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AS and A LEVEL

Topic Exploration Pack

COMPUTER SCIENCE

H046/H446
For first teaching in 2015

Theme: Boolean Algebra

Version 2

Contents

This Topic Exploration Pack should accompany the OCR resource 'Boolean Algebra' learner activities, which you can download from the OCR website.

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Boolean Algebra

Define Problems Using Boolean Logic

A good approach for teaching Boolean logic would be to start with the idea of logic gates and truth tables as these are concepts that can be easily covered and can therefore be used to build student confidence with the new material.

Activity 1

Worksheet 1 is given as an introduction to how one might define problems using Boolean algebra. Students could have a go at filling this worksheet in to get used to this style of logic.

Activity 2

You could get students to draw out truth tables for each logic gate, and then get students to combine two logic gates together, making the truth table for each. Worksheet 2 is given as a guide to Boolean algebra.

For example, the truth table for XOR, NAND and NOR is given below.

| A	B	$A \oplus B$	$A.B$	$\overline{A.B}$	$A+B$	$\overline{A+B}$
0	0	0	0	1	0	1
1	0	1	0	1	1	0
0	1	1	0	1	1	0
1	1	0	1	0	1	0

Manipulate Boolean expressions using Karnaugh maps to simplify the solution.

Manipulating Boolean expressions will require knowledge of the section on using rules to simplify statements in Boolean algebra.

In addition to manipulating and simplifying the Boolean equation and using truth tables, one other way of simplifying Boolean expressions is to use a method called Karnaugh map.

The first thing you need to do for a Karnaugh map is enter 1's for all cases where your Boolean equation is True.

This is what a simple Karnaugh map would look like for the equation $Q = \bar{A}.B + A.B$
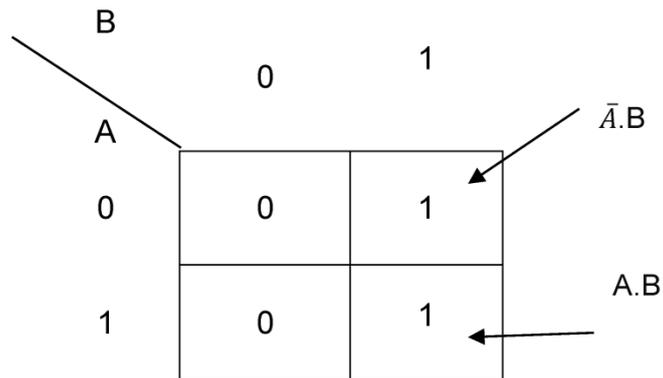
You then want to group the 1's in as few sets as possible, For example:
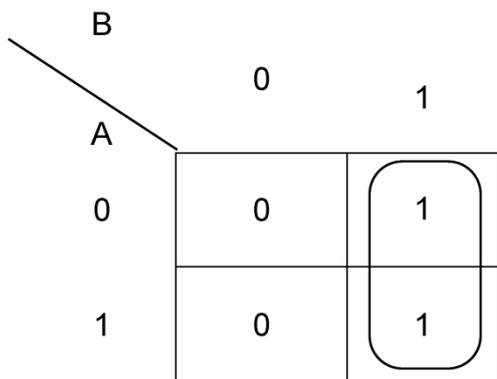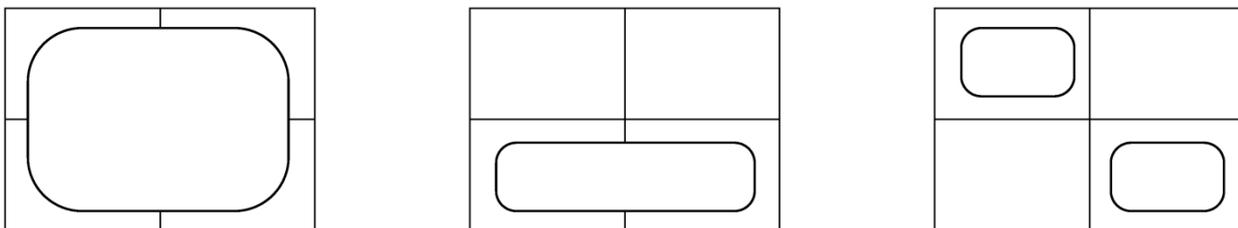
Therefore, the answer is Q = B

This guide found at the CAS resources section deals with what happens if we have three or four variables in our Karnaugh map:

http://community.computingatschool.org.uk/resources/567

Use the following rules to derive or simplify statements in Boolean algebra: De Morgan's Laws, distribution, association, commutation, double negation.

Manipulating Boolean expressions in a lot of ways is like manipulating algebraic equations. Most of the rules are the same, but some will take some getting used to. In the same way, as 4 + 2 can be replaced by 6, there are similar sorts of rules in Boolean algebra, for example A.A = A.

Once you have got to know the rules, the most tricky thing to develop is the ability to spot where different rules can be applied. It may help in some instances where different letters are being used to replace them with the ones that you are used to. The examples below use A, B and C, although you may be faced with a question that uses different letters, and some students can find this hard to grasp.

Tautology

A.A = A

A+A = A

$A+\bar{A} = 1$

$A.\bar{A} = 0$

1+A = 1

1.A = A

0+A = A

0.A = 0

Commutative

AB = BA

A+B = B+A

Associative

(A+B)+C = A+(B+C)

A.(B.C) = (A.B).C

Distributive

A.(B+C) = (A.B)+(A.C)

A+(B.C) = (A+B).(A+C)

A series of helpful worksheets and suggestions for this can be found at:

http://community.computingatschool.org.uk/resources/1788

Using Logic Gate Diagrams and Truth Tables.

