2016 Suite

OCR
Oxford Cambridge and RSA

Cambridge **TECHNICALS LEVEL 3**

Cambridge TECHNICALS 2016

ENGINEERING

Unit 1

Mathematics for engineering

L/506/7266

Guided learning hours: 60

Version 3 March 2018 - black lines mark updates

ocr.org.uk/engineering

LEVEL 3

UNIT 1: MATHEMATICS FOR ENGINEERING

L/506/7266

Guided learning hours: 60

Essential resources required for this unit: Formula Booklet for Level 3 Cambridge Technicals in Engineering, scientific calculator and a ruler (cm/mm)

This unit is externally assessed by an OCR set and marked examination.

UNIT AIM

Mathematics is one of the fundamental tools of the engineer. It underpins every branch of engineering and the calculations involved are needed to apply almost every engineering skill.

This unit will develop learners' knowledge and understanding of the mathematical techniques commonly used to solve a range of engineering problems.

By completing this unit learners will develop an understanding of:

- algebra relevant to engineering problems
- the use of geometry and graphs in the context of engineering problems
- exponentials and logarithms related to engineering problems
- the use of trigonometry in the context of engineering problems
- calculus relevant to engineering problems
- how statistics and probability are applied in the context of engineering problems

TEACHING CONTENT

The teaching content in every unit states what has to be taught to ensure that learners are able to access the highest grades.
Anything which follows an i.e. details what must be taught as part of that area of content. Anything which follows an e.g. is illustrative.

For externally assessed units, where the teaching content column contains i.e. and e.g. under specific areas of content, the following rules will be adhered to when we set questions for an exam:

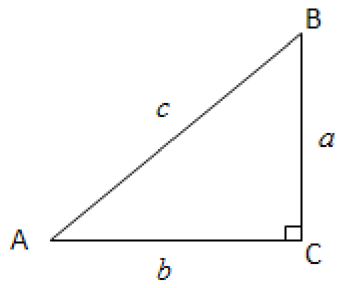
- a direct question may be asked about unit content which follows an i.e.
- where unit content is shown as an e.g. a direct question will not be asked about that example.

Learning outcomes	Teaching content	Exemplification
The Learner will:	**Learners must be taught:**	
1. Understand the application of algebra relevant to engineering problems	1.1 application of algebra i.e. • multiplication by constant • binomial expansion • removing a common factor • factorisation • using the principle of the lowest common multiple (LCM)	Learners should understand the rules of algebra to simplify and solve mathematical problems for example: • $5(3+x)=15+5x$ • $(x+3)(x+2)=x^2+5x+6$ • $(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$ • $bx+by=b(x+y)$ • $x^2+5x+6=(x+3)(x+2)$ • $\frac{x+2}{5}+\frac{x+4}{3}=\frac{8x+26}{15}$ using a LCM of 15
	1.2 simplification of polynomials i.e. • factorising a cubic • algebraic division • the remainder and factor theorems	Many engineering problems can be described by polynomials. Learners should be taught how to simplify polynomials containing cubic terms for example: $2x^3-x^2-8x-4=(x+2)(2x+1)(x-2)$ An equation is a statement that two algebraic expressions are equal and the process of finding the unknown is called solving the equation.

Learning outcomes	Teaching content	Exemplification
The Learner will:	**Learners must be taught:**	
	1.3 how to simplify and solve equations 1.4 transposition of formulae i.e. • containing two like terms • containing a root or a power	Learners should be taught to simplify and solve equations for example: • $5(x-3)-7(6-x)=12-3(8-x)$ leading to a solution that $x=5$ • given $E=\frac{mv^2}{2g}$ find v • given $T=2\pi\sqrt{\frac{k^2}{gh}}$ find K • given $Mv+mu=MV+mU$ find M or m
	1.5 how to solve linear simultaneous equations with two unknowns using: • graphical interpretation • algebraic method, i.e.: ○ elimination method ○ substitution method	Engineering problems are often described using simultaneous equations. Learners should be taught to solve simultaneous equations graphically and by calculation for example: • electrical engineering problems using Kirchhoff's laws • forces in a mechanical system using $0.7F_1+0.5F_2=9$ and $0.3F_1+0.4F_2=5$ • state that when two equations contain two unknowns such as $2x+5y=10$ and $x+2y=3$, such that only one value of x and y exist that will satisfy both equations, are called simultaneous equations

