidates appearing well-versed in applying the formula for the shortest distance between two lines. As in the previous question on vectors, there were several instances of poor sign transcription. However, it was rare for candidates not to apply a correct method when finding the vector product. One alternate method that candidates applied successfully involved finding a general vector that joined any two points on the lines, then ensuring that it was perpendicular to them by use of scalar products. Another fairly common approach was to convert the problem into finding the shortest distance between two parallel planes each containing one of the lines.

Q 7

(i) Where candidates produced good solutions, these were well laid out, with precise use of the equals sign. These scripts clearly referenced the application of De Moivre, showed their binomial expansion, were explicit about taking real and imaginary parts and made clear that they were dividing numerator and denominator by $\cos^4\theta$ to get to the final identity required. Many candidates failed to include this level of detail, whilst still accessing some of the marks available.

(ii) Many candidates were able to use the connection to part (i) to see that $\tan 4\theta = \frac{1}{\sqrt{3}}$ even if their method in getting there was often convoluted. Strong candidates were then able to find the roots with ease. Two common faults were either to presume that the roots were the four angles or to give more than four solutions. Solving $\tan 4\theta = \frac{1}{\sqrt{3}}$ is an AS Mathematics skill, yet some candidates only gave one solution, or were unfamiliar with the correct method for finding further solutions.

Q 8

(i) The best answers to this question were marked by a clear understanding of what needed to be shown, not just in terms of the basic group axioms, but particularly in terms of the fact that these axioms had to be shown to be true for elements that obeyed the condition for membership of the subgroup. The very best candidates were able to demonstrate closure of subset, existence of both inverses and identity within subset. They also knew that it was trivial that subsets of a group are associative. They showed this with clear arguments that didn't confuse the general with the particular.

As in previous years there was clear evidence that candidates had studied the topic and knew the four group axioms; however, they frequently had no concept of how to apply these in context.

Many candidates assumed closure within the group or the existence of an inverse within the group automatically led to the same in the subgroup. However, they then went to lengths to try to show associativity within the subgroup when this is genuinely inherited.
Many were confused by the term 'multiplicative group' thinking that this meant that they were dealing with the algebra of real numbers. This led them to erroneously equate the identity to 1 and the inverse to $\frac{1}{h}$

(ii) Once again, it was clarity of mathematical argument that marked out the very best answers here. Good answers systematically eliminated elements (or potential subgroups) by counter-example. Some lost a mark because they incorrectly evaluated g_1g_2 as g_2g_1 but otherwise knew how to tackle the problem. Others only considered some of the elements or talked in vague terms about the non-symmetrical nature of the combination table.

4728 Mechanics 1

General Comments:

The majority of candidates were well-prepared for this paper and routine questions were answered competently. The questions that proved to be most challenging were Q4 and Q5.

Candidates are advised to draw clear, well-labelled diagrams to aid their understanding of the problem. Repeated use of the letter *F* should be avoided, using *F* for friction and *F* for resultant force gives rise to confusion leading to errors by candidates and difficulties for examiners when deciding whether to award marks.

To avoid loss of marks through careless evaluation, candidates should state the formula they intend to use, then substitute values in to produce an unsimplified expression. Only then should evaluation and rearranging take place. Wrong answers that are not preceded by evidence of a valid method cannot be awarded marks. If not exact, final answers are expected to be accurate to three significant figures, so the use of prematurely rounded intermediate values should be avoided. Failure to work to a sufficient degree of accuracy sometimes caused the loss of a mark in Q1(iii) but many candidates made good use of their calculator memory to store values to be used in later work and thus avoided accuracy penalties.

Comments on Individual Questions:

Q. 1 Parts (i) and (ii) were almost always correct although a small minority of candidates took the initial velocity to be upwards or zero and a few were confused about the sign required with g. Part (ii) had the exact answer 6.384 but 6.38 was accepted.

Part (iii) could be solved in a variety of ways, the simplest being to use $s = vt - gt^2/2$ which seems to be the least familiar of the *suvat* equations. Any fully complete method was acceptable but rounding errors often accumulated in the multi-stage methods with 0.6 being a common incorrect answer. A diagram of the situation might have helped the few who only found the time for the first 15m or only the total time. A minority were unable to solve the quadratic equation in t they had obtained.

Q. 2 There were many excellent solutions to this questions, although some candidates showed the usual weakness when dealing with the positive and negative signs required to indicate the directions of the particles. The most common error in (i) was failure to consider conservation of momentum of the whole system. In (ii) a small minority of candidates were unable to give the correct solution to the equation $12m = 2.4$ but, if they reached this point, only lost the final mark.

Q. 3 Many candidates gained full marks for part (i) although a few found only the area above the line $v = 5$. In (ii) candidates were not always able to form the correct algebraic expressions for the total distances for A and B in terms of T to find its value by algebraic methods. An answer often appeared with little or no working but, if correct, was given full marks. The work in (iii) was sometimes simply a numerical step-by-step approach (which often also provided the answer to put in (ii)). Only one distance needed to be calculated since A and B made the same journey. If candidates calculated both, credit was given to the better answer but it was disappointing that candidates rarely reviewed and corrected earlier work in such cases.

Q. 4 This proved to be a difficult question for many candidates. For (i) there were two methods of solution, using trigonometry or by resolving; whichever approach was used the candidates who were most successful had usually draw a good diagram. The most common error made by those using trigonometry was to form a triangle by simply joining the ends of the forces instead of drawing the appropriate triangle of forces. In this case 110 instead of the correct value of 70 was used in the cosine and sine rules and a maximum of half-marks were available. Those who chose to resolve were most successful, particularly in finding the angle, if they resolved parallel and perpendicular to the 10N force. Any reference to horizontal and vertical was ignored if the candidate worked with an incorrect angle between one of the forces and the true horizontal, credit being given for correct work relative to their base directions. Using Pythagoras and basic trigonometric ratios with 10 and 6 gained no credit.

For (ii) and (iii) candidates needed to realise that the resultant was vertical. In both parts credit could be gained from correct use of the answers obtained in (i). These parts were done particularly badly, if at all, with 20 N being the most common wrong answer in (ii) and (iii) frequently not attempted.

Q. 5 Good solutions from many candidates particularly in (i) and (ii) but (iii) and (iv) proved a challenge for some who failed to think through the mechanics of the varying situations. There was some misunderstanding of the phrase 'above the horizontal' which indicated that the force was acting upwards away from the surface. Credit was given, in the parts affected, if the force was taken to be acting downwards with a small penalty applied in (iv) since this part was simplified by the misunderstanding.

Part (i) was not always completed as successfully as (ii) but was generally well done.

In (ii) most candidates understood that the normal contact force and hence the friction would now change. No credit was given if the value of friction from (i) was used.

In (iii) too many simply followed the same routine application of Newton's second law used in (ii) and gave an negative value for the acceleration having failed to appreciate that the tractive force was no longer enough to overcome friction so the acceleration would be zero.

In (iv) the applied force was now vertical so no credit was given for work which only considered horizontal motion even if this produced the answer $a = 0$. There was a general lack of understanding that the crucial question now was whether or not the applied force was sufficient to lift the object off the table.

Q. 6 The variable acceleration and hence the need to use differentiation and integration in this question was well understood with very few cases where the use of *suvat* equations was thought appropriate. The main cause of lost marks in (i) and (ii) was failure to evaluate the constant of integration in (i) which also affected the accuracy of work in (ii). Candidates should be aware that the constant of integration is not always zero.

(iii) This part was not always done correctly; answers were often incorrect because of errors made when solving a simple linear equation or because the displacement of Q was taken to be zero..

(iv) The differentiation was usually correct, although some lost the minus sign. The final step, requiring the use of conservation of momentum, was sometimes omitted. The most common error at the final stage was to use a mass of m instead of $2m$ for the after momentum.

Q. 7 There were many fully correct answers to this question although some failed to find both the quantities required in (i). Candidates should be reminded to check that they have answered a question in full.

