This Checkpoint Task should be used in conjunction with the KS4–KS5 Transition Guide – Trigonometry.

Checkpoint Task – Trigonometry

What rule should I use?

Instructions and answers for teachers

These instructions should accompany the OCR resource ‘Trigonometry’ activity which supports OCR GCSE (9–1) Mathematics.

Activity 1

Look at the following diagram and the related equations in Part 1 and Part 2 below. State whether each one is true or false. If an equation is false, state a correct version or. Then answer Part 2.

Part 1

(a) \( \sin \theta = \frac{a}{c} \)
(b) \( \tan \theta = \frac{b}{c} \)
(c) \( \sin \theta = \frac{a}{b} \)
(d) \( \cos \theta = \frac{c}{b} \)
(e) \( \cos \theta = \cos A \)
(f) \( \cos A = \cos \theta \)

Part 2

Area of \( \triangle ABC \) = \( \frac{1}{2} \times bc \sin C \)

Associated materials:

‘Trigonometry’ Checkpoint Task activity sheet.
Aim:

To assess understanding of trigonometry formulae (SOHCAHTOA) and Pythagoras’ theorem in right-angled triangles, as well as the sine and cosine rules and the area of a triangle using \( \frac{1}{2}ab \sin C \), for any triangle.

These activities are appropriate for Higher tier students at KS4. Activity 1 can be adapted so that Parts 1 and 3 can be used with Foundation tier students at KS4, by making angle \( B \) a right-angle.

The mathematics covered in this activity:

Knowing trigonometry formulae for right-angled triangles; selecting the correct trigonometric ratio and substituting the appropriate values from the diagram.

Knowing and applying Pythagoras’ theorem to find both the hypotenuse and the shorter sides.

Recalling and using the sine rule and the cosine rule to find both sides and angles.

Recalling and using the formulae for the area of a triangle (with and without trigonometry).

Activity guidance:

The activities focus primarily on substituting the appropriate values into the formulae, rather than the actual calculation of lengths and angles.

Activity 1 is suitable for whole class or group work and can be used as revision or as an example. In Parts 1 and 2 students have to decide whether the equations are true or false. If they are false, the students are expected to write a correct version with the minimal amount of changes. The statement in Part 2 is false; an extra challenge is to get students to find all the possible ways to express the area of the triangle \( ABC \). In Part 3 they are challenged to write some further different equations.

This is the first activity so if it proves to be too difficult for some groups then replace letters with numbers or draw the three triangles separately to help them; alternatively, if you only want to test SOHCAHTOA, write angle \( B \) as a right-angle and then there is no need to use the sine rule or the cosine rule.
In Activity 2 students have to complete some equations in Parts 1 and 2 using the sine and cosine rules. In Part 3 they have to decide whether they can use SOHCAHTOA or Pythagoras’ theorem in the triangle. In Part 4 they are challenged to write further different versions of the sine and cosine rules.

Activity 3 sets similar problems to Activity 2, but this time the triangle is isosceles. It is possible that students will calculate the two base angles immediately, but the intention is for them to address the questions in the order they are set. This triangle can be solved and they may do so if they wish. This is fairly typical of triangles that are seen in circles with one vertex at the centre.

Suggested questions:
• What are the trigonometry formulae?
• What do the letters $o$, $a$ and $h$ represent?
• What type of triangle can these formulae be used in?
• What other formulae can be used in this type of triangle?
• When do you use Pythagoras’ theorem and when do you use trigonometry?
• When letters are used on diagrams to represent unknowns, what is the difference between an unknown angle and an unknown side?

The sine rule and the cosine rule can be used in any triangle, how are they written when you want to find a side?

• How are the sine rule and the cosine rule written when you want to find an angle?
• Why are the rules written differently to find sides or angles?
• What formulae can be used to find the area of a triangle?
• How do you decide which area formula to use?
Answers

Activity 1
Look at the following diagram and the related equations in Part 1 and Part 2 below. State whether each one is true or false. If an equation is false, write a correct version of it. Then answer Part 3.