Learning outcomes **The Learner will:**	Teaching content **Learners must be taught:**	Exemplification
	1.6 how to solve quadratic equations i.e. • sketching of quadratic graphs • factorisation method • completing the squares • using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Engineering problems can often be described using quadratic equations. Learners should be taught to solve quadratic equations for example: • bending moment (M) of beams $M = 0.3x^2 + 0.35x - 2.6$ • fabrication of steel boxes when the volume of the box is $2(x-4)(x-4)$ where "x" is a required dimension • equations of motion $v = u + at$ $s = \frac{1}{2}(u+v)t$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$
2 Be able to use geometry and graphs in the context of engineering problems	2.1 how to use co-ordinate geometry i.e. • straight line equations i.e. ○ equation of a line through two points ○ gradient of parallel lines ○ gradient of perpendicular lines ○ mid-point of a line ○ distance between two points • curve sketching i.e. ○ graphs of $y = kx^n$ ○ graphical solution of cubic functions	The behaviour of engineering systems can be described using straight line equations. Learners should be taught how to solve problems using straight line equations for example: • force vs displacement for a linear spring or spring buffer • electrical problems using Ohm's law Learners should be taught to sketch mathematical functions in order to visualise (and sometimes to solve) problems for example: • $y = -3x^2$ • $f(x) = x(x-1)(2x+1)$ • $m(x) = (2-x)^3$ This might present an opportunity for the use of ICT e.g. spreadsheets to plot and solve cubic functions using trend lines.

Learning outcomes **The Learner will:**	Teaching content **Learners must be taught:**	Exemplification
	• graphical transformations i.e. o translation by addition o transformation by multiplication i.e.: - stretches - reflections	Learners should be taught graphical transformations for example: • translation in the y direction by adding a whole number to the whole function • translation in the x direction by adding a whole number to x • multiplying the whole function by a whole number • multiplying x by a whole number
3 Understand exponentials and logarithms related to engineering problems	3.1 problem solving using exponentials and logarithms i.e. • $y = e^{ax}$ • $y = e^{-ax}$ • $e^{y} = x$ • $\ln x = y$ 3.2 how to use inverse function and log laws	Learners should be taught how to solve problems involving exponential growth and decay including use of the exponential and logarithmic functions and the log laws. Learners should be taught both how to produce and interpret sketch graphs showing exponential growth and decay. Many engineering systems and devices can be characterised, and problems solved using exponentials and logarithms for example: • voltage and current growth in capacitor circuits (RC circuits) $V_C = V_S(1 - e^{-\frac{t}{RC}})$ • voltage and current decay in capacitor circuits (RC circuits) $V_C = V_S e^{-\frac{t}{RC}}$ • stress-strain curves for certain engineering materials $\sigma = K e^{n}$ Learners should be taught how to use inverse function and log laws for example: • $y = \sin x \Rightarrow x = \sin^{-1} y$ • $y = e^{x} \Rightarrow x = \ln y$ • $\log(ab) = \log a + \log b$ • $\log(a^{b}) = b \log a$

Learning outcomes	Teaching content	Exemplification
The Learner will:	**Learners must be taught:**	
4. Be able to use trigonometry in the context of engineering problems	4.1 angles and radians i.e. • define the terms angle and radian • the formulae x radians = $180°x/\pi$ degrees x degrees = $\pi x/180$ radians	Learners should be taught to solve problems involving angles and radians for example: • a wheel rotating at the rate of 54 revolutions per minute. Determine the angular speed in radians per minute • a shaft rotating at 100 revolutions per minute. Express this in radians per second
	4.2 problem solving with arcs, circles and sectors i.e. • the formula for the length of an arc of a circle • the formula for the area of a sector of a circle • the co-ordinate equation of a circle $(x-a)^2+(y-b)^2=r^2$ to determine: ○ centre of the circle ○ radius of the circle	Learners should be taught to solve problems involving arcs, circles and sectors in an engineering context e.g. calculating the length of a braking surface based on the radius of the arc of the brake lining and the angle subtended. • **length of arc** $S=\theta r$ and $S=\dfrac{\pi r\theta^o}{180}$ • **Area of sector** $A=\dfrac{r^2\theta}{2}$ and $A=\dfrac{2\pi r^2\theta^o}{360}$
	4.3 problem solving involving right-angled triangles i.e. • what is meant by the term "solution of a triangle" • Pythagoras' Theorem • use of sine, cosine and tangent rule for right-angled triangles • the formulae for the area of a right-angled triangle	Learners should be taught to solve problems involving right-angled triangles in an engineering context for example: $c^2=a^2+b^2$ $\sin A=\dfrac{a}{c}$ $\cos A=\dfrac{b}{c}$ $\tan A=\dfrac{a}{b}$ Area = $\dfrac{ab}{2}$ 