In (i) use of the combined approach is still quite common and some credit was given if the value of *a* was found this way. To gain full marks a correct Newton's second law equation had to be obtained for at least one of the particles so that *a* could be used to find *T.* A number of candidates forgot or neglected to find *T* having found *a.*

Parts (ii) and (iii) did not depend on work from (i) but blank spaces on some scripts suggest some candidates did not appreciate this. When attempted these parts were well done and most candidates scored full marks on both parts.

(iv) Here it was necessary to use Newton's second law on each particle separately. In both parts the signs used needed to be consistent with the direction of motion in order to gain accuracy marks. In (a) the motion of particle *P* should have been used; most candidates realised this but a few used the wrong mass or had different masses on the 2 sides of the equation. In (b), to find the friction, the motion of *Q* should have been used but candidates who used the combined approach were also given credit. Again there were some muddles with the masses and signs but most scored well here.

4729 Mechanics 2

General Comments:

Many candidates were very well-prepared for the rigours of this paper and scored very highly. It was also pleasing to see an improved standard of presentation, including good force diagrams, where appropriate.

Candidates are reminded about the necessity of showing enough detail in their solution in instances where the answer is given in the question. The use of graphical calculators to solve equations was more apparent in this session, which is welcome, but when the final answer is incorrect, examiners find it difficult to award method marks.

Comments on Individual Questions:

Question No. 1

(i) Most candidates scored full marks for this part.

(ii) This was not approached well by the majority of candidates. This is in fact an energy type problem, with many candidates assuming constant acceleration instead – a few of these gained B for calculating the work done, usually amongst some completely wrong working. A small number gained only the B1 for the change in KE. Even those who used an energy approach sometimes omitted one of the energy terms.

Question No. 2

This question proved a good source of marks for many candidates. The only significant error that was made was for candidates to use the angle with the horizontal and not all candidates went on to give the angle with the vertical as requested.

Question No. 3

The majority of candidates completed the usual approach of using Newton's second law to produce two simultaneous equations successfully. However a minority didn't progress much further by failing to replace their driving force with *P*/16 and *P*/25. The solving of the simultaneous equations presented the usual opportunities for algebraic slips from the weaker candidates, with sign errors cropping up.

Question No. 4

(i) A significant number of correct solutions were seen to this standard type of question, but there were two common errors seen. Firstly the centre of mass of the semi-circle was seen as 3 cm from use of the wrong formula, ${}^{3r}/_{8}$, and was also seen as 4 cm. Secondly, a significant number of candidates had an incorrect area for the semi-circle with values of 64π and $8\ \pi$ commonly seen. Some also worked out the distance of the centre of mass from *AE* here (or in part (ii)) even though this was not required.

(ii) This question requested two values, but it was very common to see only one value given. The key points *A* and *E* appeared to be missed by many candidates. There wereas also many candidates who appeared confused between mass and weight, such that it was common to see the final answer as mass.

Question No. 5

(i) The vast majority of candidates scored full marks here. Of those who didn't, the common error was to use 5 m s^{-1} as the speed when hitting the ground.

(ii) Many candidates who were able to answer (i) correctly went wrong here. The common error was not to deal correctly with the vector aspect of the momentum of the sphere with a difference of two scalar quantities being seen significantly often.

(iii) This was done relatively poorly. The problem was either through not considering all the energy terms needed (ignoring PE or KE) or using far too many (bringing in the 7.507 and/or the 5.255).

Question No. 6

(i) This question was answered correctly by nearly all candidates. A common mistake made was for a sign error in $s = ut + ½at^2$ using 9.8 instead of -9.8.

(ii) A minority of candidates started again but most realised they simply had to substitute into the trajectory equation given in (i). There were a few cases where y = 0 was used; the more common error was to use 100 instead of -100. A necessary substitution was widely known. Some sign errors occurred when expanding $-k(1 + \tan^2\theta)$. The second solution of -44.6 was frequently converted to 135 but usually ISW could be applied and the mark awarded.

Question No. 7

This proved to be the most difficult question on the paper. Candidates would help themselves greatly if they had a full force diagram for the forces acting on the ladder. Also some indication of the point that is being used to take moments about would be of assistance to examiners.

(i) Some very elegant answers were seen to this part, but they were a rarity. For those who knew what they were doing, this was a standard resolve and take moments request. The usual place to take moments about was *A*, but a significant number of candidates used the alternative of *B*. The most common error here was failing to include all the necessary terms when taking moments about *B*. Other errors included missing out a distance from one or more terms (often the 4 was omitted), putting a vertical force at *A*, putting the force at *A* perpendicular to the ladder instead of perpendicular to the wall and using an inexact value for θ instead of dividing through so they could make use of the given value of $\tan\theta$.

(ii) Again some candidates took moments about *B*, when the easier root was to take moments about *A*. Invariably those who found part (i) difficult also struggled here, omitting terms from their moments equation or having additional terms from placing the extra weight in the incorrect place. A few managed to reach $a = 3$ but then failed to give their answer as a range of values.

Question No. 8

(i) Not all candidates recognised that since the answer was given there was a need for them to show their working clearly. There were many good solutions but a disappointing number wrote a formula and then gave a value for ω or v^2 without showing the values used to obtain it. A few candidates did not know how to proceed and seemed to think the coefficient of restitution was relevant in this part.

(ii) The majority of candidates scored full marks on this part. For those who guessed the wrong direction for one of their velocities, the issue was resolved at the end and speed was stated successfully.

(iii) This part was poorly done by a significant majority of candidates, who were unable to find the force required to proceed. Many thought that 4.9 N was the force towards the centre. Other common errors seen were for the force towards the centre to (4.9 – 0.4g) N, as well as some who thought the 4.9 N force was at an angle of 45^{o} to the horizontal.

4730 Mechanics 3

General Comments:

This paper proved easy for many of the candidates, with a large proportion scoring more than 50 of the 72 marks available. There were, however, some scripts gaining fewer than 20 marks.

Candidates should be made aware that they should show full and complete working when establishing results given in the question paper. For example, in question 5(i), it is not enough to state the results gained by using the Law of Conservation of Momentum and Newton's Experimental Law and then state the given result for $a\cos\alpha$; candidates are expected to show fully all the steps needed.

A number of cases was seen where candidates had some correct work in an answer, but then misread their own writing and so arrived at a wrong answer. This is penalised, and is not counted as a genuine misread. There were also cases where candidates had used a correct value from the question paper at the start of a part question, or in an earlier part, but then gone on to use a different value. Again, this is penalised and is not regarded as a genuine misread.

The presentation of the scripts was extremely good in many cases, and generally acceptable, though with a few exceptions. Most of the candidates who used additional paper did so because they genuinely needed to make an extra attempt at a part question; however, the parts candidates needed extra paper for varied considerably.

Comments on Individual Questions:

1) (i) Almost all candidates did this part completely correctly, though not always by efficient methods. Some candidates found the answer to part (ii) on their way to the answer to this part.

(ii) This part was almost always done correctly, though a small number of candidates ignored the vector nature of momentum and impulse and used $I = mv - mu$, with I =0.25, m = 0.2 and u = 3.

2) (i) Very few candidates had any problem at all with this part, though there was a very small number of candidates who made no real attempt at this question.

(ii) About half the candidates took moments about *A* for the two rods together. Other candidates first worked out the horizontal and vertical components of the force acting on rod *AB* at *B* and then took moments about *A* for rod *AB*. (A variation was to find the horizontal and vertical components of the force acting on *AB* at *A*, and take moments about *B*). A small number of candidates using the former method made errors in the values of one or more of the trigonometric expressions used, and a small number of candidates using the latter method made errors in the directions of the components.

3) (i) This part proved quite straightforward, though some candidates did not know the formula for the elastic potential energy stored in a stretched string, and a very small number did not work out the **total** elastic potential energy stored in the two strings.

(ii) This part proved to be one of the more challenging questions on the paper. Most candidates sensibly considered a general position where each string was extended by a length *x* (m) from the equilibrium position. Other candidates used *x* to represent the total length of the stretched string, while a small number used other definitions. To gain full marks candidates were expected to state that an equation of the form $\ddot{x} = -\omega^2 x$ represented SHM. Where a candidate arrived at an equation of the form $\ddot{x} = -\omega^2(x - c)$, they were expected to explain what the constant represented; most did this appropriately. Candidates were also expected to show that *P* performs SHM **throughout the subsequent motion**; only a minority of candidates gained the mark for this. Few candidates had any difficulty with the period.