Logic gates would be a good topic to start this part of the specification as they are the fundamental building blocks when moving onto Boolean algebra.

From knowing simple logic gates, we can then go on to understanding how to construct truth tables for these fundamental symbols. These are easy for students to grasp what is going on. Please see Worksheet 2 for a full run down.

You may choose to combine practice examples and questions with constructing the logic circuits on a simulator such as 'Logic gate simulator' by Steve Kollmansberger at.

http://www.kolls.net/gatesim/

The Logic Associated with D-Type Flip Flops, Half and Full Adders

After studying Boolean addition, students should be able to appreciate the purpose of the half and full adder circuits. The purpose of a half adder is to add together two 1-bit binary numbers and outputs the sum and a carry bit, whereas a full adder takes in 3 inputs. The full adder is simply two half adders combined with an OR gate.

Some nice animations and truth tables which illustrate what is going on in a half adder and a full adder circuit can be found at:

http://isweb.redwoods.edu/INSTRUCT/CalderwoodD/diglogic/half-add.htm

In general terms, a flip-flop is an electronic component that has two stable states.

A D-type flip flop is an electronic component that accepts two inputs. One is the input 'D' and the clock. There are two outputs, Q and $\bar{Q}$. $\bar{Q}$ is simply the inverse value of Q (as we already know from Boolean algebra!).

The input at D (let's say the current output at Q is 0 and D is 1) does not cause a change right away, but waits for a rising clock edge (see diagram below).
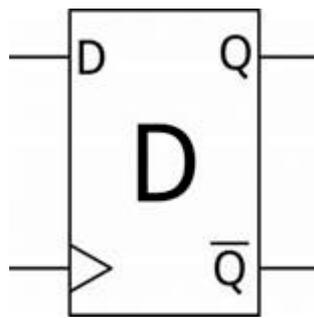
Q	D	Q(t+1)
0	0	0
0	1	1
1	0	0
1	1	1
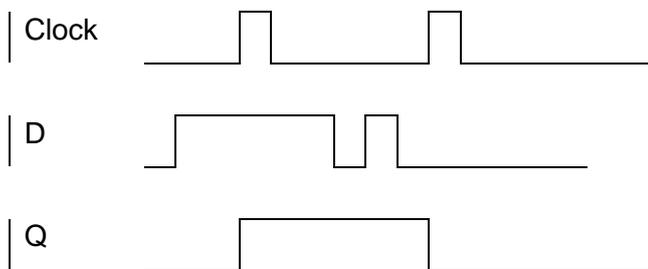
In this waveform diagram, there are a few things to notice that help us with understanding how a D-type flip-flop works.

See how D rises, but yet Q doesn't change until the 'rising edge' of the clock pulse.

Also notice how D rises and falls and then rises again. During this period Q doesn't change. That is because Q will only change on the rising clock edge.

At the second clock rising edge, we can see that D is now low, which forces Q to be low.

Teacher preparation

In terms of teacher preparation the best thing to do would be to firstly have a good grasp of the subject matter yourself and the specific methodologies, hints and tricks that fit your experience of what makes things easier to understand. A guide – Worksheet 1– has been provided for use by both the teacher and students to get used to the key concepts in Boolean algebra quickly.

A list of tasks that you may want to get familiar with in order are:

1. Understand what a logic gate is and why we use them

2. How to write down simple Boolean equations (see Worksheet 1)

3. How to construct truth tables for simple 1 or 2 gate logic setups (see Worksheets 2 and 3). At this point you may want to test these out using software, or if you are really adventurous, constructing the actual circuits.

4. Simple Boolean identities and their proof using truth tables (see Worksheet 2)

5. More complex Boolean laws (Commutation, Association, Distribution, Double negation and De Morgan's). Prove these using truth tables.

6. Be able to use a combination of these rules to simplify increasingly complex Boolean equations. Prove that these are correct using truth tables.

7. Be able to draw out logic circuit diagrams based on a Boolean equation and vice versa (see Worksheet 1 of the delivery guide).

8. Understand how Karnaugh maps can also be used to simplify Boolean logic. Again, you can check whether you are correct by manipulating the algebra, creating truth tables and checking via software.

9. Understand the concepts behind how half adders, full adders and flip-flops work.

You may choose to get some of the software installed in advance (or test the web-based versions to see if they work properly) and familiarise yourself with how these operate.

It is then a case of getting sufficient worksheets prepared with examples and enough variety to stretch the most able students. Of course, there are many examples available online too.

Student Sheets

Worksheet 1 – Defining problems

This worksheet provides a basis for relating Boolean algebra to everyday life. You will first need to cover the key terminology and symbols associated with this sort of algebra, however, as it is quite different from what students would be used to GCSE mathematics.

Worksheet 2 – Boolean algebra guide

The Boolean algebra guide aims to give a walkthrough of each law and proof using truth tables. Rather than students being told what a certain law is, they can prove it using the truth table and this will help them find the activity far more convincing.

Fully worked examples are given at the end of the guide that students can try to work out.

Worksheet 3 – Examples of using logic gates

This worksheet will get students thinking about how logic gates can be combined to give different outcomes. The activity leads on to creating truth tables for each series of gates.

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