Learning outcomes	Teaching content	Exemplification
The Learner will:	**Learners must be taught:**	
	4.4 problem solving involving non-right angled triangles i.e. • sine rule • cosine rule • area	Learners should be taught to solve problems involving non-right angled triangles for example: • **lengths and angles**: ○ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ ○ $a^2 = b^2 + c^2 - 2bc\cos A$
	4.5 common trigonometric values i.e. • $\sin 60^o = \frac{\sqrt{3}}{2}$ • $\cos 60^o = \frac{1}{2}$ • $\tan 60^o = \sqrt{3}$ • $\tan 45^o = 1$ • $\sin 45^o = \cos 45^o = \frac{1}{\sqrt{2}}$ • $\sin 30^o = \frac{1}{2}$ • $\cos 30^o = \frac{\sqrt{3}}{2}$ • $\tan 30^o = \frac{1}{\sqrt{3}}$	where A, B and C are angles within the triangle and a, b, and c are the lengths of the three sides • **area**: ○ $Area = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height ○ $Area = \frac{1}{2}bc\sin A$ where b and c are the lengths of two sides and A is the angle opposite the third side ○ $Area = \sqrt{s(s-a)(s-b)(s-c)}$ where a, b, and c are the lengths of the sides of the triangle and $s = \frac{1}{2}(a+b+c)$

Learning outcomes The Learner will:	Teaching content Learners must be taught:	Exemplification
	4.6 common trigonometric identities i.e. • $\sin A = \cos(90 - A)$ • $\cos A = \sin(90 - A)$) • $\tan A = \dfrac{\sin A}{\cos A}$ • $\sin^2 A + \cos^2 A = 1$	
	4.7 sine, cosine and tangent operations i.e. • graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for a range of angles for 0^0 to 360^0 • determine the sine, cosine and tangent of any angle between 0^0 and 360^0	Learners should be taught to interpret and produce graphs from sine, cosine and tangent for example: • An alternating e.m.f. is represented by $v = 25 \sin x$. Determine the value of v when x equals (a) 30° ,(b) 60°, (c) 90° , (d) 180° (e) 210°, and (f) 270°
5 Understand calculus relevant to engineering problems	5.1 problem solving involving differentiation i.e. • determine gradients of a simple curve using graphical methods • the rule to differentiate simple algebraic functions • determine the maximum and minimum turning points and the co-ordinates of the turning points by differentiating the equation twice	Learners should be taught to solve problems involving differentiation for example: • given that the surface area S of a cylindrical water tank is given by $S = 2\pi(r^2 + \dfrac{6750}{r})$, calculate the dimensions of the tank so that its total surface area is a minimum. • given that an alternating voltage $v = 20 \sin 50t$ where v is in volts and t in seconds, calculate the rate of change of voltage at a given time • differentiate displacement to get velocity • differentiate velocity to get acceleration, where possible problems should be presented in an engineering context

Learning outcomes	Teaching content	Exemplification
The Learner will:	**Learners must be taught:**	
	• differentiate functions of the form $y = x^n$ $y = \sin ax$ $y = \cos ax$ $y = \tan ax$ $y = e^{ax}$ $y = \ln ax$ $y = a^x$ $y = \log_a x$	• Learners should be taught how to draw a graph and derive the differentiation of $\sin ax$ and $\cos ax$ • Learners should be taught to solve problems involving exponentials and logarithms e.g. if $y = ae^{bx}$ then $\frac{dy}{dx} = bae^{bx}$ if $y = ae^{-bx}$ then $\frac{dy}{dx} = -bae^{-bx}$ if $y = \ln x$ then $\frac{dy}{dx} = \frac{1}{x}$ if $y = \ln 3x$ then $\frac{dy}{dx} = \frac{1}{x}$ if $y = 4\ln 2x$ then $\frac{dy}{dx} = \frac{4}{x}$
	5.2 solve problems involving indefinite integration i.e. • define indefinite integration • recognise the symbol $\int$ for integration • the rule to integrate functions of the form $\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$ for $n \neq 1$ $\int \frac{1}{x} dx = \ln\|x\| + C$ $\int e^{ax} dx = \frac{e^{ax}}{a} + C$ $\int \sin ax dx = \frac{-\cos ax}{a} + C$ $\int \cos ax dx = \frac{\sin ax}{a} + C$	Problems using indefinite integrals for example: Indefinite integration is the reverse process to differentiation and state that an indefinite integral does not reveal a calculated value. Integrate the following example functions with respect to x ○ $x^3 + 3x^2 + x$ ○ $x^{1.4} + \frac{1}{x^3}$ ○ $6x^4 + \sqrt{x} - e^x$ ○ $\sin 3x + \cos 2x$ $\int_2^4 6x dx = [3x^2 + C]_2^4$ The numerical values of 2 and 4 mean that x = 2 and x = 4. When x = 4, integral $= 3x^2 + C = 48 + C$ When x = 2, integral = $3x^2 + C = 12 + C$ So $\int_2^4 6x dx = (48 + C) - (12 + C) = 36$; i.e. $\int_2^4 6x dx = 36$