A significant minority of candidates was unable to do this question; usually these candidates did not appreciate the need to use Newton's Second Law at a general point. It is possible to do this question by a consideration of energy at a general position, but only an extremely small number of successful solutions by this method were seen.

4) (i) A small number of candidates made errors with this part, including an error with the minus sign, using $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ and using limits the wrong way round.

(ii) About half the candidates used the equation they had found in part (i), and wrote it as $\frac{dy}{dx} = 10e^{-\frac{t}{8.2}}$, though some took a long time to get to this stage. Other candidates used $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$, and had a rather more straightforward calculation to do. In both cases, a small number of candidates made sign errors, or omitted the arbitrary constant of integration.

5) (i) Almost all candidates used the Law of Conservation of Momentum and Newton's Experimental Law correctly, though there were some sign errors. Most candidates correctly derived the required result, though a small number did not do this convincingly and so did not gain full marks. A small number of candidates also quoted a result gained by using the Law of Conservation of Momentum perpendicular to the line of centres. Usually this was not used, and so was ignored by markers.

(ii) Almost all candidates realised that this required the use of the result quoted in part (i) and $a\sin\alpha = 2$ and efficiently and correctly found the values of *a* and *α*. A small number of candidates was unable to make any meaningful attempt at this part. A minority of candidates also worked out *b* and *β*.

6) (i) Although most candidates found this introductory part straightforward, the amplitude was sometimes given as 1.2 m or 0.3 m and ω as 8 s^{-1} or 2 s^{-1}. Almost all realised that the maximum velocity was given by $a\omega$, though some used $v^2 = \omega^2(a^2 - x^2)$.

(ii) Most candidates used the correct formulae to find the value of *x* and the velocity, though some used the wrong trigonometric function. A few candidates employed $\omega t + \varepsilon$, though rarely correctly. A small number of candidates found the correct value of *x* and then went on to add 0.6 m to it. Candidates using $v^2 = \omega^2(a^2 - x^2)$ were expected to make the direction of the velocity clear.

(iii) Quite a number of candidates found the values of *t* from a consideration of the motion of the particle, with a lot of helpful diagrams seen. Any mistakes made in finding *a* or *ω* earlier restricted the mark in this part. Some candidates did this part by solving equations such as $v = -a\omega\sin\omega t$; these candidates could only gain marks in this part if they had correct values in earlier parts.

7) (i) Most candidates had little trouble with this part question, though some did not show sufficiently clearly how they derived the given answer. A small number of candidates, having initially failed to arrive at the given answer, did the question again, or went back and corrected their work.

(ii) Quite a number of candidates, including some with high marks, assumed that the particle would be stationary at the highest point, and arrived at the wrong answer of 4.43 $m\,s^{-1}$. Other candidates thought the value of θ at the uppermost point was 0, $\pi/2$ or 2π

(iii) The most common error on this part was to make a sign slip and arrive at $\cos\theta = \frac{1}{6}$. A significant minority of candidates found the angle θ and then stopped before finding the speed.

4731 Mechanics 4

General Comments:

The work on this unit was generally of a very high standard. Many of the candidates were very competent and demonstrated a sound understanding of the principles of mechanics covered in this module. However, a small number of candidates struggled with the majority of the paper and were not able to apply principles appropriate to the situations. Candidates seemed to be particularly confident when using calculus to find the x - coordinate of the centre of mass of a lamina, applying the principle of conservation of mechanical energy, finding the magnitude of the force acting at an axis, and using energy to investigate stability of equilibrium. Topics that were found more challenging included relative velocity, finding the y -coordinate of the centre of mass of a lamina and using calculus to find the moment of inertia of a uniform solid cylinder. Candidates appeared to have sufficient time to complete the paper. The standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions:

Question No. 1

Nearly all candidates found this first question straightforward and the vast majority found the correct angular acceleration of the turntable in part (i), and the time taken for the turntable to increase its angular speed in part (ii). The most common error was stating the angular displacement as 4 rather than 8π . It is worth pointing out that candidates are advised, where possible, to use methods that do not rely on previous obtained results. For example, in part (ii) a number of candidates used the formula $\omega = \omega_0 + \alpha t$ with an incorrect value of α to calculate t . Candidates could have bypassed this incorrect value altogether by using the formula $\theta = \frac{1}{2}(\omega_0 + \omega)t$ which is independent of the angular acceleration.

Question No. 2

Most candidates in part (i) correctly found the x - coordinate of the centre of mass of the lamina.

The ability of the candidates to find the exact value of k in part (ii), however, was significantly more varied. It was expected that candidates would first find the y - coordinate of the centre of mass of the lamina by consider the integral $\frac{1}{2}\int_1^2 y^2 \mathrm{d}x$ but it was relatively common for the factor of $\frac{1}{2}$ to be missing. A number of candidates tried to integrate about the y -axis but this was rarely successful as many stated that the required integral was $\frac{1}{\sqrt{k}}\int_k^{4k} y^{3/2}\mathrm{dy}$, forgetting the factor of a $\frac{1}{4}$, which is required as $y = kx^2 \Rightarrow \mathrm{d}y = 2kx\mathrm{d}x$. Those candidates who did find the y - coordinate of the centre of mass correctly usually went on to find the correct value for k .

Question No. 3

Relative velocity remains a difficult and challenging topic for many and a number of candidates left both parts of this question blank. However, there were a significant number of candidates who answered both parts of this question correctly. The most succinct and efficient solutions were from those candidates who adopted a velocity triangle approach to find both the bearing and the shortest distance between the two planes in part (i).

Part (ii) was also answered well with the majority of candidates having the correct approach for finding the total distance travelled by both planes. A significant number, however, only found the two distances travelled by the planes, and they either neglected to (or forgot to) add these two values together.

Question No. 4

Part (i) was almost always answered correctly.

In part (ii) many candidates correctly derived, by integration, the required moment of inertia for the uniform solid cylinder, and the presentation of these candidates' work was generally sound. A number of candidates, however, seemed unprepared for a question of this nature and did not progress any further than simply stating the mass per unit volume of the cylinder. It was expected that candidates would:

- state (using part (i)) the moment of inertia of an elemental disc about its diameter,
- find an expression for the moment of inertia of this elemental disc about an end face using the parallel axis rule,
- integrate this expression between the limits of 0 and h.

Many candidates, however, did not appreciate that the parallel axis rule had to be used and many tried to incorrectly add Mr^2 (their answer to part (i)) to $\frac{M}{h}\int_0^h x^2 \mathrm{d}x$. A significant number of candidates either missed off the final demand in part (ii) to find the moment of inertia about a diameter through the centre of the cylinder, or they incorrectly added, rather than subtracted $M\left(\frac{h}{2}\right)^2$ from $M\left(r^2+\frac{1}{3}h^2\right)$ (the given result for the moment of inertia of the cylinder about a diameter of an end face).

Question No. 5

In part (i) most candidates correctly derived the given expression for the total potential energy with only a small minority having difficulty with calculating the distances *HB* and *HP*.

Candidates found part (ii) the most challenging part of the paper with only a small minority scoring all four marks. While the majority of candidates obtained the two correct critical values of λ, only a small minority appreciated that $1 \le \lambda < 2$; when $\lambda = 2$ the values of $\cos\theta$ and $\sin\theta$ are equal and therefore there would only be the one equilibrium position.

In part (iii) nearly all candidates stated that when the first derivative of V was zero then either $\sin\theta = 0$ or $\cos\theta = \frac{3}{4}$. Most candidates correctly went on to find the second derivative of V and they knew that they had to find the sign of the second derivative for their values of θ. In general, the work for $\sin\theta = 0$ was very good, but many struggled to make any progress with $\cos\theta = \frac{3}{4}$, often because they did not rewrite their expression for the second derivative of V in terms of $\cos\theta$.

