

Cambridge TECHNICALS 2016

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Cambridge **TECHNICALS LEVEL 3**

ENGINEERING

Unit 23

Applied Mathematics for Engineering $\tau = k \cdot (\chi)$

R/506/7270 Guided learning hours: 60 Version 2

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LEVEL 3

UNIT 23: Applied mathematics for engineering

R/506/7270

Guided learning hours: 60

Essential resources required for this unit: Formula Booklet for Level 3 Cambridge Technicals in Engineering, scientific calculator and a ruler (cm/mm)

This unit is externally assessed by an OCR set and marked examination.

UNIT AIM

Once the key mathematical techniques needed for engineering are learnt, they need to be applied to engineering problems. Understanding mathematics in an applied engineering context is what distinguishes the engineer from the pure mathematician.

The aim of this unit is to extend and apply the knowledge of the learner gained in Unit 1 Mathematics for engineering. It is therefore strongly recommended that learners have completed Unit 1 Mathematics for engineering prior to commencing the study of this unit.

By completing this unit learners will:

- be able to apply trigonometry and geometry to a range of engineering situations.
- be able to apply knowledge of algebra, equations, functions and graphs to engineering problems.
- be able to use calculus to analyse a range of problems.
- understand applications of matrix and vector methods.
- be able to apply mathematical modelling skills.

TEACHING CONTENT

The teaching content in every unit states what has to be taught to ensure that learners are able to access the highest grades. Anything which follows an i.e. details what must be taught as part of that area of content. Anything which follows an e.g. is illustrative.

For externally assessed units, where the teaching content column contains i.e. and e.g. under specific areas of content, the following rules will be adhered to when we set questions for an exam:

- a direct question may be asked about unit content which follows an i.e.
- where unit content is shown as an e.g. a direct question will not be asked about that example.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
 Be able to apply trigonometry and geometry to a range of engineering situations 	 1.1 how to decompose composite shapes into triangles, circles, circle segments and other shapes 	Learners should be taught how to use and apply standard formulae to solve engineering problems for example: Calculate the perimeter of a cam:
	 1.2 the use of standard formulae to calculate the volume and surface area of solids with straight and curved sides i.e. prisms spheres cones cylinders rectangular pyramids 	Tangential at these points

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
The Learner will:	 Learners must be taught: 1.3 the concepts of frequency, amplitude and phase angle in periodic functions 1.4 the principle of simple harmonic motion 	 Calculate the surface area of a funnel:
	1.5 how to apply common trigonometric identities i.e. • $\tan A = \sin A / \cos A$ • $\cos^2 A + \sin^2 A = 1$ • $\cos(-A) = \cos A$ • $\sin(-A) = -\sin A$ • $\sin A = \cos(A - \pi/2)$ • $\sin(2A) = 2\sin A \cos A$ • $\cos(2A) = \cos^2 A - \sin^2 A$	 Express the AC voltage 25 sin(2πft + π / 4) as A(cos α + sin α) and determine A and α. Express a sin θ + b cos θ in terms of A sin(θ + α) and determine values of A and α for given values of a and b. Learners must be able to use trigonometric identities given in the list provided.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	1.6 how to determine relationships between angles and lengths in given geometric configurations	• For the following diagram show that: $\theta_2 = \tan^{-1} \left(\frac{3 - \sin \theta_1}{2 + \cos \theta_1} \right)$ 3 m C Pulley θ_2 Pulley θ_2 Counterweight θ_1 B Concrete block 200g
 Be able to apply knowledge of algebra, equations, functions and graphs to engineering problems 	 2.1 how to evaluate composite expressions including polynomials, trigonometric terms, exponential terms, logarithmic terms and terms involving negative and fractional powers 2.2 how to use indices and logarithms with different bases 2.3 how to manipulate and rearrange algebraic equations using fundamental laws of algebra 	 Learners should be taught how to analyse the mathematics associated with engineering problems for example: The displacement, <i>x</i>, of a mass in a particular arrangement with a spring and a damper is given by: x = e^{-αt} (A cos βt + B sin βt) Calculate <i>x</i> given specific values of <i>A</i>, <i>B</i>, α, β and <i>t</i>. Express aⁿ/a^m in terms of a^{n-m}. Express log_a Aⁿ in terms of n log_a A.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	2.4 how to solve equations involving one unknown using basic algebraic manipulation and evaluation	Teaching should be set in engineering contexts : • In crank mechanism the distance, <i>x</i> , between the centre of the crankshaft and the centre of the gudgeon pin is given by $x = r \cos \omega t + l \sqrt{1 - \frac{r^2}{l^2} \sin^2 \omega t}$ Rearrange the formula to make <i>l</i> the subject. • In a particular electrical circuit involving two resistors with resistances $r \Omega$ and $2r \Omega$, the following relationship holds: $\frac{1}{5} = \frac{1}{r} + \frac{1}{2r}$ Calculate the value of <i>r</i> .
	2.5 how to solve quadratic equations by factorisation, completing the square and by using the standard formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 2.6 how to solve linear simultaneous equations with up to two unknowns	Quadratic equations should be taught in an engineering context. (Learners should understand that a negative discriminate leads to a complex result.) Teaching should be set in engineering contexts, e.g.: • In an electrical circuit involving currents I_1 and I_2 the following equations are satisfied: $12I_1 + 6I_2 = 11$ $6I_1 + 9I_2 = 8$ Calculate the values of I_1 and I_2 .