Learning outcomes The Learner will:	Teaching content Learners must be taught:	Exemplification
	5.3 problem solving involving definite integrals i.e. • the rule for a definite integral • the notation for definite integration $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$ • the interpretation of a definite integral • integrate functions of the form i.e. $\int_a^b x^n dx$ $\int_a^b \sin x dx$ $\int_a^b \cos x dx$ $\int_a^b e^{ax} dx$ $\int_a^b \frac{1}{x} dx$	• Awareness that in all calculations for definite integrals the constant C will disappear when an upper and lower limit are given • The evaluation of integrals such as: ○ $\int_0^2 4x dx$ ○ $\int_1^2 3x + 2\ dx$ ○ $\int_1^4 2x^3 + x^2\ dx$ Interpretation of a definite integral that it represents the area between the function f(x) and the x axis between the limits given Learners should be taught to solve problems using definite integrals e.g. • Find the area between the line y = x and the x axis between the values x = 0 and x =10 $\int_0^{10} x dx = \left[\frac{x^2}{2}\right]_0^{10} = \frac{10^2}{2} - 0 = 50$ square units
6 Be able to apply statistics and probability in the context of engineering problems	6.1 the terms “data handling” and “sampling”	Statistics and probability are often used in engineering in the areas of quality control, component and system reliability and reliability-centred maintenance. Learners should be taught statistics in the context of engineering problems where possible e.g. • The diameters of 30 components were measured in millimetres with a micrometer, with the following results: 5.8 6.2 6.0 6.2 etc Construct a table showing a tally diagram and then draw a (a) histogram (b) frequency polygon and (c) cumulative frequency diagram • The tensile strength for 15 samples of tin are: 34.16 34.75 34.04 etc

Learning outcomes	Teaching content	Exemplification
The Learner will:	**Learners must be taught:**	
	6.2 problem solving involving histograms, frequency polygons and cumulative frequency curves 6.3 problem solving for a set of data i.e. • normal distribution • arithmetic mean • mode • median • percentiles • quartiles • distribution curve • positive skew • negative skew • variance • standard deviation	Determine the mean, mode and median • In a study exercise components being assembled by a group of technicians were timed in seconds as shown: 56 61 68 59 etc Construct a histogram and a frequency polygon to represent the data. Determine the (a) median (b) lower quartile and (c) the upper quartile.

Learning outcomes	Teaching content	Exemplification
The Learner will:	**Learners must be taught:**	
	6.4 problem solving using probability i.e.: • expectation • dependent event without replacement • independent event with replacement 6.5 the addition law of probability and the multiplication law of probability	**Probability** • The probability of a resistor failing in one year due to excessive temperature is 1/25, due to excessive vibration is 1/30 and due to excessive humidity is 1/55. Determine the probabilities that over one year a resistor fails due to excessive (a) temperature and vibration (b) vibration or humidity • How Venn diagrams are used to calculate probability • The expectation of an event happening is defined as the product of the probability of an event happening and the number of attempts made • Two events, A and B, are independent if the fact that A occurring does not affect the probability of B occurring • Two events are dependent if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed • With Replacement: the events are Independent - the chances don't change. • Without Replacement: the events are dependent - the chances change

ASSESSMENT GUIDANCE

All Learning Outcomes are assessed through externally set written examination papers, worth a maximum of 60 marks and 1 hour and 30 minutes in duration.

Learners should study the design requirements, influences and user needs within the taught content in the context of a range of real engineered products. Exam papers for this unit will use engineered products as the focus for some questions, however it is not a requirement of this unit for learners to have any detailed prior knowledge or understanding of particular products used. Questions will provide sufficient product information to be used, applied and interpreted in relation to the taught content. During the external assessment, learners will be expected to demonstrate their understanding through questions that require the skills of analysis and evaluation in particular contexts.

LEARNING OUTCOME WEIGHTINGS

Each learning outcome in this unit has been given a percentage weighting. This reflects the size and demand of the content you need to cover and its contribution to the overall understanding of this unit. See table below:

LO1	30-40%
LO2	10-20%
LO3	5-15%
LO4	10-25%
LO5	10-20%
LO6	10-20%

To find out more
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