Question No. 6

In part (i) the moment of inertia of the pendulum was almost always found correctly and almost all candidates correctly derived the given expression for ω^2 in part (ii).

The majority of candidates correctly differentiated ω^2 with respect to time to find the angular acceleration in part (iii) although a significant number adopted the more time consuming approach of using the rotational form of Newton's second law.

Candidates found parts (iv) and (v) demanding and only a few succeeded in getting both of these parts correct. In part (iv) only a minority of candidates, when trying to find the magnitude of the force acting on the pendulum at the axis of rotation, derived the correct equations of motion involving the radial and transverse components of the acceleration. The most common errors included sign errors, using a mass of m rather than $3m$ and using a radius of $2a$ rather than $\frac{4}{3}a$. The radial and transverse accelerations, however, were almost always correct.

In part (v) candidates were asked to show a certain definite integral would give the time taken for the pendulum to rotate between two given positions. This demand was unfamiliar to candidates and very few appreciated that re-writing ω as $\frac{\mathrm{d}\theta}{\mathrm{d}t}$, and using the given result from part (ii) together with the identity $2\sin^2 X \equiv 1-\cos 2X$, would yield the required integral. It was disappointing that so many able candidates could not correctly state that $\int \operatorname{cosec}\left(\frac{x}{2}\right)\mathrm{d}x = 2\ln\left|\tan\frac{1}{4}x\right|(+c)$, which meant that only a small minority of candidates obtained the correct final answer.

4732 Probability & Statistics 1

General Comments

This paper was found to be perhaps more accessible than usual. Most candidates scored well on the standard calculations such as those in questions 1(i), 3(i) and 4(i). A few questions contained relatively non-standard requests (eg 2(ii), 5(iii), 6(ii)(c) and 8(iii)) and some candidates could not handle the slightly different approaches that were needed. In particular, in question 8(iii) many candidates were unable to identify all the relevant cases and many were unsure when to add, and when to multiply, probabilities. Most candidates were surprisingly weak in the combinations questions, 6(ii(a), (b) and (c).

Answer given in words

The questions that required answers given in words were fairly well-attempted, although in some cases (especially in questions 1(ii), 1(iii), 2(v) and 8(iv)) answers obviously learnt by rote failed to reflect what was actually asked. In question 1(iii) candidates struggled to find a second convincing reason to give. On the other hand, many candidates had clearly been taught the two standard answers to this type of question and were able to recite them without difficulty. In question 3(ii) many candidates gave answers that were correct as far as they went, but this question required a little more interpretation than is given by a standard answer learned by rote, such as ‘There is good correlation between age and quality’. The mark was awarded only if candidates made clear the implication that ‘Older is better’.

Rounding

Centres should note the rubric about giving answers correct to three significant figures. A large minority of candidates lost marks by premature rounding or by giving their answer to fewer than three significant figures without having previously given an exact or a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or to keep intermediate answers correct to several more significant figures.

Two errors in rounding occur frequently. If the third significant figure is zero, candidates often omit it; and some candidates think that, for example, 0.92 is actually three significant figures.

Drawing

In question 2(iv) many candidates' drawings were rather faint and therefore difficult to mark. Candidates should be made aware that their answers will be scanned and read on a computer screen and therefore some clarity may be lost, unless they draw clearly and not too faintly.

Use of statistical formulae and tables

The formula booklet, MF1, was useful in questions 1(i), 3(i), 4(i) and 7(i)(a) & (b) (for binomial tables). Candidates generally used the formula booklet more successfully than has sometimes been the case in the past. In questions 1(i) and 4(i) very few candidates quoted their own (incorrect) formulae for r and/or b, rather than using the ones from MF1. A small number of candidates thought that, eg, $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. In question 4(i) some candidates tried to use the formula $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1 but almost all candidates who used this formulae misunderstood the Σ notation. For example they interpreted $\Sigma(x - \bar{x})^2$ as $(\Sigma x - \bar{x})^2$. Candidates should be encouraged to use the formula $b = \frac{S_{xy}}{S_{xx}}$ instead.

In question 3(i), Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ or even $(\Sigma d^2)^2$and the formula was sometimes misquoted as $\frac{6\times\Sigma d^2}{n(n^2-1)}$ or $\frac{1-6\times\Sigma d^2}{n(n^2-1)}$ or $1-\frac{6\times\Sigma d^2}{n}$ or $1-\frac{6\times\Sigma d^2}{n^2(n-1)}$, despite the formula being given clearly in MF1.

In question 7, some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities. Also in 7(i)(b), many candidates used the formula rather then the table, which is quite understandable, but leads to a somewhat longer method than necessary.

Use of calculator functions

Increasingly nowadays, calculators can provide answers using statistical functions, binomial functions and others, without the need to quote a formula and substitute values into it. The problem here is that if candidates write down their answer with no working, they can only score either full marks or no marks, with no possibility of gaining any credit for partially correct working. In most cases, the use of such functions saves very little time and it is advisable to show working instead. However, if candidates wish to use these functions, they should input all the relevant data twice in order to check their answer.

It should also be noted that there are sometimes questions (e.g. 5(ii)) in which a correct answer without working may not gain full marks.

Other points

There was one 'Show that' question, 9(i). Some candidates seemed not to be aware that in such questions, each essential step must be shown and nothing should be assumed or implied.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers that are acceptable in the examination context, centres should refer to the published mark scheme.

Comments on Individual Questions

1) (i) This question was well-answered, with only a few candidates showing some of the errors mentioned above, including giving only an answer correct to two significant figures, rather than the standard three.

(ii) This was also well-answered, although a significant minority lost the mark by not setting their answer in context. Some others lost the mark by referring to goals instead of points. There were some imaginative answers such as 'The value of *r* shows whether the players were worth what they got paid' but these did not gain the mark. Some candidates failed to understand the significance of the small value of *r*, giving answers such as 'There is a small correlation between the top salary and points scored. As the top salary increases, the points scored increases.'

(iii) Most candidates gave at least one of the two rote answers (Extrapolation and poor correlation), but many gave two answers that amounted to the same thing. Some gave the rote answer 'Correlation does not imply causation', which did not gain a mark.

2) (i) A generous range was allowed here (30 to 40), but a few candidates made errors in reading the graph and gave answers outside this range. The fact that the scales on the two axes are different may have confused some candidates, A few candidates just read off the cumulative frequency for a mass of 45 g. Others found the average of the two cumulative frequencies for 40 g and 45 g.

(ii) Almost all candidates answered this question correctly. A few omitted to subtract the cumulative frequency from the total (400). Others thought the total frequency was 450. Some found the correct decimal (0.125) but not the percentage.

(iii) Many candidates thought that the highest mass was 100 g. Many incorrect answers were given for the reason why the exact values cannot be read off. Some simply restated the question (‘Exact values cannot be read off the graph.’). Other reasons included the following ‘Because it is a curve’; ‘Because it is cumulative frequency’;, ‘Masses start from 5 and level off after 80’; ‘Cumulative frequency graph does not show the range or the frequency;, ‘The resolution is too low’; ‘The scale is too small; and ‘Because we don't know the full range of masses’.

(iv) The diagram was often correct, although large minority of candidates thought that the median was above 40, perhaps because they assumed that the total was 450. Some candidates reduced their chances of marks by drawing free-hand, thus making it unclear at which values their lines were drawn. A few drew the maximum line at 100, even though the value for the highest mass, given in their answer to part (ii), was, e.g., 90 g or 88 g.

(v) The requirement here was to note either the longer whisker at the top or the fact that there were more masses in the lower half of the range than in the upper part. Many candidates gave inadequate answers such as ‘The spread is fairly even’ or ‘There is wide spread of masses’ or ‘The IQR is nearer the lower end’ or ‘The majority are between 25 and 55’. The concept of ‘skew’ is not in the specification, but candidates could gain the mark by stating that the data had positive skew. Many stated (wrongly) that there was negative skew. ‘Positive correlation’ was not infrequently seen.

3) (i) This question was well-answered by many. The mistakes mentioned above were seen infrequently. Occasionally a candidate made a mistake in the ranking. Some ignored the instruction that years should be ranked from the earliest (1) to the latest (9).