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	2.7 how to draw graphs of functions of a single independent variable	 The speed, v m s⁻¹, of a car t s after the brakes have been applied in is modelled by the equation: v = e^{-t}(20+t) Sketch a graph of v against t for t between 0 and 10.
	2.8 how to plot graphs given numerical data and interpret values from graphs	• The relationship between the power coefficient C_p and the tip speed ratio λ of a particular wind turbine is summarised in the following table.
		λ 4 6 8 10 12
		Sketch a graph of C_p against λ and estimate the value of C_p when $\lambda = 5$.
	2.9 how to calculate constants in functions of a known general form given sufficient numerical information	• Calculate constants <i>a</i> , <i>b</i> and <i>c</i> in the following functions y = ax + c $y = ax^{2} + bx + c$
		$y = a \sin(bx + c)$ given values of y for corresponding values of x.
	2.10 how to derive equations of straight lines that are tangential to and normal to a given function at a given point	 The gradient of a roller coaster track at a point with coordinates (30, 20) is -0.35. Find the equations of the straight lines that are: (i) tangential to (ii) normal to this point.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	2.11 how to represent inequality relationships in graphical form	
	2.12 how to identify poles, zeroes and asymptotes of functions	Teaching should be set in engineering contexts for example:
	2.13 how to express functions that have a polynomial denominator as a sum of partial fractions	• The transfer function of a particular dynamic system is expressed as $Y(s) = \frac{1}{s(s^2 + 5s + 4)}.$
	 2.14 the principles of complex numbers i.e. know that j = √-1 calculate powers of j express complex numbers in the form z = a + jb plot z = a + jb on an Argand diagram express z = a + jb in the form r(cos θ + jsin θ) express z = a + jb in the form re^{jθ} determine the conjugate of a complex number simplify complex expressions manipulate expressions and solve equations involving complex numbers 	Express this function as a sum of partial fractions. • The transfer function associated with a particular electrical circuit is given by $\frac{8(j+1)}{j^2(j0.5+1)}$ Express this in terms of (i) $a + jb$ (ii) $r(\cos \theta + j\sin \theta)$ (iii) $re^{j\theta}$ and plot this on an Argand diagram.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
3. Be able to use calculus to analyse a range of	3.1 Learners will be taught standard derivatives i.e.	Where standard derivatives are required in the examination, formulae will be provided.
problems	$f(x)$ $\frac{df(x)}{dx}$ C 0 x^n nx^{n-1} $sin(ax)$ $a cos(ax)$ $cos(ax)$ $-a sin(ax)$ e^{ax} ae^{ax} $\ln(ax)$ $\frac{1}{x}$ $\log_a x$ $\frac{1}{x \ln a}$ $tan(x)$ $sec^2 x$ 3.2how to apply differentiation methods to a range of functions and applications i.e.•differentiation of functions containing trigonometric, exponential and logarithmic terms•differentiation of functions involving products, quotients and functions of a function•identification of stationary points of a function•identification of stationary points of a function•using second derivatives to identify local maximum and minimum values	 Learners must be able to apply differentiation theory to a range of problems for example: Identify the coordinates of the stationary points of the function y = 2x³ + 3x² - 36x + 12. The height, h m, of a projectile above sea level t s after it has been projected from the top of a cliff is given by h = -4.9t² + 25t + 20. Calculate maximum height of the projectile. Determine the coordinates of any local maximum and minimum points of the following function. y = 2x³ + 3x² - 36x + 12