Part 1
(a) \( \sin C = \frac{h}{y} \)  
    False \( \sin C = \frac{h}{a} \)

(b) \( \tan A = \frac{h}{x} \)  
    True

(c) \( a^2 = h^2 + y^2 \)  
    True

(d) \( \frac{\sin A}{a} = \frac{\sin B}{x + y} \)  
    True

(e) \( \cos A = \frac{b^2 + c^2 - a^2}{2bc} \)  
    True

(f) \( h^2 = c^2 + x^2 \)  
    False \( h^2 = c^2 - x^2 \)

(g) \( (x + y)^2 = a^2 + c^2 - 2ac \cos C \)  
    False \( (x + y)^2 = a^2 + c^2 - 2ac \cos B \)

(h) \( \frac{c}{\sin C} = \frac{h}{\sin A} \)  
    False \( \frac{c}{\sin C} = \frac{a}{\sin A} \)
Part 2

Area of triangle $ABC = \frac{1}{2}cy \sin A$

**False**

A number of alternatives:

Area of triangle $ABC = \frac{1}{2}c(x + y)\sin A$

Area of triangle $ABC = \frac{1}{2}ab \sin C$

Area of triangle $ABC = \frac{1}{2}ac \sin B$

Area of triangle $ABC = \frac{1}{2}a(x + y)\sin C$

Area of triangle $ABC = \frac{1}{2}hx + \frac{1}{2}hy$

Area of triangle $ABC = \frac{1}{2}h(x + y)$

Area of triangle $ABC = \frac{1}{2}bh$

Part 3

There are three triangles in the diagram on the previous page, $ABC$, $ABD$ and $BCD$. For each triangle write a different trigonometric equation. How many different equations are there? Can you write one that your partner has not written?

Numerous possible answers. Students to check each others’ work.
Activity 2
Use the diagram below to answer the following parts.

![Diagram of a triangle with sides labeled 15, 27, and an angle of 42 degrees.]

Part 1
Use the sine rule to complete these equations.

(a) \( \frac{\sin C}{a} = \frac{\sin 42}{15} \)
(b) \( \frac{27}{\sin 42} = \frac{a}{\sin B} \)

Part 2
Use the cosine rule to complete these equations.

(a) \( 27^2 = 15^2 + a^2 - 2 \times 27 \times 15 \times \cos 42 \)
(b) \( \cos C = \frac{a^2}{54a} \)

Part 3
(a) Can you use SOHCAHTOA on this triangle as it stands? Explain your answer.
   No, because SOHCAHTOA refers to the ratio of sides in right-angled triangles.
(b) Can you use Pythagoras’ theorem on this triangle as it stands? Explain your answer.
   No, because Pythagoras’ theorem refers to the sides of a right-angled triangle.

Part 4
(a) Can you write a different version of the cosine rule about this triangle that will only involve one unknown?
   \( a^2 = 27^2 + 15^2 - 2 \times 27 \times 15 \times \cos 42 \)
(b) Can you write a different version of the sine rule about this triangle that will only involve one unknown? Explain your answer.
   No, because you need information about one angle and its corresponding side.
Activity 3
Use the diagram below to answer the following parts.

Part 1
(a) Can you use SOHCAHTOA on this triangle as it stands? Explain your answer.
   No, because SOHCAHTOA refers to the ratio of sides in right-angled triangles.

(b) Can you use Pythagoras' theorem on this triangle as it stands? Explain your answer.
   No, because Pythagoras' theorem refers to the sides of a right-angled triangle.

If your answer is “No” to either (a) or (b), describe what you have to do to this triangle to be able to use that technique.

   Draw the line of symmetry through the isosceles triangle to form the perpendicular bisector and create two congruent right-angled triangles.

Part 2
Write one equation using the sine rule and one using the cosine rule that can be used to find $x$.

Using sine rule \[ \frac{x}{\sin 73^\circ} = \frac{27}{\sin 34^\circ} \]

Using cosine rule \[ x^2 = x^2 + 27^2 - 54x \cos 73^\circ \]
Part 3
Write an expression for the area of this triangle.

\[ \text{Area} = \frac{1}{2} x^2 \sin 34 \]

\[ \text{Area} = 13.5 \times \sin 73 \]

Part 4
Can you work out the values of any of the unknown angles or sides?

\[ \text{Angle } A = \text{Angle } B = 73^\circ \]

\[ x = 46.2 \text{ (1dp)} \]