(ii) Many answers omitted the context. Attempts at rote-learned answers often failed to make it clear that the good rank correlation suggests that, generally speaking,’Older is better.’

4) (i) This question was very well-answered. Occasionally the errors mentioned above were seen and sign errors sometimes crept in.

(ii) Most candidates scored full marks on this question.

(iii) Some candidates knew what was required, but many gave answers that suggested that they did not understand that the two regression lines both pass through the point $(\bar{x}, \bar{y})$. The most common incorrect answer was (0, 0). Some candidates gave the answer ‘The midpoints of the two lines’. A common answer was (0, 7.18). Some candidates just quoted one or two of the points given in the table. Some substituted $y = x$ into the equation found in part (i) and solved to find x and y. Many gave coordinates without any indication of how they had been obtained.

(iv) The most frequent error was not recognising that the x on y regression line is required, and many incorrect answers began ‘Rearrange the equation . . .’. A few candidates correctly stated that the x on y line must be used but omitted to say that $y = 5.8$ needs to be substituted into the equation or read off from the line. Some candidates mentioned a non-specific line of best fit.

5) There was some confusion between the binomial and geometric distributions in this question.

(i)(a) This was generally well answered. A few candidates introduced a binomial coefficient while others interchanged the 0.27 and 0.73.

(i)(b) Many candidates found $1 - 0.27^8$ instead of $(1 - 0.27)^8$. Others found $1 - 0.73^8$ or 1 - (their answer to part (i)(a)) or $0.73^8 \times 0.27$. A few candidates introduced a binomial coefficient. Some candidates (unwisely) used the long method: 1 - P(X = 1, 2, 3, 4, 5, 6, 7 or 8) giving themselves ample scope for omissions or arithmetical errors.

(ii) Some candidates omitted the binomial coefficient.

(iii) Some candidates realised that their answer to part (ii) was useful. Others started from scratch. Many candidates found incorrect probabilities such as P(3 tickets in 9 attempts) and P(4 tickets in 12 attempts) and then either left these as two separate answers, or added or multiplied them. Other incorrect methods involved P(3 tickets in 11 attempts) or P(1 ticket in 3 attempts).

6) Many errors in all parts of this question were due to candidates either ignoring the fact that one digit is repeated or ignoring the phrase ‘without regard to order’. Candidates who attempted to list the possibilities were often unsuccessful.

(i) Many candidates recognised the repeated digit and correctly found $\frac{7!}{2}$ = 2520. But many then considered the repeated digit again and gave $\frac{2}{2520}$ instead of $\frac{1}{2520}$. Others ignored the fact that one digit was repeated and found $\frac{1}{7!}$.

(ii)(a) Some incorrect methods seen were these: ${}^5C_2 \times 2$, ${}^7P_3$, ${}^7C_3 \times {}^2C_2$, $\frac{3!}{2!}$ and $\frac{3! \times 5}{2!}$. Some candidates considered different orders, and either made a list of 15 possibilities or found ${}^5C_2 \times 3$.

(ii)(b) A large number of different methods were seen, many giving rise to answers in the hundreds. Some examples are as follows. $5! \times 2$, $\frac{5!}{3!}$, ${}^6C_2 \times {}^2C_1$, $20 \times 3! \times 2$, ${}^7C_3 \times 3!$, $\frac{{}^7P_3}{2!}$, ${}^6C_3$ and ${}^5C_2 \times {}^5C_1$,

(ii)(c) Most candidates attempted to calculate this in one step, (often just ${}^6C_3$ or ${}^7C_3$ or ${}^7P_3$) without realising that there are separate cases to be considered (or that two of those cases have already been considered in the previous two parts). A few candidates did appreciate this and correctly used their previous two answers in this part, even if those previous answers were incorrect. These candidates gained full credit. Answers such as 2520 or 15120 or 88200 apparently gave candidates no qualms.

7) (i)(a) This question was usually answered correctly, usually by the formula and sometimes using tables. A few candidates omitted the binomial coefficient while others only read one value (0.7759) from the table, i.e P($X \leq 3$)). Others found 1 - 0.7759.

7) (i)(b) A few candidates found 1 - 0.7759 (i.e. P($X > 3$), or just gave 0.7759 as the answer. Others found 1 - P($X = 2$). Some (correctly) used the formula to find 1 - P(X = 0, 1 or 2). This is not a particularly long method, but it is significantly longer than finding 1 - P($X \leq 2$) using the tables. Others attempted various methods not involving the binomial distribution at all.

(ii) A good number of candidates answered this question correctly. Many, however, found this very typical question difficult, not appreciating its two-layered structure. These candidates gave answers such as $\frac{1}{6}\times{}^{10}C_3\times0.25^3\times0.75^7$.

Other candidates used their answer to part (i)(b) (thus gaining one mark) but used it in incorrect ways, such as dividing it by 6 or by raising it to the power of 6. Some subtracted it from 1, which is correct, but failed to take the next two steps correctly. A few candidates used their answer to part (i)(a) instead of (i)(b).

8) (i) Most candidates answered this part correctly. Some gave the products of probabilities at the end of some of the branches. This was unnecessary, but did not lose any marks, as long as the $\frac{2}{3}$ and $\frac{1}{3}$ were correctly placed throughout as well. A few labelled the branches B, R, BB, BR, BBB, BBR etc, which is incorrect. Some drew 30 branches, ignoring the fact that the turn stops when blue is obtained. A small number omitted the labels "R" and "B". A few others gave probabilities that reflected a 'without replacement' situation (i.e. $\frac{4}{6}\to\frac{3}{5}\to\frac{2}{4}\to\frac{1}{3}$ etc). These candidates were presumably following a rote method for drawing a probability tree, without reference to the actual context.

(ii) The most common error in this part (which was often carried forward into the next part as well) was thinking that P(4 throws) = $(\frac{2}{3})^4$, instead of $(\frac{2}{3})^3$. Many understood this, but used the longer method $\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$ (which made the method for the next part rather laborious).

(iii) This question was answered well by some candidates. Candidates who started by listing all six possible ways in which Adnan throws more times than Beryl, were much more likely to gain marks than those who did not. Many scored only one or two marks and a good number failed to score at all. A common error was to list the possibilities for the two players separately, find the total of their separate probabilities, and then to multiply (or add) the two totals. No marks could be gained unless candidates linked together individual possible results for the two players. Another common error was to interpret 'Adnan throws the die more often than Beryl' to mean only the following pairs of numbers of throws:
4&3, 3&2 and 2&1. Most of the more successful candidates listed all six possible pairs systematically (2&1, 3&1, 3&2, 4&1, 4&2, 4&3). Some of these obtained the correct probabilities for the first three of these, but made an error on the three pairs that include 4. The error was in thinking that P(4) = P(RRRB) rather than P(RRRR) + P(RRRB), or more simply, just P(RRR). Some candidates tried to consider three cases, instead of six, namely (>1&1), (>2&2) and (4&3). This can lead to an elegant correct method, which a few candidates gave, that is P(R&B or RR&RB or RRR&RRB). But more commonly it led to an incorrect method, with overlapping probabilities e.g. $\frac{2}{3}\times\frac{1}{3}+(\frac{2}{3})^2\times\frac{1}{3}+(\frac{2}{3})^3\times\frac{1}{3}$. A few candidates used the neat method of $\frac{1}{2}$(1 - P(1&1 or 2&2 or2&3 or 4&4)), usually successfully, although some thought incorrectly that P(4&4) = $(\text{P(RRRR)})^2$ +$(\text{P(RRRB)})^2$

(iv) Most candidates answered this correctly. A few reproduced a rote answer such as 'The probability of success must be constant' or 'Each turn must not be affected by the previous turn'.

9) (i) This is a 'Show that' question and therefore each essential step needs to be given to gain both marks. Most candidates showed sufficient working and gained the marks. A few gave an inadequate response such as the following.

$a + b + a + 2b + a + 3b = 3a + 6b$.
$\Rightarrow 3a + 6b = 1$ (i.e. the given answer just copied).