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	3.3 how to relate first and second order derivatives to physical rates of change such as speed and acceleration	• If x is distance, v is speed and a is acceleration, each expressed as a function of time t, express v and a in terms of derivatives with respect to t i.e. $v = \frac{dx}{dt} \text{ and } a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
	3.4 Learners will be taught standard integrals i.e. • $\int a dx = ax + C$ • $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$ $\int \frac{1}{x} dx = \ln x + C$ • $\int e^{ax} dx = \frac{e^{ax}}{a} + C$ • $\int e^{ax} dx = \frac{a^x}{a} + C$ • $\int sinax dx = \frac{-\cos ax}{a} + C$ • $\int cos ax dx = \frac{\sin ax}{a} + C$	Where standard integrals are required in the examination, formulae will be provided.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	 3.5 how to apply integration methods to a range of functions and applications i.e. integration of a range of algebraic functions including those containing trigonometric and exponential terms integration by parts using the formula ∫uv'dx = uv - ∫u'vdx integration by substitution integration using partial fractions how to calculate the value of definite integrals and apply this to the calculation of areas, volumes and other physical properties ∫_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a) 	Learners must be able to apply integration techniques to a range of problems within an applied engineering context. Examples of integral problems: • $\int x^3 + e^{x/2} - \sin 3x dx$ • $\int \frac{2}{x^2} - \frac{1}{x} dx$ • $\int x \cos ax dx$ • $\int e^x \cos x dx$ • $\int (3x-2)^3 dx = \int u^3(\frac{1}{3}du)$ where $u = (3x-2)$ • $\int \frac{x+1}{x^2-3x+2} dx$ • Evaluate the following. $\int_{1}^{2} 3x^2 + x + 1 dx$ • Given that the volume of rotation $V = 2\pi \int_{a}^{b} xy dx$ calculate V when $y = e^x$, $a = 1$ and $b = 2$.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	 3.6 how to solve simple differential equations by direct integration and separation of variables 3.7 how to use initial conditions to evaluate constants of integration in the general solution of differential equations 	• Examples of differential equation problems: $\frac{1}{x} \frac{dy}{dx} = a - e^{-x}$ $\frac{d^2 y}{dx^2} = ax$ $\frac{dy}{dx} = (1 + x)(1 - y)$ $1200 \frac{dy}{dx} + y^2 = 400$ • The speed of a falling object is modelled by the following differential equation. $\frac{dv}{dt} = g - cv$ • Derive an algebraic expression for v in terms of t given that $v = 0$ when $t = 0$. • Find the solution to $\frac{dy}{dx} = x^2 + 2$ given that $y = 2$ when $x = 1$. • Find the solution to $\frac{d^2 y}{dx^2} = e^{-4x}$ given that $\frac{dy}{dx} = \frac{1}{2}$ and $y = 1$ when $x = 0$. • Given that the solution to a differential equation is $y = Ae^{-x} + Be^{-2x}$. calculate A and B when $y = 5$ and $\frac{dy}{dx} = -50$ when $x = 0$.

The Learner will: Learners must be taught:	
4. Understand applications of matrix and vector methods4.1 Learners will be taught matrix notation i.e.: • what is meant by a rectangular matrix • what is meant by a column vector and a row vector• Understand that a matrix is a rectangular matrix elements with <i>n</i> rows and <i>m</i> columns • Understand that a square matrix has the same nu rows as it has columns• the notation for an element in the <i>i</i> th row and <i>j</i> th column in a matrix is a_{ij} • Understand that a square matrix has the same nu rows as it has columns• the notation for an element in the <i>i</i> th row and <i>j</i> th column in a matrix is a_{ij} • Learners will need to be able to identify the value elements and calculate column vectors for example $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mathbf{C} = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}$ $A = \begin{bmatrix} 3 & -4 & 7 \\ -2 & 1 & 5 \\ 4 & 3 & -6 \end{bmatrix}$ I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ • the transpose of a matrix, \mathbf{A}^T (i) Identify the value of element a_{23} .	mber of s of le: econd

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	4.2 how to represent linear simultaneous equations with two unknowns in matrix notation	
	4.3 how to calculate the value of a 2 by 2 determinant	For example: $\begin{vmatrix} 4 & -2 \\ -3 & 5 \end{vmatrix}$
	4.4 how to perform addition, subtraction and multiplication of matrices and vectors with two rows and two columns	For example: (i) $\begin{bmatrix} 3 & -6 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -5 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$
	4.5 the concept of the matrix inverse A^{-1}	
	4.6 how to determine the inverse of a 2 by 2 matrix	For example: • If $\mathbf{A} = \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix}$ determine \mathbf{A}^{-1}
	4.7 that $A^{-1}.A = A .A^{-1} = I$	