The missing step was
$a + b + a + 2b + a + 3b = 1$.

Some candidates appeared not to understand the notation in the question, and persevered with an x through several steps. Others tried to work backwards from the given answer. These candidates appeared to have missed the essential point about a probability distribution, that $\Sigma p = 1$.

(ii) This question was well-answered by a fair number of candidates. A few candidates did not use the formula for $E(X)$ and so were unable to proceed. Some tried to work with an x, rather than the given values of x. Some tried to introduce binomial probabilities. Others found the correct equation $6a + 14b = \frac{5}{3}$, but did not proceed to combine this with the equation given in part (i). A few wrote both equations together but did not show any attempt to solve them. This who did attempt a solution sometimes made arithmetical errors.

4733 Probability & Statistics 2

General Comments:

As last year this paper was found accessible and many excellent scripts were seen, with a large number of candidates able to get correct numerical answers to most of the questions. There has been a steady increase in the number of candidates who can deal accurately with technical details such as continuity corrections or when to include the $\sqrt{n}$ divisor. Conclusions to hypothesis tests are now very often well stated, although it is again emphasised that it is wrong to conclude that there is evidence that the null hypothesis is true. The correct statement requires a double negative, for example 'there is insufficient evidence that the mean number of failures has been reduced'.

The least well-answered questions remain those requiring verbal answers. Many candidates attempt to write down an answer to a different question, presumably one that they have seen before; questions and mark schemes are not sympathetic to this approach.

As mentioned under question 2(ii), a substantial number of candidates write that, in order to use a Poisson distribution, the number of events occurring in a fixed interval must be constant. This is clearly incorrect.

There are persistent misunderstandings as to when the normal distribution can be assumed. Some think that all continuous distributions are normal; some think that any distribution that is not binomial or Poisson must be normal; some think that the central limit theorem applies only to distributions that are normal in the first place; some fail to realise that two different distributions are involved, the parent distribution and the distribution of the sample mean. In any case, candidates are advised to read each question carefully to identify whether a particular distribution is stated in the question.

This year's paper made it plainer than ever that many candidates a clearer understanding of the concept of a probability density function (even if they can often get marks for calculations).

This year there was an unexplained growth in the number of candidates who, in answering hypothesis tests based in the binomial or Poisson distributions, gave a method that could not be clearly identified, apparently half way between the critical region method and the probability method. This is hard to credit appropriately as well as being poor practice.

There was also an increase in the number of candidates using extra unnecessary continuity corrections, usually of the form $1/2n$. These are called for only when using a sample mean derived from a discrete distribution and then a normal approximation (thus *not* anywhere on this year's paper).

In order not to disadvantage candidates whose calculators do not give, for instance, Poisson probabilities directly, questions increasingly insist on working, such as 2(iii), which used the words 'Use the formula'. Candidates who put the numbers straight into their calculator get no marks here.

Comments on Individual Questions:

Question No.

Q.1 A very confident start to the paper by many. Fully correct answers were common, though inevitably there were some who made sign errors or who failed to use the tables in reverse.

Q.2(i) Candidates could usefully clarify their understanding of the issues involved in words such as ‘random’. The words ‘events occur randomly’ mean nothing more than that events occur at unpredictable intervals, or without a pattern. They carry no implication of independence, or constant average rate; those are extra assumptions needed for the Poisson distribution to be valid. Likewise, candidates who wrote that ‘events had to be independent and with equal probability’ were confusing the idea of *events occurring randomly* with the properties of a *random sample*. Candidates should focus on answering the question that is asked, and not one vaguely like it that they have seen before.

Q.2(ii) All that was needed here was to say that the instances of dead rabbits are independent of one another, so that if one occurs it does not affect that probability of others occurring. The constant average rate condition can be used, but it must be noted that constant rate (without the word average) is wrong. The substantial number of candidates who write quite explicitly that events must occur at exactly regular intervals, or that ‘there must always be the same number in any stretch of road’ (an exact quotation from a script) is deeply worrying; obviously there is a major misconception here.

Some candidates failed to explain what their statement meant, particularly if they had used the constant average rate condition. The best way of explaining this is to indicate that the mean number of events in any given length (of road, in this context) is proportional to the length.

Q.2(iii) Almost everyone got the right answer. However, as mentioned above, those candidates who went straight from Po(2.75) to the answer without showing the formula scored only 1 out of 3. Candidates with calculators that will do this must take careful note of the wording of the question: “Use an appropriate formula”.

Q.3(i) Most knew what to do here but there were frequent algebraic errors. Very common was a tendency for the a^3 factor to migrate from the bottom of the fraction to the top. Weaker candidates omitted the limits altogether.

Q.3(ii) This question had a number of poor answers. A number of candidates used the same limits (3 and –3) that they had just used in part (i). Many candidates forgot all about the mean, or failed to square it. As has been observed before, the attempt to use a single massive formula for the variance, including the square of the integral for the mean, is beyond most candidates and is not recommended.

Those who attempted to find the mean by integration often got it wrong; they should have been aware that $\mu = 0$ by symmetry, even if they were not confident enough to use 0 straight away. Various algebraic errors, such as the migration of a^3 mentioned in part (i), also hindered candidates.

Q.3(iii) This question confirmed the suspicion that only a small proportion of candidates have a clear idea as to what a probability density function is. The simple, correct answer, x is a value taken by X, was less often seen than answers to questions that have been asked in the past, such as ‘values at the extremities are more likely than those at the centre’. A statement such as ‘X is a continuous random variable’ seems to cause problems; candidates seem to think that X is something that does or does not happen, dependent upon the value of x. It might be helpful to focus on the idea that X is the output of a process – when the variable X is observed, what comes out is a number, one particular value of which is denoted by x.

Q.4(i) Weaker candidates struggled to see what was going on here, but the vast majority realised that they had to find P($\le$ 6) from the distribution B(90, 0.05), using the approximation Po(4.5). Many, however, omitted one or both parts of the necessary conditions for the approximation. Both *n* large (or "n > 50") and *p* small (or "np < 5") were needed. Some quoted spurious extra conditions such as nq > 5. If candidates choose to give numerical conditions, they must be those given in the specification, so that n > 30 loses a mark.

Some candidates with powerful calculators found P($\le$ 6) from the exact binomial distribution. This lost most of the marks. The specification includes knowledge and use of the approximations, and the question required use of an appropriate approximation.

Q.4(ii) Again, some candidates used the exact B(90, 0.35) and lost most of the marks. The vast majority, however, recognised both the initial distribution and the approximating distribution N(31.5, 20.475), and it was particularly pleasing that so many included the right continuity correction.

Q.5(i) This unfamiliar request was in fact answered well. About half the candidates wrote that it avoids (or reduces) bias, or that it ensures a representative sample (this latter statement is not really true but it was given credit). Sophisticated answers seen included 'use of random numbers allows distributions such as the binomial to be used'.

Q.5(ii) This was a standard binomial hypothesis test and many scored full marks, although some poor conclusions were seen. As usual, weaker candidates using the probability method attempted to use P(> 8) or P(= 8) as opposed to the correct P($\ge$ 8). It is not, however, sufficient to write down two probabilities, tell us that one is > 0.05 and one is < 0.05, and then say 'do not reject H_0'; it is not clear whether this is using the critical value method or the probability method. Using the probability method, only one probability can be given; using the critical region, an explicit statement and comparison such as CR is $\ge$ 9, and 8 < 9 is essential.

The final conclusion was usually well stated, though it is incorrect to say that 'there is evidence that the proportion of 1's *is* 25%'. A double negative is required: 'there is insufficient evidence that the proportion of 1's is not 25%'.

Q.6(i) Another standard question, if lengthy, and generally well answered. It is pleasing to note how few candidates gave their hypotheses in terms of the sample mean (H_0: μ = 11.76 instead of the correct H_0: μ = 11.0). Most, too, remembered to multiply the variance by 120/119. However, quite a few omitted the $\sqrt{120}$ in the denominator of the standardisation. Conclusions were well stated.

Q.6(ii) As so often, a question that asked whether the normal distribution had to be assumed was met with a range of bafflingly self-contradictory answers. 'Yes because we can use the central limit theorem' was probably typical. Perhaps the misunderstanding stems from what the word assume means, perhaps from a failure to distinguish between the two different distributions in the question. The question asked whether the *consultation times* (that is, the parent population) had to be normal, whereas the calculation involves the *sample mean*. The distribution of the parent population does *not* have to be normal, because the central limit theorem tells us that the distribution of the sample mean *is* (approximately) normal.

Q.7(i) Most candidates found this straightforward, although some made the usual mistakes in a Poisson hypothesis test (calculation of P(= 2) instead of P($\le$ 2) was the most common). Some, however, were confused by the rates given (1 per 3 days, what happens in 15 days?); the use of Po(5/3) was occasionally seen.

Q.7(ii) Generally very well done. Omission of the continuity correction (easier to forget in this context than in the more usual one) was the most common error, so that 38 was seen more often than the correct 37. Some used the wrong tail of the distribution, and several, having quoted the Normal-to-Poisson condition 50 > 15, then unfortunately used N(15, 15). Good candidates realised that they had to round down. Some who obtained 38 as their answer then very intelligently checked the corresponding probability using a continuity correction and changed their answer to 37.

Q.8(i) The vast majority of candidates got the correct answer, with a few going for $1 - P(\leq 8) = 0.0083$ instead of $1 - P(\leq 7) = 0.0315$. There is no particular need to give the answer as 3.15%, and it is wrong to correct this to a familiar number like 5%.

Q.8(ii) A challenging final question on the paper nevertheless received a pleasing number of completely correct answers. The usual error with this type of question is that candidates attempt to recalculate the critical region for a different value of *p* (trying to find a new set of values with a probability just below 0.05) instead of using the critical region given in the question (which remains the same whatever the value of *p*). Some found the complement of the probabilities of the Type II errors. Many, however, obtained both 0.6047 and 0.0933, and many ended with an appropriate binomial calculation. The usual failing among better candidates was the failure to calculate the probability that a single test results in a Type II error, which is $\frac{1}{3} \times 0.6047 + \frac{1}{3} \times 0.0933 = 0.2327$.

4734 Probability & Statistics 3

General Comments:

There were about 350 candidates, which is similar to recent years.
There was no evidence of candidates running short of time.
Conclusions to hypothesis tests were usually given in context and were not over-assertive.
The modal mark for every question, except Q5(ii), was full marks.
The modal mark for Q5(ii) was zero.

Comments on Individual Questions:

Question No.

1(i) Usually answered correctly, but many made errors in the variance - usually using 4^2 and 3^2.

1(ii) Usually answered correctly by those answering part (i) correctly. The others often scored two out of three for $\frac{x-28}{\sqrt{(i)}}$ and 2.326.

2 Usually answered correctly. Many did not pool the samples and usually scored five out of seven, as in SC2 in the RH column of the mark scheme.

3(i) The numerical part of the question was almost always answered correctly. Some candidates lost the mark(s) for the hypotheses and/or the assumption.

3(ii) This was quite a challenging question, but over 40% of the candidates scored full marks. Others lost marks for giving $\bar{d}$ =1-k, rather than 1+k, changing the variance to complicated expressions in k and sometimes changing the CV.

4(i) Most scored full marks. Others lost marks by using z(usually 1.96). A few did not answer to three decimal places.

4(ii) Another challenging question was generally well-answered. Most scored at least the first three marks, but could not convert t=1.637 to 80% confidence interval. 20% and 90% were common wrong answers.

5(i) Most candidates scored full marks. Many lost marks for using the wrong row of the Poisson table.

5(ii) Many candidates had no idea how to answer this question, often attempting a normal approximation. Of those who saw how to answer it, many did not pair A=0 with B=5 and usually scored two marks.

6(i)(ii)(iii) Almost all candidates answered these parts correctly.

6(iv) Just over one-third of the candidates scored full marks. The most common error was to use an incorrect CV, usually 7.815. These candidates usually scored four out of six. Many candidates scored the first three marks but then chose a CV from the wrong tail of the distribution, obviously confused by the small value of the test statistic.These candidates did not gain any more marks.

7(i)(ii) Almost all candidates answered these parts correctly.

7(iii) Most candidates scored full marks. The most common mistake was not to add 0.6 to $\frac{k}{2}\int_2^X (4-x)^2 dx$.

7(iv) Candidates whose used $\int (4-x)^2 dx = -\frac{(4-x)^3}{3}$ did better than those who tried to solve $0.05x^3-0.6x^2+2.4x-2.2=0.75$. Candidates with graphical calculators were at an advantage when using this method, but there were many correct solutions using Newton-Raphson and Iteration. Over half the candidates scored full marks.

4735 Probability & Statistics 4

General Comments:

There were 64 candidates, similar to recent years. As usual, many produced extremely good scripts. Approx. one-third scored 70 or more out of 72.

There was no evidence of candidates running out of time.

The modal mark for every part was full marks.

Comments on Individual Questions:

Question No.

1(i) Almost all gained full marks. Those who did not usually obtained the answer 0.5.

1(ii) Those who were correct in (i) were also correct here.

1(iii) Almost all the candidates earned this mark. Those who were incorrect in (i) usually scored the mark on follow through.

2 Some gave incorrect hypotheses. H_1:m<30 was quite common. The numerical part of the question was answered correctly by all but two or three candidates. Those who lost the first mark for the hypotheses usually lost the mark for the conclusion as well. Most candidates gained the mark for the assumption.

3(i) Almost always answered correctly. A few omitted 20C1.

3(ii) Almost always answered correctly.

4(i) Almost always answered correctly. Some turned the p.g.f. into a probability distribution and answered using techniques learned in S1.

4(ii) Almost all knew that they needed to find the square root of the p.g.f., but many were unable to do this.

4(iii) Almost all the candidates knew that they had to write down the coefficient of t^2.

5 Almost always answered correctly. Most used the product rule to find the second derivative.

6 Most scored at least eight or nine. The lost mark was usually due to there being no continuity correction.

7 Most scored full marks, but a significant minority assumed independence. They usually gained one mark in part (i), one in part (ii) and five in part (iii).

8(i) Most scored at least six out of eight. The marks lost were for answers to the variances not being simplified sufficiently. Some weaker candidates omitted the suffices and were only allowed method marks.

8(ii) Most scored full marks. There were many ingenious methods showing T_2 was the more efficient estimator, but all used $(n_1+n_2)^2$ at some stage. Weaker candidates sometimes gained the first mark, but no more.

4736 Decision Mathematics 1

General Comments:

Several excellent scripts but a number of candidates with almost illegible handwriting, in particular writing figures ambiguously or so small that they cannot be read, even when the script is enlarged. The vast majority of candidates were able to make an attempt every question. Few candidates needed additional sheets; additional sheets should only be used if a candidate cannot fit their answer in the space provided (having crossed through the attempt that has been replaced). When work is done on additional sheets, even if it is only rough work (crossed through), candidates should indicate the question and part number.

Comments on Individual Questions:

Question No. 1

Most candidates were able to get the result of the first and fourth passes through bubble sort and many recognised that in this case there was no early termination so 15 comparisons were needed in total. A few candidates seemed to think that a 'check pass' is always required at the end, instead of only as a mechanism for knowing when bubble sort terminates without carrying out every pass. Some candidates forgot that some values get fixed and passed through the entire list each time, giving a total of 25 comparisons instead of 15.

With shuttle sort, some candidates did not seen to understand what constituted a pass and went past the required results for the first and fourth passes, or counted comparisons that had not been needed. The majority of candidates realised that in this case shuttle sort was more efficient because it used fewer comparisons and some were able to support this with a correct calculation of the comparisons.

Question No. 2

The first part of this question concerned a spanning tree for a network from which a vertex and the arcs joined to it have been removed and asked under what circumstances the residue of the spanning tree will form a spanning tree for the reduced network. Several candidates chose to answer some other question - some candidates described the use of a reduced spanning tree to find a lower bound for the travelling salesperson problem, some described the conditions for a spanning tree but did not use the fact that they had started with a tree, and some worried about whether the vertices were odd or even. All that was needed was to ensure that the removed arcs did not split the tree into two (or more) parts, so the residue was still connected. For this to happen the removed vertex must have been connected to only one other vertex in the original tree.