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	4.8 how to solve linear simultaneous equations with two unknowns using matrix methods	For example: • In an electrical circuit voltages V_1 and V_2 satisfy the following linear simultaneous equations. $\frac{V_1}{4} - \frac{V_2}{6} = 10^{-3}$ $-\frac{V_1}{6} + \frac{V_2}{3} = -4 \times 10^{-3}$ Represent these equations in matrix notation and use matrix methods to find the values of V_1 and V_2
	4.9 how to represent position vectors and direction vectors in component form i.e. $\mathbf{z} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$	Learners must be able to perform calculations involving vectors for example: • Starting from position O defined by the position vector $2i + 3j$ a body is moved through three sequential steps defined by the direction vectors 5i - 2j -10i + 7j 6i - 2j Draw a diagram to represent these vectors and determine the final position of the body.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
	 4.10 how to perform vector operations of addition, dot product and cross product and calculate the magnitude of a vector a+b a-b ab (dot product) a×b (vector product) a+b 4.11 how to use vector notation and vector operations to solve problems involving spatial position, velocity and forces 	For example: • If vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and vector $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ Calculate $\mathbf{a} + \mathbf{b}$ $\mathbf{a} - \mathbf{b}$ $\mathbf{a} \cdot \mathbf{b}$ $ \mathbf{a} + \mathbf{b} $ • Forces represented by the vectors $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j}$ act on a point mass. Calculate the resultant force vector and its horizontal and vertical components. • Calculate the torque on a nut when a particular force vector is applied to a wrench of a given length.

Learning outcomes	Teaching content	Exemplification
The Learner will:	Learners must be taught:	
5. Be able to apply mathematical modelling	5.1 how to represent aspects of physical problems in terms of abstract mathematical formulae	Where laws of physics are required in the examination, relevant formulae will be provided.
skills	 5.2 how to manipulate given formulae to derive mathematical models and solve problems 5.3 how to formulate and solve mathematical models of practical problems using common laws of physics including Newton's laws of motion, Ohm's law, Kirchhoff's laws, Hooke's law, Newton's law of cooling and the principles of energy conservation 	Learners will be expected to solve a range of engineering problems for example: • Given $I = V/R$ where <i>I</i> is current, <i>V</i> is voltage and <i>R</i> is resistance, calculate the voltage across a 100 Ω resistor when a current of 0.25 A is flowing through it. • The total resistance R_s across <i>n</i> resistors connected in series is $R_s = R_1 + R_2 \dots R_n$ and the total resistance R_p across <i>n</i> resistor connected in parallel satisfies $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \dots \frac{1}{R_n}$
	context of the problem being solved	Calculate the total resistance of a circuit involving several resistors connected in different series and parallel combinations.
	5.5 the need to reflect on results in order to verify their feasibility and validity	 Calculate potential energy and kinetic energy of bodies and use the principle of energy conservation.
	5.6 the importance of recognising the implications of simplifying assumptions in mathematical models	 Calculate the speed and distance travelled by a body falling under the influence of gravity with and without aerodynamic drag. Calculate force vectors to maintain a suspended body in the state of equilibrium. Analyse problems involving the flow of liquid in pipes and tanks. Apply moments of forces to leavers, beams and other physical structures.

ASSESSMENT GUIDANCE

All Learning Outcomes are assessed through externally set written examination papers, worth a maximum of 80 marks and 2 hours in duration. Learners should study the design requirements, influences and user needs within the taught content in the context of a range of real engineered applications. Exam papers for this unit may use engineered products as the focus for some questions, however it is not a requirement of this unit for learners to have any detailed prior knowledge or understanding of particular products used. Questions will provide sufficient information to be used, applied and interpreted in relation to the taught content. During the external assessment, learners will be expected to demonstrate their understanding through questions that require the skills of analysis and evaluation in particular contexts.

LEARNING OUTCOME WEIGHTINGS

Each learning outcome in this unit has been given a percentage weighting. This reflects the size and demand of the content you need to cover and its contribution to the overall understanding of this unit. See table below:

LO1	10-20%
LO2	25-35%
LO3	20-30%
LO4	5-15%
LO5	15-25%

***SYNOPTIC ASSESSMENT AND LINKS BETWEEN UNITS**

Ten per cent of the marks in each examination for this unit will be allocated to synoptic application of knowledge. There'll be questions that draw on knowledge and understanding from Unit 1 Mathematics for engineering and/or Unit 2 Science for engineering that then has to be applied in the context of this unit.

To find out more ocr.org.uk/engineering or call our Customer Contact Centre on 02476 851509

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