The application of Kruskal's algorithm was usually correct, although some candidates used nearest neighbour or some mixture of Kruskal and nearest neighbour, and some did not give the total weight of the minimum-spanning tree. The method for the lower bound was usually correct, only a few candidates used the shortest arc twice instead of the two shortest arcs from the removed vertex. The nearest neighbour route usually started correctly but several candidates did not close their route to find a cycle (as specifically asked for in the question).

Question No. 3

Most candidates found the optimum by checking the value of P at each vertex, a few gave the optimal value of P without saying which was the optimum vertex or gave the optimum vertex without giving the corresponding optimal value of P. In part (ii), some candidates gave integer valued points that were not in the feasible region.

In part (iii) it was not enough to note that the points A and B have larger values of y than C and D, the x values needed to be considered as well. Some candidates only explained why the optimum could not be at C (instead of not being at C or at D). Part (iv) was best done using an algebraic method to compare the expression for P at A with that at B and at D.

Question No. 4

Many correct answers. Some candidates could not find all the constraints and tried to use $y \leq 5$ (or sometimes $y \geq 5$), even so they were often able to gain credit for what they did know how to do and were not severely penalised for early errors. In part (iv), candidates were asked to write down the values of x, y and z from the given tableau, and to advise on how many acres of each crop to plant and how many acres to leave unplanted. Some candidates only gave the values of x and z, some did not give the number of acres left unplanted and some assumed that acres (which they knew were units of area) were discrete quantities that could only take integer values.

Question No. 5

Answered well when candidates had read the questions carefully. In parts (i) and (ii) the routes were not always given, although the lengths of the routes were usually stated. In part (iv) some candidates just gave the length of the shortest route but did not show their working to reject the other ways of pairing the odd nodes. In part (v), several candidates gave distances that were too long or that missed out an arc. Nadia started at E and needed to travel twice along the arcs that have not yet been used, this gave a distance of 2x(150+150+150+200+270+200+100) with Nadia ending at E again. She then needs to return to A having used AC = 500 on the way. The shortest way to get from E to C is to use the arc EC = 80, so this gives a total distance of 2440+580=3020 metres.

Question No. 6

There were many good answers, and some very confused ones, in particular in part (iv). In part (v) some candidates seemed to assume that the subgraph had to be connected, whereas the issue was that even with G as a separate loop there was no valid way to deal with Y, B and R. Quite a few candidates achieved the two possible colourings in part (vii).

Outside an examination context, the complete solution of the puzzle would then just require the consideration of which way up to put each cube to deal with the two long sides of the tower.

4737 Decision Mathematics 2

General Comments:

Many excellent scripts but a few candidates with almost illegible handwriting, in particular writing figures ambiguously or so small that they cannot be read, even when the script is enlarged. The vast majority of candidates were able to make an attempt at every question. Fewer candidates used additional sheets than on previous papers, although some candidates used additional sheets for rough working even when there was plenty of space in the answer booklet. Candidates should always attempt their answer in the printed answer booklet first and only use additional sheets if they cannot fit their answer in the space provided (having crossed through the attempt that has been replaced). When work is done on additional sheets, even if it is only rough work (crossed through), candidates should indicate the question and part number.

Comments on Individual Questions:

Question No. 1
Several fully correct answers. Most candidates were able to draw the bipartite graph and the incomplete matching. A few candidates answered some parts of the question in the wrong answer space, but when this happened they usually indicated which part they were answering. Some candidates appeared to have used coloured pens to answer part (ii); these do not show up when the scripts are scanned. In part (iii) some candidates had used a longer path than was necessary, the question asked for the shortest alternating path and candidates should have written this path as well as the resulting incomplete matching. A few candidates assumed the stem to part (iv) when answering part (iii), and started their alternating path from the statement that *E* was paired with *S* (instead of *M* for the shortest path).

Question No. 2
Most candidates were able to list the immediate predecessors, apart from activity *H* (which appeared to be confused with the end vertex by some candidates, and was assumed to have all the activities as immediate predecessors by others). The forward pass was usually correct but some candidates did not deal correctly with decisions on the backward pass, particularly when a dummy activity was involved. Several candidates did not take the clue that the answer space for (iv)(b) was tiny because only a brief answer was wanted – the easiest approach was to recognise that the total time needed for the activities was 28 hours so 2 workers could not possibly complete the project in 13 hours. Some candidates realised that, using the early start times, the first time at which two workers were not enough was at 5 hours, but they did not follow through the implications of delaying, say, activity *D* on subsequent activities. Starting from the end, completion in 13 hours requires *F*, *G* and *H* overlapping for at least one hour between time 9 hours and the end. Many answers were vague or just repeated the wording in the question. In part (v) the delay could be modelled as an activity bolted on to the front of activity *C* and lasting 2 hours, this meant that the critical activities are now *B*, *C* and *F* taking 14 hours. Some candidates did not say that *B* was still critical - or thought that *B* was no longer critical, having not appreciated that the timing of *B* affects the new (delayed) start time of *C*.

Question No. 3
The Hungarian algorithm in part (i) was done well by most candidates. A few candidates made numerical slips, and some reduced rows first even though the instruction to reduce columns first was in bold in the question. Some candidates augmented past the solution, crossing out using four or even five lines and then continuing to augment. In part (ii), some candidates gave the new total time instead of the increase (‘how long’ instead of ‘how much longer’). In part (iii), the reduced matrix needed to be seen before finding the new solution.

Question No. 4
Most candidates were able to draw a correct network, although these were not always shown with directed arcs and sometimes the network joined back into itself instead of having two vertices for Jeremy's home ((0; 0) and (4; 0) or *J* and *J*', for example). There were several correct dynamic programming tabulations, and quite a few that were correct apart from the labelling of the actions. The action value is the state number for the stage above. For example, at (2; 2) (which represents *P*) the actions are action 1 (corresponding to the arc from (2; 2) to (3; 1), which is arc *PS*) and action 2 (corresponding to the arc from (2; 2) to (3; 2), which is arc *PT*). The dynamic programming tabulation should work backwards (from Monday to Sunday to Saturday to Friday) starting with stage 3 (actions from states in stage 3 to states in stage 4). Candidates who solved a maximisation problem but worked forwards (from Friday through to Monday) were able to gain partial credit.

Question No. 5
Some candidates were not able to interpret the information represented by the labelling procedure, but most realised that the flow along the route was 3 litres per second. Some candidates only wrote 3 *l* (meaning 3 litres) and this could be confused with the number 31. Parts (ii) and (iii) were often done well, and candidates often identified the maximum flow as being 13 litres per second (although some though that it was 15). The cut was quite difficult to find, and was best described by listing the set of vertices on the source side and the set of vertices on the sink side of the cut. The usual misconception that the value of a cut can be calculated from the flows rather than the capacities was seen quite a few times. In part (v), the arc represents a vertex restriction and means that at most 2 litres per second flows through vertex E. Finally, the diagram showing the flow in part (vi) needed the vertices E_1 and E_2 merged into a single vertex E with directed arcs entering E from S, D and F and leaving E to C, G and T.

Question No. 6
The calculation of the excesses in part (i) was usually correct, but in part (ii) the interpretation of the value (-5) required recognising that, after the game, R has 5000 fewer soldiers than C (rather than having lost 5000) and, correspondingly, C has 5000 more than R (rather than having 'gained' 5000). Most candidates were able to identify the play-safe strategies and recognise that column 1 could be removed using dominance. Answers to parts (v) to (viii) were sometimes vague or muddled, although there were several good attempts, particularly at part (vii). Candidates needed to be able to translate between the mathematical model and the physical situation to describe what the various strategy choices represented (in terms of how many divisions are sent North) and to interpret the values of m and M in the various cases discussed.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

PART OF THE CAMBRIDGE ASSESSMENT GROUP